Turkish J. Eng. Env. Sci.
29 (2005) , 171 – 183.

A Comparison of Recursive Least Squares Estimation and Kalman Filtering for Flow in Open Channels

Ömer Faruk DURDU
Adnan Menderes University, Faculty of Agriculture,
Aydın-TURKEY
e-mail: odurdu@adu.edu.tr

Received 13.10.2004

Abstract

An integrated approach to the design of an automatic control system for canals using a Linear Quadratic Gaussian regulator based on recursive least squares estimation was developed. The one-dimensional partial differential equations describing open channel flow (Saint-Venant) equations are linearized about an average operating condition of the canal. The concept of optimal control theory is applied to drive a feedback control algorithm for constant-level control of an irrigation canal. The performance of state observers designed using the recursive least squares technique and the Kalman filtering technique is compared with the results obtained using a full-state feedback controller. An example problem with a multi-pool irrigation canal is considered for evaluating the techniques used to design an observer for the system. Considering the computational complexity and accuracy of the results obtained, the recursive least squares technique is found to be adequate for irrigation canals. In addition, the recursive least squares algorithm is simpler than the Kalman technique and provides an attractive alternative to the Kalman filtering.

Key words: Optimal control, Least squares estimation, Kalman filters, Open-channel flow.

Introduction

With increasing demand for food and competing use within the water sector, the pressure is on irrigation professionals to manage water efficiently. In response to this, strategic decisions and interventions need to be made on a continuous basis. These decisions should cover the full spectrum of the irrigation water supply system, from diversion and distribution to on-farm application down to the crop root zone. Supply-oriented systems have not been able to provide the needed flexibility in terms of water quantity and timing to achieve improved crop yields and water use efficiency. This calls for a demand delivery of water to the farmers in the command area of an irrigation project. Demand delivery offers the maximum flexibility and convenience to the water user (Reddy, 1990). In a demand delivery schedule, under constant level control, since the demands (disturbances or flow rate changes) are not known in advance, the effect of the random disturbances on the system variables must be measured and used in the feedback loop to control the system. The variation in the depths of flow is used in the closed loop (feedback loop). During the last few decades, several control algorithms have been developed to derive the relationship between the deviations in the system variables (flow depth and flow rate) and the change in gate opening (gate-control algorithm). Linear quadratic optimal control theory has been applied for deriving closed-loop control algorithms for real-time control of open irrigation canals (Reddy, 1999; Reddy *et al.*, 1999; Seatzu *et al.*, 2000; Seatzu, 2002; Durdu, 2003; Durdu, 2004). However, when lumped parameter models are used to derive control algorithms for irrigation canals, the number of state variables (flow depths and flow rates) that must be used in the feedback loop is large. Consequently, it is costly to measure flow depths and flow rates at several points in a multi-pool irrigation canal. Therefore, to minimize the cost of implementing feedback control algorithms, the number of measurements per pool must be kept to an absolute minimum. Since 1 or 2 flow depths per pool are normally measured in practice, it is preferable (and possible) to estimate values for the state variables that are not measured. This is done by using an observer. Reddy (1995) demonstrated Kalman filtering for the estimation of state variables in the control system. The objective of this paper is to develop an LQG regulator using the recursive least squares method and to demonstrate the performance of this regulator in comparison to full-state feedback and a Kalman filter based LQG regulator.

Basic Equations

The Saint-Venant equations, presented below, are used to model flow in a canal

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - q_l = 0 \qquad (1)$$

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = gS_0 - gS_f - g\frac{\partial x}{\partial y} \qquad (2)$$

in which Q = discharge, m^3/s; A = the cross-sectional area, m^2; q_l = lateral flow, m^2/s; y = water depth, m; t = time, s; x = longitudinal direction along the channel, m; g = the acceleration due to gravity, m^2/s; S_o = canal bottom slope (m/m); and S_f = frictional slope, m/m, and is defined as

$$S_f = Q|Q|/K^2 \qquad (3)$$

in which K = hydraulic conveyance of canal = $AR^{2/3}/n$; R = hydraulic radius, m; and n = Manning friction coefficient, s/m$^{1/3}$. Lateral canals in the main canal are usually scattered throughout the length of the supply canal. The mathematical representation of flow through these structures is given as follows (Reddy, 1999):

