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The Stochastic Modeling of GPS Observations

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Abstract

In global positioning system (GPS) applications several error sources affect the observations. Tropospheric delay that occurs during the propagation of the wave through the troposphere and multipath taking place as a result of signal reflection are the most important error sources. Both errors increase as the satellite elevation cut-off angle decreases. Thus, in practice, observations over 15° -20° are used and therefore those distorted by the multipath and tropospheric delay effects are not taken into account. In this case, the accuracy required for many engineering applications is easily achieved. However, for high precision applications this accuracy may not be adequate. In particular, the accuracy of a height component obtained by GPS is quite low for high precision geodetic or non-geodetic applications. One way to overcome this problem is to introduce of new stochastic models that enable us to process low elevated GPS observations too. In this study, the applications of 2 stochastic models are presented. The results show that these models improve the coordinate solutions.

Key words: GPS, Stochastic model, Sigma-ε, Cos^2(z).

Introduction

There have been enormous developments in the global positioning system (GPS) and its applications since its installation as a navigational system. GPS is actually a positioning technique initially developed for military requirements. However, it later started to be used by civilian users also for high precision positioning applications as well as in activities such as trekking or mountaineering. Today GPS is widely used in many fields of civilian life.

In GPS applications several error sources affect the observations. These error sources can be given as satellite and receiver clock errors, satellite orbit errors, ionospheric delay, tropospheric delay, multipath, receiver noise and geometric dilution of precision (GDOP).

Satellite-receiver clock errors can be minimized by the use of appropriate processing techniques. Because satellite orbit information has been more precisely obtainable by means of several permanent stations in recent years, satellite orbit errors have become negligible. The reduction in receiver errors due to technological advances and improvements in the satellite constellation with the operation of new satellites have decreased the effects of these error sources, especially in local studies. Ionospheric delay is also almost completely eliminated through dual frequency observations. Therefore, the most important error sources in GPS measurements are the multipath effect, taking place when the electromagnetic wave reflects from the reflecting surfaces around the receiver, and the delay effect that occurs during the propagation of the wave through the troposphere because of the refraction of light. Both errors increase as the satellite elevation cut-off angle, i.e. the angle between the satellite and the receiver on the horizontal plane, decreases. Because of this, in practice, observations over 15°-20° are used. Therefore, observations distorted by the multipath and tropospheric delay effects are not taken into account. In this case, the accuracy required for many engineering applications is easily achieved. However, for high accuracy applications this accuracy may not be adequate.

A way of improving the accuracy of point positioning is to include more observations in the processing. This may be possible by including low elevated observations in the processing. In this case, the satellite geometry will also improve. Furthermore, studies show that the inclusion of low elevated observations in the processing also improves the accuracy of the height component, the accuracy of which is about 3 times worse than the accuracy of the horizontal components because of the multipath and tropospheric delay (Herring, 1992).

In recent years many studies have focused on the subject of processing low elevated observations. One of these studies involved the development of mapping functions produced for modeling the tropospheric delay with respect to meteorological data or mathematical methods. The computed delay values are applied as corrections to the observations. The problem here is that the nature of the troposphere layer of the atmosphere is not known properly. In recent years, thanks to developments in meteorology parallel to technological progress and the increasing amount of atmospheric data and information, important progress in this field has taken place; and, through the use of regional or global meteorological data, many mapping functions have been developed. These functions, or in other words atmospheric models, yield better results then do conventional atmospheric models. The most common and widely applied tropospheric mapping functions are those of Saastamoinen (1973) and Hopfield (1969). Most commercial GPS processing software uses these mapping functions. The tropospheric models and developed mapping functions are especially important for the observations of medium-and long-range GPS baselines. For short baselines conventional atmospheric models or any of the new models give identical results. Some recent studies have focused on comparing newly developed mapping functions. The model comparisons show that the mapping functions developed by Lanyi (1984), Ifadis (1986), Herring (1992) and Niell (1996) give satisfactory results for geodetic applications. This conclusion was also mirrored in the International Earth Rotation Service (IERS) conventions on tropospheric models for radio techniques. It states that if information is available on the vertical temperature distribution in the atmosphere, the Lanyi mapping function should be used. Otherwise, one of the mapping functions derived by Ifadis, Herring or Niell should be used (McCarthy, 1996). Janes *et al.* (1991), MacMillan and Ma (1994), Mendes and Langley (1994), Niell (1996) and Bisnath *et al.* (1997) give comprehensive analyses of different mapping functions.

