

The Fuzzy Finite Element Stress Analysis of Adhesive-Bonded Single Lap Joints

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Abstract

An adhesive-bonded single lap joint is analyzed using a new fuzzy finite element model. In the model, Young's moduli and Poisson's ratios of the joint materials are taken as fuzzy numbers in order to take the uncertainty of the material properties into account. The fuzzy numbers are modeled using linear triangular membership functions. At a selected material point in the adhesive layer, the possibility distributions for the displacements and shear stresses are depicted by graphics.

Key words: Fuzzy finite element method, Adhesive joints, Stress analysis

Introduction

Traditional approaches to stress analysis assume that all the problem data are known with mathematical precision. In practice, however, most real problems are imprecise in nature and their mathematical models can be best represented with some level of imprecision. The relations and statements used for the description of real problems are in general imprecise because of inherent fuzziness in the problems. There are several ways by which imprecise parameters can be represented and manipulated in engineering calculations. The simplest approach is to choose single values for each parameter and record a single value output using the system governing equations. The process is repeated in order to explore a given space. This method is very simple but can be extremely expensive from a computational point of view.

Interval analysis is another method for carrying out computations with imprecise parameters (Moore, 1979). In this case, an interval is used to represent a problem variable. The results are also represented by 2 numbers at the end points of the

interval. The result characterized in this manner indicates the range of possible values for outputs but provides no information on the performance of the result within the interval.

The theory of fuzzy sets and its logic offer a new approach in solving engineering problems where crisp information and engineering knowledge can be integrated to arrive at an approximate reasoning on the problem. Several applications of the fuzzy set theory in numerical analysis can be found in Aydemir *et al.* (2002), Chao and Ayyub (1996), Muhanna and Mullen (1999) and Valliappan and Pham (1993). Several probabilistic methods are also used in finite element analysis and a comparison of the methods is given in the work of De Lima and Ebecken (2000).

Before carrying out a finite element stress analysis some properties of the material must be obtained. For a linear elastic finite element stress analysis the properties are Young's modulus and Poisson's ratio of the materials. These values are obtained via several experiments, and in all test results there is a degree of scatter present due to variations in the

specimen preparation, testing errors and certainly not least to variability in the materials tested. With adhesives the scatter is proportionately greater than that with metals. With metals the coefficient of variation, i.e. standard deviation divided by the sample mean, is often less than 5%; with adhesives it is usually 10% or more (see the work of Benson (1967)).

In this paper, a fuzzy finite element stress analysis of an adhesive-bonded single lap joint is carried out. For the analysis, a new fuzzy strain-stress matrix is defined. The mechanical properties, i.e. Young's moduli and Poisson's ratios, of the joint materials are modeled as fuzzy numbers.

Fuzzy Modelling of the Mechanical Properties

In a classical finite element stress analysis the mechanical properties are taken as crisp values. Since the input data are imprecise, no matter what techniques are used the solution will not be reliable. In fuzzy finite element stress analysis, the mechanical properties are considered fuzzy parameters in order to take the uncertainty into account. The fuzzy numbers are characterized by their membership functions. In this paper, linear triangular membership functions are used for the fuzzy numbers. Although brief information will be given below, more information on the membership functions can be found in the work of Valliappan and Pham (1995).

Before constructing a membership function 2 extreme values L' , which is the minimum possible value of the parameter, and H' , which is the maximum possible value of the parameter, are defined. The extreme values are calculated using the definition below:

$$L' = \begin{cases} P - 2(P - L) & \text{if } P \geq 2(P - L) \\ 0 & \text{if } P \leq 2(P - L) \end{cases} \quad (1)$$

$$H' = P + 2(H - P) \quad (2)$$

where L , P and H are expert estimates of the mechanical properties for low, possible and high values, respectively. A linear triangular membership function $\mu(r)$ can now be constructed using the piecewise function defined below:

$$\mu(r) = \begin{cases} 0 & \text{if } r \leq L' \\ \frac{r - L'}{P - L'} & \text{if } L' \leq r \leq P \\ \frac{H' - r}{H' - P} & \text{if } P \leq r \leq H' \\ 0 & \text{if } r \geq H' \end{cases} \quad (3)$$

where r is a value.