$$q_l = C_d b_l w_l (2g(Z - Z_l))^{1/2} \quad \text{for submerged flow} \qquad (4)$$

$$q_l = C_d b_l w_l (2g(Z - E_s))^{1/2} \quad \text{for free flow} \qquad (5)$$

in which q_l = lateral discharge rate, m^3/s; C_d = outlet discharge coefficient; b_l = width of outlet structure, m; w_l = height of gate opening of outlet structure, m; Z= water surface elevation in the supply canal, m; Z_l = water surface elevation in the lateral canal, m; and E_s = sill elevation of the head regulator, m. In the regulation of the main canal, decisions regarding the opening of gates in response to random changes in water withdrawal rates into lateral canals are required to maintain the flow rate into laterals close to the desired value. This is accomplished either by maintaining the depth of flow in the immediate vicinity of the turnout structures in the supply canal constant or by maintaining the volume of water in the canal pools at the target value (Reddy, 1999). When the latter option is used, the outlets are often fitted with discharge rate regulators. The water levels or the volumes of water stored in the canal pools are regulated using a series of spatially distributed gates (control elements). Hence, irrigation canals are modeled as distributed control systems. Therefore, in the solution of Eqs. (1) and (2), additional boundary conditions are specified at the control structures in terms of the flow continuity and the gate discharge equations, which are given by (Reddy, 1990)

$$Q_{i-1,N} = Q_{gi} = Q_{i,1} \quad \text{(continuity)} \qquad (6)$$

$$Q_{gi} = C_{di} b_i u_i (2g(Z_{i-1,N} - Z_{i,1}))^{1/2} \quad \text{(gate discharge)} \qquad (7)$$

in which, $Q_{i-1,N}$ = flow rate through downstream gate (or node N) of pool $i-1$, m^3/s; Q_{gi} = flow rate through upstream gate of pool i, m^3/s; $Q_{i,1}$ = flow rate through upstream gate (or node 1) of pool i, m^3/s; C_{di} = discharge coefficient of gate i; b_i = width of gate i, m; u_i = opening of gate i, m; $Z_{i-1,N}$ = water surface elevation at node N of pool $i-1$, m; $Z_{i,1}$ = water surface elevation at node 1 of pool i, m; and i = pool index (i = 0 refers to the upstream constant level reservoir). The Saint-Venant open-channel equations, Eqs. (1) and (2), are linearized about and average operating conditions of the canal to apply the linear control theory concepts to the problem. After applying a finite-difference approximation and the Taylor series expansions to Eqs. (1) and (2), a set of linear, ordinary differential equations is obtained for the canal with control gates and turnouts (Durdu, 2004)

$$A_{11}\delta Q_j^+ + A_{12}\delta z_j^+ + A_{13}\delta Q_{j+1}^+ + A_{14}\delta z + 1 = A'_{11}\delta Q_j + A'_{12}\delta z_j + A'_{13}\delta Q_{j+1} + A'_{14}\delta z_{j+1} + C_1 \quad (8)$$

$$A_{21}\delta Q^+ j + A_{22}\delta z_j^+ + A_{23}\delta Q_{j+1}^+ + A_{24}\delta z_{j+1}^+ = A'_{21}\delta Q_j + A'_{22}\delta z_j + A'_{23}\delta Q_{j+1} + A'_{24}\delta z_{j+1} + C_2 \quad (9)$$

where δQ_j^+ and δz_j^+ = discharge and water-level increments from time level $n+1$ at node j; δQ_j and δz_j = discharge and water-level increments from time level n at node j; and A_{11}, A'_{21}, ... A_{12}, A_{22} are the coefficients of the continuity and momentum equations, respectively, computed with known values at time level n. Similar equations are derived for channel segments that contain a gate structure, a weir or some other type of hydraulic structure. From the matrix form of Eqs. (8) and (9), the state of system equation at any sampling interval k can be written in a compact form as follows (Durdu, 2003)