Another subject that concerns the scientific community is, as mentioned before, the multipath effect that takes place due to the reflection of the electromagnetic wave from the objects on the Earth, causing it to have a longer path to reach the receiver. So far the factors causing the multipath phenomenon have not been thoroughly determined; but studies on this subject point out that the multipath has a similar effect on observations on sequential days. This fact has oriented those people who have concentrated on this topic to work on the modeling of the multipath effect. Examples are Ge *et al.* (2000a, 2000b) and Han and Rizos (2000), who applied some filtering algorithms for multipath mitigation to be used for continuous GPS measurements, and Ray (2000), who studied the mitigation of GPS code and carrier phase multipath effects using a multi-antenna system. Although the accuracy achieved may not be adequate for some engineering applications, important progress has been made in these studies. In particular, when the permanent stations, established for the purpose of real-time monitoring of displacements and collecting data continuously, are considered the modeling of multipath effect and its elimination through corrections have great importance.

The third subject is the development of new stochastic or weighting models, which enable us to process low elevated GPS observations too. Conventionally GPS observations are equally weighted in the adjustment procedure. With suitably chosen weighting algorithms the low elevated data can also be used in least squares adjustment. These weighting algorithms are based on the satellite elevation cut-off angle or on some signal quality measures. In recent years many researchers have also drawn attention to the importance of the stochastic modeling of GPS data (Barnes and Cross, 1998; Hartinger and Brunner, 1998a; Hartinger and Brunner, 1998b; Wang *et al.*, 1998; Lau and Mok, 1999; Tiberius and Kenselaar, 2000). In this study, 2 stochastic models or, in other words, weighting algorithms and their applications in GPS processing, are discussed. The aim of the study is to investigate the effects of stochastic models on the precision of GPS coordinates. The day-to-day repeatability of the coordinate solutions has also been investigated.

GPS Data Processing

For all high precision geodetic applications, GPS carrier phase observations are used in GPS data processing algorithms. These algorithms are usually based on least squares estimation. It is well known that there are 2 aspects to optimal GPS processing, the definitions of the functional model and the corresponding stochastic model.

The functional model is formed through the relationship between observations (i.e. the code ranges and the carrier phases) and the unknown parameters and possibly atmospheric delays, as well as the other parameters like clock errors and carrier phase ambiguities. Carrier phase measurements used for precise positioning are expressed as follows (Leick, 1995; Teunissen and Kleusberg, 1998):

$$\Phi = \rho + d\rho + c(d_t - d_T) + \lambda N - d_{ion_1} + d_{trop} + d_e + \varepsilon_\Phi + \varepsilon_{M_\phi} \quad (1)$$

where Φ is the measured carrier phase (m), ρ is the geometric range between the satellite and receiver antennas (m), $d\rho$ is the orbital error (m), c is the speed of light in a vacuum (m/s), d_t is the satellite clock error (s), d_T is the receiver clock error (s), λ is the carrier wavelength (m), N is the integer cycle ambiguity (cycles), d_{ion} is the ionospheric delay error (m), d_{trop} is the tropospheric delay error (m), d_e is the satellite and receiver equipment delay (m), ε_Φ is the receiver carrier noise and ε_{M_Φ} is the carrier phase multipath error (m).