On the basis of the definitions given above, the mechanical properties of the joint can be represented in an interval of confidence as

$$E = [E_L \quad E_R] \quad (4)$$

$$\nu = [\nu_L \quad \nu_R] \quad (5)$$

where the subscripts L and R stand for the left and right of the relevant fuzzy number, respectively. The given type of fuzzy numbers lead to the fuzzy arithmetic computations at each-level cut set. At an α -level, the intervals given above can be rewritten as

$$E^\alpha = [E_L^\alpha \quad E_R^\alpha] \quad (6)$$

$$\nu^\alpha = [\nu_L^\alpha \quad \nu_R^\alpha] \quad (7)$$

In the following, a fuzzy finite element formulation will be given based on the definition above.

Fuzzy Finite Element Formulation

In the finite element analysis, the load-displacement relation is written as

$$\mathbf{K}\mathbf{Q} = \mathbf{F} \quad (8)$$

where \mathbf{K} is the stiffness matrix, \mathbf{Q} is the nodal displacement vector and \mathbf{F} is the nodal force vector. The stiffness matrix is defined by

$$\mathbf{K} = \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV \quad (9)$$

where \mathbf{B} is the strain-displacement matrix, \mathbf{D} is the elasticity matrix and V is the element volume. If the finite element displacement method is used, the displacements are first calculated from Eq. (8) as

$$\mathbf{Q} = \mathbf{K}^{-1} \mathbf{F} \quad (10)$$

in which the inverse of the elasticity matrix \mathbf{D}^{-1} will be involved in the fuzzy mathematical modeling. The inverse of the elasticity matrix is the elastic compliance matrix \mathbf{C} , i.e. $\mathbf{C} = \mathbf{D}^{-1}$.

The matrix \mathbf{C} is material dependent and for a homogeneous material it includes 36 elastic constants. Strain energy considerations can be used to show that for fully anisotropic crystalline materials the number of independent material constants can be as

large as 21 (Ugural and Fenster, 1995). For a homogeneous isotropic material, the constants must be identical in all direction at any point. It is observed later that if the material is isotropic the number of essential elastic constants reduces to 2, namely Young's modulus and Poisson's ratio. In the 2-dimensional case, \mathbf{C} can be written as below

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \quad (11)$$

where C 's are the material dependent constants. Since the strain energy is always a positive value; the terms of \mathbf{C} are not arbitrary. For example, the terms on the principal diagonal must always be positive, for more information see the work by Alers and Neighbours (1957).

For an isotropic material \mathbf{C} can be written in terms of the mechanical properties. Since the mechanical properties are considered fuzzy parameters, the elastic compliance matrix needs to be fuzzified. A 3-dimensional version of the fuzzy elastic compliance matrices was given by Aydemir (2001). Here, this matrix will be given for the plane-strain case. For an isotropic material, in the case of the plane-strain, the fuzzy elastic compliance matrices are defined as

$$\mathbf{C}_{L,R}^{\alpha} = \begin{bmatrix} \frac{1-(\nu_{R,L}^{\alpha})^2}{E_{R,L}^{\alpha}} & -\frac{\nu_{R,L}^{\alpha}(1+\nu_{R,L}^{\alpha})}{E_{L,R}^{\alpha}} & 0 \\ -\frac{\nu_{R,L}^{\alpha}(1+\nu_{R,L}^{\alpha})}{E_{L,R}^{\alpha}} & \frac{1-(\nu_{R,L}^{\alpha})^2}{E_{R,L}^{\alpha}} & 0 \\ 0 & 0 & \frac{2(1+\nu_{L,R}^{\alpha})}{E_{R,L}^{\alpha}} \end{bmatrix} \quad (12)$$

In a plane-strain fuzzy finite element calculation, if \mathbf{C}_L^{α} is used the lower-bound displacements (\mathbf{Q}_L^{α}) will be calculated. Similarly, if \mathbf{C}_R^{α} is used the upper-bound displacements (\mathbf{Q}_U^{α}) will be calculated.

In the case of stress-deformation problems, the nodal displacements are the first results and the stresses \mathbf{S} are computed from the displacements. Then the fuzzy stress vectors at the same α -level will be

$$\mathbf{S}_{Q_L}^{\alpha} = \mathbf{D}_{Q_L}^{\alpha} \mathbf{B} \mathbf{Q}_L^{\alpha} \quad (13)$$

$$\mathbf{S}_{Q_U}^{\alpha} = \mathbf{D}_{Q_U}^{\alpha} \mathbf{B} \mathbf{Q}_U^{\alpha} \quad (14)$$

It should be noted that the vector of fuzzy secondary quantities (stresses) calculated from Eq. (13) may not always be smaller than those calculated from Eq. (14). Their magnitudes depend on the values of fuzzy material parameters, and the vector of the related fuzzy primary quantities (displacements).