In geodetic applications through the GPS, the differencing, which is described as a way to eliminate or reduce most of the errors, is carried out. In this approach, the GPS observables are first differenced between different satellites. After that the differenced observables are differenced between the receivers. This procedure is called double-differencing. Most GPS processing software evaluates double-differenced carrier phase observations. From the carrier phase observations given in Eq. (1), double-differenced (DD) carrier phase observations are formed as follows:

$$\begin{aligned}\Delta\nabla\Phi_{ab}^{ij} &= \Delta\Phi_{ab}^{j} - \Delta\Phi_{ab}^{i} \\ &= \Delta\nabla\rho_{ab}^{ij} + \Delta\nabla d_{\rho_{ab}}^{ij} + \lambda\nabla\Delta N_{ab}^{ij} - \Delta\nabla d_{ion_{ab}}^{ij} + \\ &\quad \Delta\nabla d_{trop_{ab}}^{ij} + \Delta\nabla\varepsilon_{\Phi_{ab}}^{ij} + \Delta\nabla\varepsilon_{M\Phi_{ab}}^{ij}\end{aligned} \quad (2)$$

In the above equations superscript "i" refers to the satellite i and "a" and "b" refer to the receivers a and b respectively. The term Δ represents a *between-receiver single difference* and ∇ a *between-satellite single difference*. Through differencing many errors almost vanish. Using this functional model the least squares estimation is followed for computations. In Figure 1, the relationship between the DD residuals and the elevation cut-off angles is shown. Vertical components in the figure are both the DD residuals in millimeters and the elevation cut-off angles in degrees with respect to time. The lower the elevation cut-off angles, the higher the DD residuals.

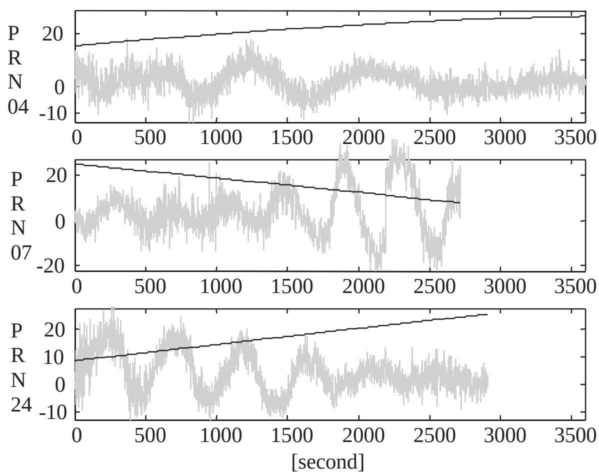

Figure 1. DD residuals (gray, in mm) and elevation cut-off angles (black, in degrees) of PRN04, PRN07 and PRN24.

Since random noise affects both GPS pseudoranges and carrier phase observations, the random behavior of the noise effect should be taken into account in order to get the desired information from the contaminated measurements (Tiberius *et al.*, 1999). Therefore a stochastic model describing the noise characteristics should be introduced to perform the processing under such principles. In order to realize these principles proper data processing models and suitable weighting algorithms should be specified. The main objective for the use of suitable weighting schemes is to minimize the impact of tropospheric delay and the site-specific multipath or signal diffraction effects on GPS carrier phase measurements.

Stochastic Properties of GPS Observations

In comparison with data processing models, less effort has been made for stochastic models. Conventionally, in most geodetic applications the GPS carrier phase data are equally weighted. However, the need for higher accuracy in high precision geodetic applications encouraged the scientific community to concentrate on some weighting algorithms for GPS data.

The observation weights, which are collected in a weight matrix, allow one to specify by how much individual observations should contribute to the overall solution. For instance, it is sensible to give lower weights to the noisier observations and higher weights to the less noisy observations (Teunissen *et al.*, 1998). The choice of weights is optimal when the weight matrix equals the inverse of the variance-covariance matrix of the observations with the variance of unit weight being equal to 1. In that case the balance between the relative weights is such that the best possible precision is obtained in the computed solution. In order to specify the variance covariance matrix adequately, one needs to know the stochastic properties of the actual data (Tiberius, 1999). In recent years, some weighting algorithms reflecting the noise characteristics of observations have been developed. Most of the developed models are based either on signal quality measures or elevation cut-off angles.

Signal Quality Measures

The power of a GPS signal is the basic measure of its quality. The power levels of GPS signals are usually specified in terms of decibels with respect to 1 watt of power (dBw). The minimum received power levels of the GPS signals for the users on Earth are –160 dBw and –166 dBw for L_1 and L_2, respectively (NAVSTAR GPS, 1995). The most common signal quality measures that can be used for weighting are the signal-to-noise ratio (SNR) and the carrier-to-noise power density ratio (C/N_0).

The SNR is the ratio of the amplitude of the desired signal to the amplitude of noise signals at a given point in time. It is expressed as 20 times the logarithm of the amplitude ratio, or 10 times the logarithm of the power ratio. The SNR is generally used as a measure of the noise level that can contaminate a GPS observation. This value can be compared with the power of a GPS signal. The ratio of the power of a received signal, S, and the noise power, Ns, can be considered a measure of strength. The SNR, or as in some references S/R, is expressed as follows:

$$\frac{S}{Ns} = \frac{P_S}{P_N} \tag{3}$$

where P_S is the signal power in watts and P_N is the noise power in watts. It is clear that the larger the SNR value, the stronger the signal.

The C/N_0 in GPS receivers is the ratio of the power level of the signal carrier to that of the noise in a 1 Hz bandwidth. Nominal GPS receiver C/N_0 values often are in the 40 to 50-dB-Hz range. Compared to the SNR the C/N_0 describes the ratio of the power level of the signal carrier to the noise level in an influence on the C/N_0 value. It is a key parameter in analyzing GPS receiver performance and directly affects the precision of the receiver's pseudo-range and carrier phase observations (Langley, 1997). The individual C/N_0 value is a function of the quality of the received signal. For example, if an antenna is set up in a high multipath environment, usually the C/N_0 oscillates around an expected C/N_0 value. A formula for the phase variance as a function of the C/N_0 in the following form was derived by Cohen (1996), Langley (1997), and Braasch and Van Dierendonck (1999):

$$\sigma_{\phi_i}(m) = \sqrt{\frac{B}{c/n_0}} \cdot \frac{\lambda}{2\pi} \tag{4}$$

where the subscript i indicates the L_i signal (L_1 or L_2), B is the noise bandwidth of the carrier tracking loop (Hz), λ is the wavelength of the carrier (m), and c/n_0 is the carrier-to-noise density expressed as a ratio ($=10^{\frac{C/N_0}{10}}$ for C/N_0 expressed in dB-Hz). This equation gives a nominal value for the L_1 carrier phase noise of 0.2 mm for a 2 Hz bandwidth loop and a C/N_0 value of 45 dB-Hz. Typically this error is dominated by multipath and signal diffraction and is smaller than 1 mm for typical tracking parameters (Cohen, 1996).

SNR or C/N_0 values are recovered from the GPS receiver. However, currently manufacturers are not obliged to provide these values. The C/N_0 information used in this study has been extracted from the observations by additional software provided by the receiver manufacturer.

Stochastic Models

The signal quality measures given in the previous section can be used for weighting the GPS observations. Ward (1996) gives the following formula expressing the phase variance σ_i^2 in mm^2 as a function of the measured C/N$_0$ values:

$$\sigma^2_{\phi_{\varepsilon_i}} = C_i 10^{-(C/N_{0\mathrm{measured}})/10} \tag{5}$$

The Sigma-ε model is based on Eq. (5) expressing the estimation of phase variances. Using the phase variances, the weights are determined as follows:

$$w_{\phi_{\varepsilon_i}} = \frac{1}{\sigma^2_{\phi_{\varepsilon_i}}} \tag{6}$$

Weights for the carrier phases are computed using the above formula. Through the use of the Sigma-ε model, the measured C/N$_0$ values are used for the calculation of the variances of DD carrier phase observations. From the epoch-by-epoch C/N$_0$ variances the DD variances are computed. This approach is based on the law of the propagation of variances leading to DD-C/N$_0$ variances (Brunner *et al.*, 1999). Hence the DD phase values get proper weights, and the mathematical correlations are appropriately determined. Brunner *et al.* (1999) suggests a value of 1.61 x 10^{-4} mm^{2}for C_i. In this study the same value is applied in the GPS processing of GPS data.

In addition to this stochastic model, another model based on elevation cut-off angles has been applied on some longer baselines. The model, as introduced by Rothacher *et al.* (1998), is given below:

$$w(z) = Cos^2(z) \tag{7}$$

where z stands for the zenith angle of the satellite.