A Case Study: Adhesive-Bonded Single Lap Joint

Although there are many types of adhesive joints the single lap joint is the most famous and most commonly studied adhesive-bonded joint. Its stress distribution is open to some doubt and certainly a complex combination of stresses. The pioneering work dates back to the early 1940s when Volkarsen (1938) published the first known stress analysis. Over the past 60 years there have been numerous studies conducted and information collected on the stress analysis of adhesively-bonded single lap joints. In most cases, analytical expressions are developed that attempt to better predict the stress state within the joint these and are then compared to past research or finite element results. However, with finite element analysis, solutions can be obtained that are totally intractable by classical analytical methods; see the work of Lang and Mallick (1998).

In the following, the defined fuzzy finite element model will be used to carry out the stress analysis of an adhesive-bonded single lap joint.

Joint configuration

The joint dimensions and applied loads are given in Figure 1. In the joint, the adhesive thickness, the overlap length and the length of each identical adherend are 0.15 mm, 13 mm and 51 mm, respectively. A horizontal tension stress with the magnitude of 1 MPa is applied to the non-overlapped ends of each adherend.

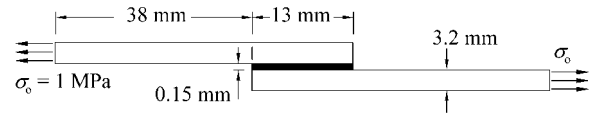


Figure 1. Joint dimensions and applied loads.

Material properties

It is often possible to obtain expert knowledge about the values of the mechanical properties in the form of low, probable and high. It is already said that based on this kind of subjective information the membership functions of the mechanical properties can be constructed. Although the construction of membership functions is still a big issue and a debatable

one in the fuzzy set theory, in this paper we use linear triangular membership functions. Here, it should be noted that the shape of the membership function is subjective and depends on its application. The choice of the shape of the function will depend on the expert's opinion, since it cannot be inferred that any type of possible shape is more suitable than the other.

In the joint, the adherends are identical and made from aluminum. Assume that low (L), possible (P) and high (H) values for the elastic material properties of the joint are given as below:

for the adherends,

$$\begin{Bmatrix} E_L \\ E_P \\ E_H \end{Bmatrix} = \begin{Bmatrix} 68 \\ 70 \\ 72 \end{Bmatrix} \text{ GPa}$$

$$\begin{Bmatrix} \nu_L \\ \nu_P \\ \nu_H \end{Bmatrix} = \begin{Bmatrix} 0.29 \\ 0.30 \\ 0.31 \end{Bmatrix}$$

for the adhesive,

$$\begin{Bmatrix} E_L \\ E_P \\ E_H \end{Bmatrix} = \begin{Bmatrix} 2.80 \\ 2.87 \\ 2.94 \end{Bmatrix} \text{ GPa}$$

$$\begin{Bmatrix} \nu_L \\ \nu_P \\ \nu_H \end{Bmatrix} = \begin{Bmatrix} 0.37 \\ 0.38 \\ 0.39 \end{Bmatrix}$$

Now, using the definitions given by Eqs.(1-3), the linear triangular membership functions of the mechanical properties can be constructed. They are given in Figures 2-4.

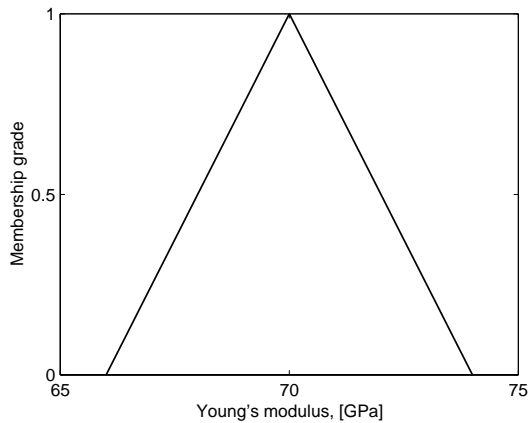


Figure 2. The membership function for Young's modulus of the adherend.

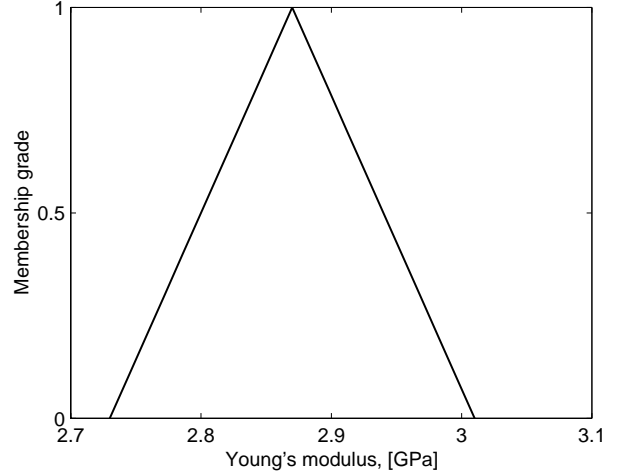


Figure 3. The membership function for Young's modulus of the adhesive.

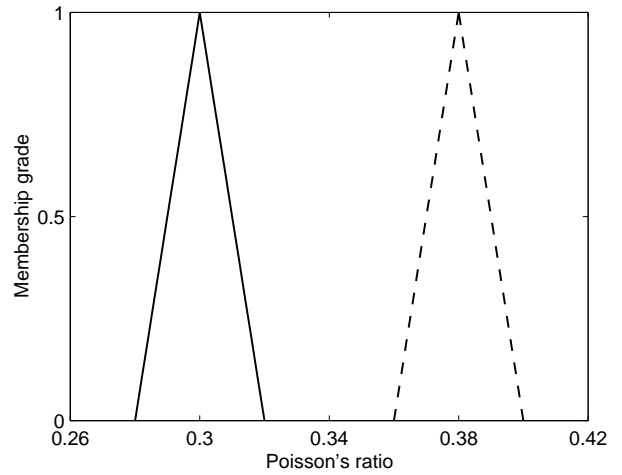


Figure 4. The membership functions for Poisson's ratios. The solid line is for the adherend material and the dotted line is for the adhesive material.

The finite element model

The boundary conditions and the coordinate system are shown in Figure 5. The coordinate system is placed in the center of the adhesive layer. Point A is fixed and point B is free to move in the x -direction. In this figure, (x, y) coordinates of points A and B are also given.

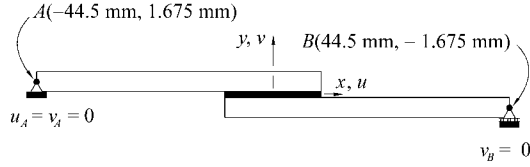


Figure 5. Boundary conditions and coordinate system.

For the solution, the plane strain assumptions are used. The finite element mesh and its details are shown in Figure 6. In the mesh, quadrilateral 8-node isoparametric finite elements are used. In a finite element stress analysis of single lap joints, it is essential to use a suitable mesh, which will be able to simulate the real happenings in the material interfaces as much as possible. Since the mechanical properties of the joint materials are highly different from each other, an intensive mesh discretization is prepared through the overlap length, especially in the adhesive layer.

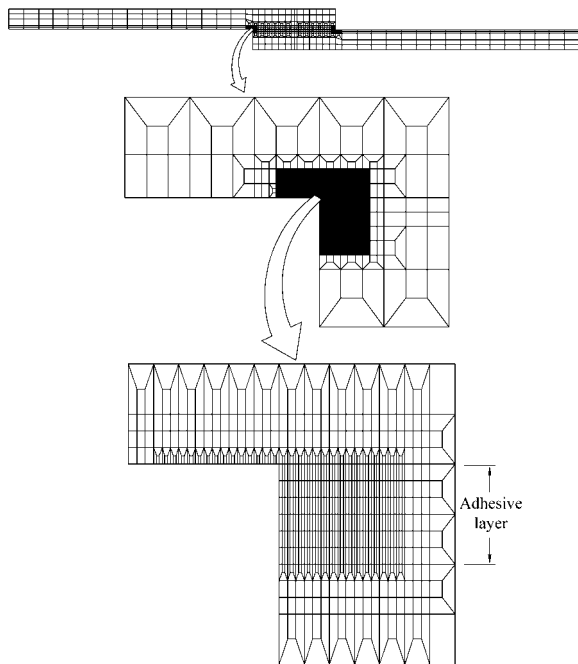


Figure 6. Details of the finite element mesh.

Results

Three different cases are considered: *i*) only E is fuzzy, *ii*) only ν is fuzzy, and *iii*) both E and ν are

fuzzy. The displacements and the stresses are calculated at each α -level. To show the results, the point where von Mises equivalent stress is maximum in the centerline of the adhesive layer is selected. The coordinates of the selected point is (-6.42 mm, 0 mm). It should be noted that any result obtained from fuzzy finite element stress analysis will have an interval; but, in the following, only the intervals for the vertical displacements and shear stresses are shown.