Both models mentioned above are applied to GPS data at zero difference level, where the observations are undifferenced. Applications of such stochastic models are a current task among geodesists. For example, the second model given above was implemented into the latest (4.2) version of Bernese GPS processing software (Hugentobler *et al.*, 2001). For elevation cut-off angle dependent models, there is no limitation. They can be easily implemented into GPS processing. However, it is not the same for signal quality based models. The main drawback about the applicability of such models is the lack of standards in reporting the information of signal quality. At the moment there is no common standard about the signal quality measures. Through the developments in the ongoing standardization process over reporting signal quality measures, such models might become widely applicable in the near future.

Numerical Examples

The GPS observations used in this study consist of 2 parts. One part was collected on short baselines. The other observation set consists of data provided by the International GPS Service on the Internet and belongs to longer baselines.

Examination on Short Baselines

For this purpose, at 2 stations, HEI1 and HEI2, GPS data were collected using Leica SR399 receivers and Ashtech Dorne Morgolin choke ring antennas. The measurements were performed on 2 consecutive days to investigate the repeatability of the coordinate solutions. The measurement length is 1 h and the sampling rate is 1 s. The distance from HEI1 to HEI2 is 119.81 m. The coordinates of HEI2 were computed with respect to those of HEI1. In the processing of the baseline the Sigma-ε method was applied. The software used for the processing was Grazia, which was developed at Graz University of Technology for continuous landslide monitoring on considerably shorter baselines (Gassner *et al.*, 2001). The software enables us to use the Sigma-ε weighting algorithm as given in Eq. (6). The observations were carried out in 2 sessions on 2 consecutive days. The process was carried out using 5°, 10° and 15° minimum elevation cut-off angles separately. Figure 2 shows the epoch-to-epoch coordinate solutions for the baseline HEI1-HEI2 obtained by the introduction of the Sigma-ε weighting algorithm.

These solutions indicate that the patterns of the variations of all coordinate components are quite similar in both sessions. This is also an indicator for the day-to-day repeatability of the solutions. The semilogarithmic scale plots of the power spectral density estimates of the coordinate components are visualized in Figure 3. In the plots the product of the frequency (F$_s$) and the product of power spectral density (P$_x$) versus semilogarithmic scale of F$_s$ is shown.

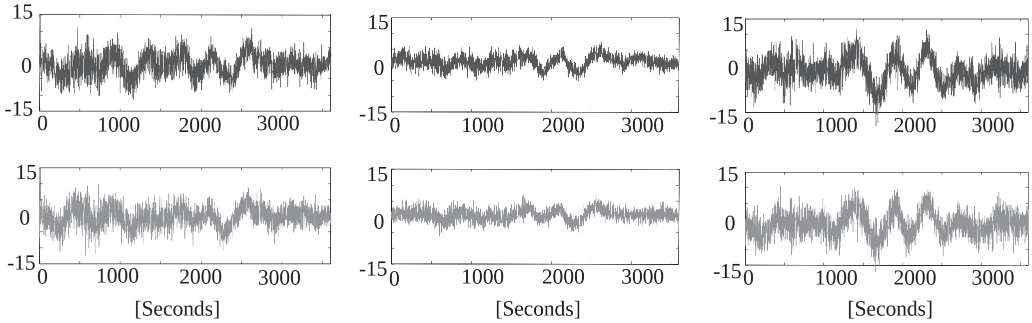

Figure 2. North (left), east (middle) and height (right) components of point HEI2 in the first (upper) and second (lower) sessions.

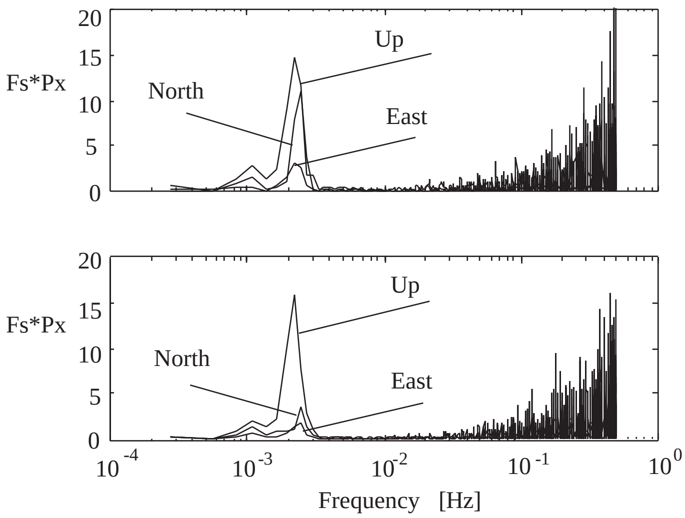

Figure 3. Spectral power density for the epoch-to-epoch coordinates of point HEI2 in the first (top row) and second (bottom row) sessions.

The area below the $F_s{*}P_x$ curve is proportional to its contribution to the variance of the signal. In the figures for all coordinate components, 2 clear periodic signals are seen at frequencies of about 0.001 Hz ($16^{\min}40^s$) and 0.002 Hz ($8^{\min}20^s$). Except for the signal for the height component, which remains constant in both sessions, the amplitudes of these signals are lower in the second session. As seen from the figure, the patterns of the spectral power density values have the same characteristics as the coordinate solutions.

In addition to the Sigma-ε model, the GPS data were also processed in the traditional way in which all observations are considered as equally weighted. For the baseline HEI1-HEI2, the standard deviations from both solutions are shown in Figure 4 (Özlüdemir, 2002).

As seen from the above figure, by the application of the signal quality based Sigma-ε weighting algorithm even for low elevated data better coordinate solutions are achieved.

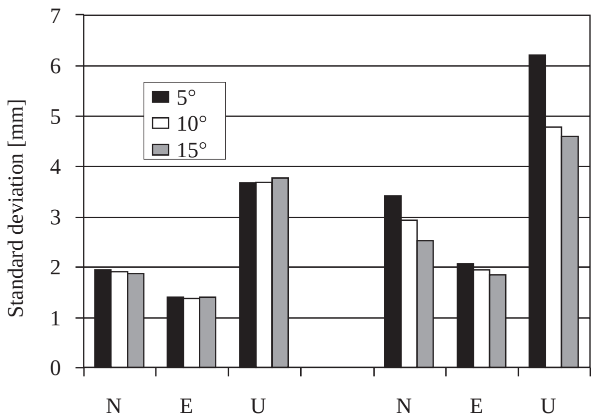

Figure 4. The standard deviations of the solutions by Sigma-ε (left) and equal-weighting algorithms (right) for coordinate components north (N), east (E) and up (U).

Examination on Long Baselines

For the processing of long baselines, 3 IGS stations were chosen. These stations are WTRS (Westerbork Synthesis Radio Telescope), KOSG (Kootwijk Observatory for Satellite Geodesy) and POTS (Potsdam GeoForschungs Zentrum). The stations WSRT, KOSG and POTS have Rogue SNR-12 RM, AOA SNR-12 ACT and AOA SNR-8000 ACT receivers, respectively. The stations WSRT and POTS have AOAD/M_T antennas while KOSG has AOAD/M_B.

By taking the coordinates of WSRT as fixed, the baselines WSRT-KOSG and WSRT-POTS, which are 98073.3 m and 441226.7 m in length, respectively, were processed by Bernese software (Hugentobler *et al.*, 2001). Bernese software enables us to apply the $\mathrm{Cos}^2(z)$ weighting algorithm. The baseline solutions were realized using $\mathrm{Cos}^2(z)$ and equal weighting algorithms. Through GPS processing, identical results were obtained for the baselines. Figures 5 and 6 (Özlüdemir, 2002) show the solutions for the baseline WSRT-KOSG and WSRT-POTS.