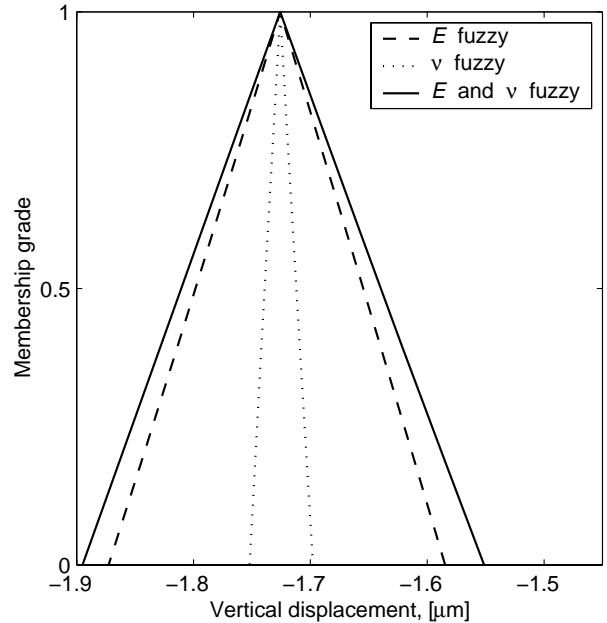


Figure 7. Possibility distributions of vertical displacements.

The possibility distributions for the vertical displacement are shown in Figure 7. It is observed that even small changes in the value of E will contribute to large changes in the value of the displacements. The distributions of the displacements are almost symmetric around the possible values. The largest possible distribution range is obtained for the case where both E and ν are fuzzy, and the smallest possible distribution range is obtained for the case where only ν is fuzzy.

The possibility distributions for shear stresses are shown in Figure 8. The stress distributions are not symmetric around their possible values. It can be seen that even small changes in the value of ν will contribute to large changes in the shear stress values. The largest possible distribution ranges are obtained for the case where both E and ν are fuzzy. For the shear stress, the smallest possible distribution range is obtained for the case where only E is fuzzy.

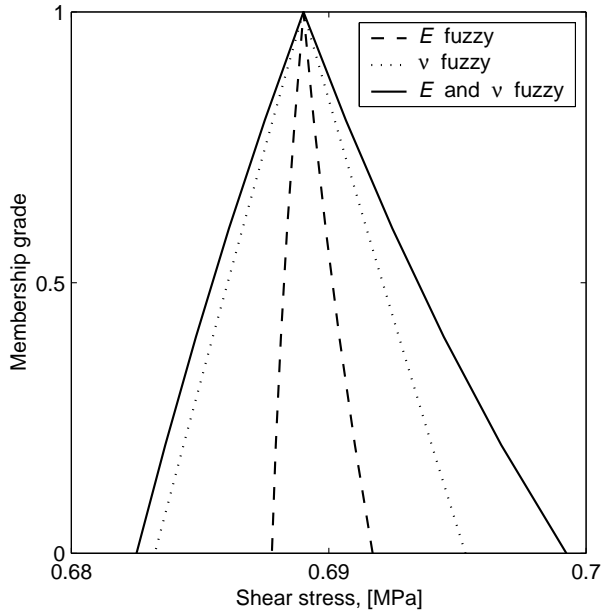


Figure 8. Possibility distributions of shear stresses.

Conclusions

The fuzzy finite element method proposed in this paper is a new technique for solving problems subjected to fuzzy parameters. It is important to note that the resulting elasticity matrices are different from those given in the literature by Valliappan and Pham (1993).

Finite element solutions are presented in terms of lower and upper bounds at different membership

grades. To be aware of the possible interval of the results is undoubtedly very important for an engineer, and the fuzzy set theory makes this possible within a reasonably shorter calculation time. To adopt the fuzzy finite element procedures to an existing finite element code is also quite simple.

This is the first time that the fuzzy set theory has been used for the finite element plain strain analysis of an adhesive-bonded single lap joint. The method has vast potential in the finite element stress analysis of adhesive-bonded joints. Furthermore, the method shown here is not limited to this specific application (i.e. adhesive joint). It can be applied to any kind of linear-elastic plain strain problem.

Furthermore, it is well known that the accuracy of finite element analysis results is highly affected by the reliability of the mesh discretization. Although, in the case study here, a suitable mesh is used, the results are still mesh dependent. Several adaptive mesh refinement procedures have been developed to improve the reliability of the finite element results. Among them, a procedure suitable for use in a fuzzy finite element stress analysis was proposed by Valliappan and Pham (1993). Therefore, the results might be further elaborated to take the mesh dependency into account.

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