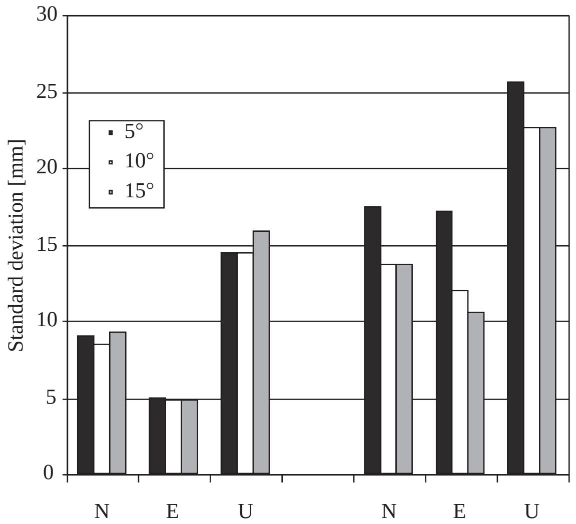

Figure 5. The standard deviations of the solutions of WSRT-KOSG by $\mathrm{Cos}^2(z)$ and equal-weighting algorithms (right) for coordinate components north (N), east (E) and up (U).

Through the application of the elevation cut-off angle dependent weighting algorithm, the precision of coordinate solutions becomes better. The solutions are similar to those obtained with the signal quality based Sigma-ε algorithm. Downweighting the GPS data with the Sigma-ε or $\mathrm{Cos}^2(z)$ stochastic models improves the precision of coordinate solutions. In particular the repeatability of the solutions becomes much better than that of the conventional equal weighting approach.

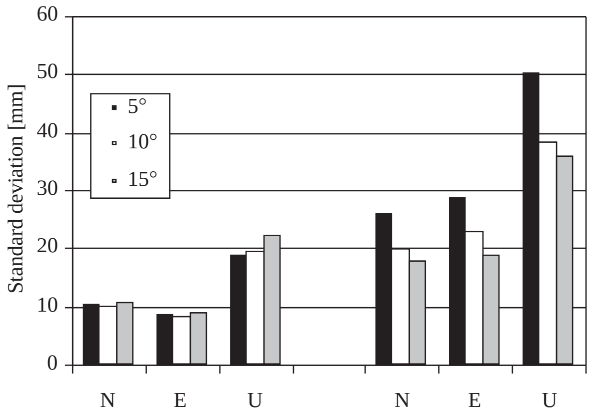

Figure 6. The standard deviations of the solutions of WSRT-POTS by $\mathrm{Cos}^2(z)$ and equal-weighting algorithms (right) for coordinate components north (N), east (E) and up (U).

Conclusion

The basic limiting factors of the attainable accuracy through GPS observations are the multipath and tropospheric delay effects. The multipath and tropospheric delay effects increase when the line of sight approaches the horizon. In other words, the lower the satellite elevation angles the higher the carrier phase noise. Therefore, a relation between measurement noise and all these effects can be formed. The stochastic models or weighting algorithms applied in this study are based on the characterization of the noise effect. The effectiveness of these stochastic models was evaluated with 2 data sets and the results obtained were analyzed in comparison with those obtained by non-weighting processing. The analyses show that the precision of coordinate solutions, especially the day-to-day repeatability of the solutions, becomes much better with the introduction of these models. The advantage of these models is their sensitivity against multipath and tropospheric delay effects. However, whether or not these models improve the accuracy of the coordinates needs further investigation.

The elevation dependent $\mathrm{Cos}^2(z)$ model can be implemented in any software. However, it is not easy to implement the Sigma-ε model that uses C/N_0 values. Currently manufacturers are not obliged to provide SNR (or C/N_0) values and those providing this information often report it in a reduced format. The common format for GPS observations is the RINEX (Receiver INdependent Exchange) format. In the current version of the RINEX format, i.e. version 2.1, SNR reporting is defined. This will enable the GPS people to convert receiver-generated values in the raw data stream to some agreed-upon definition, e.g., C/N_0 values.

Through the investigation of stochastic models, it was proven that the applied models can be used suitably for weighting GPS carrier phase data, therefore enabling us to use low elevated data. By the inclusion of low elevated data, the satellite geometry and height determination accuracy improve.

Since the applied models reflect the multipath effect, based on the day-to-day repeatability of the coordinate solutions, they can also be used for the development of multipath mitigation techniques.

It should be noted that the stochastic models introduced in this study were applied in the post-processing of GPS data. These models are especially important for real-time applications on permanent stations. It is also feasible to implement these models in real-time applications as well.

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