$$A_L\delta x(k+1) = A_R\delta x(k) + B\delta u(k) + C\delta q(k) \quad (10)$$

where $A = nxn$ system feedback matrix, $B= nxm$ control distribution matrix, $C = pxn$ disturbance matrix, $\delta x(k) = nxn$ state vector, $\delta u(k) = mxn$ control vector, $\Delta\delta q$ = variation in demands (or disturbances) at the turnouts, m^2/s, n = number of dependent (state) variables in the system, m = number of controls (gates) in the canal, p = number of outlets in the canal, and k = time increment, s. The elements of the matrices A, B, and C depend upon the initial condition. The dimensions of the control distribution matrix, B, depend on the number of state variables and the number of gates in the canal. The dimensions of the disturbance matrix, C, depend on the number of disturbances acting on the canal system and the number of dependent state variables (Malaterre, 1997). Equation (10) can be written in a state-variable form along with the output equations as follows

$$\delta x(k+1) = \Phi\delta x(k) + \Gamma\delta u(k) + \Psi \quad \delta q(k) \quad (11)$$

$$\delta y(k) = H\delta x(k) \quad (12)$$

where $\Phi = (A_L)^{-1} * A_R$, $\Gamma = (A_L)^{-1} * B$, and $\Psi = (A_L)^{-1} * C$, $\delta x(k) = nxn$ state vector, $\delta y(k) = rxn$ vector of output (measured variables), $H = rxn$ output matrix, and r = number of outputs. The elements of the matrices Φ, Γ, and Ψ depend upon the canal parameters, the sampling interval, and the assumed average operating condition of the canal. In Eq. (11), the vector of state variables is defined as follows

$$\delta x = (\delta Q_{i,1}, \delta Z_{i,2}, \delta Q_{i,2}, \ldots \delta Z_{i,N-1}, \delta Q_{i,N-1}, \delta Q_{i,N}) \quad (13)$$

Full-State Feedback Regulator

The full-state feedback (Linear Quadratic Regulator (LQR)) control problem is an optimization problem in which the cost function, J, to be minimized is given as follows

$$J = \sum_{i=1}^{K_\infty} [\delta x^T(k) Qx_{nxn} \delta x(k) + \delta u^T(k) R_{mxm} \delta u(k)] \quad (14)$$

subject to the constraint that:

$$-\delta x(k+1) + \Phi\delta x(k) + \Gamma\delta u(k) = 0 k = 0, \ldots, K_\infty \quad (15)$$

where K_∞= number of sampling intervals considered to derive the steady state controller; Qx_{nxn} = state cost weighting matrix; and R_{mxm} = control cost weighting matrix. The matrices Qx and R are symmetric, and to satisfy the non-negative definite condition, they are usually selected to be diagonal with all diagonal elements positive or zero. The first term in Eq. (3) represents the penalty on the deviation of the state variables from the average operating (or target) condition, where the second term represents the cost of control. Equations (3) and (15) constitute a constrained-minimization problem that can be solved using the method of Lagrange multipliers (Reddy, 1999). This produces a set of coupled difference equations that must be solved recursively backwards in time. In the optimal steady-state case, the solution for change in gate opening, $\delta u(k)$, is of the same form as

$$\delta u(k) = -K\delta x(k) \quad (16)$$

where K is given by

$$K = [R + \Gamma^T S\Gamma]^{-1}\Gamma^T S\Phi \tag{17}$$

S is a solution of the discrete algebraic Riccati equation (DARE)

$$\Phi^T S\Phi - \Phi^T S\Gamma[R + \Gamma^T S\Gamma]^{-1}\Gamma^T S\Phi + Qx = S \tag{18}$$

where $R = R^T > 0$ and $Qx = Qx^T = H^T H \geq 0$. The control law defined by Eq. (16) brings an initially disturbed system to an equilibrium condition in the absence of any external disturbances acting on the system. In the presence of these external disturbances, the system cannot be returned to the equilibrium condition using Eq. (16). An integral control, in which the cumulative (or integrated) deviation of a selected output variable is used in the feedback control loop, is required to return the system to the equilibrium condition in the presence of external disturbances (Kwakernaak and Sivan, 1972). Integral control is achieved by appending additional variables of the following form to the system dynamic equation

$$-\delta x_n(k+1) = D\delta x(k) + \Gamma\delta x_n(k) \tag{19}$$

in which δx_n =integral state variables; and D = integral feedback matrix. This produces a new control law to the form (Reddy *et al.,* 1992)

$$\delta u(k) = -K\delta x(k) - K_n \delta x_n(k) \tag{20}$$

The first term in Eq. (20) accounts for initial disturbances, whereas the second term accounts for external disturbances. Equation (20) predicts the desired gate openings as a function of the measured deviations in the values of the state variables (Reddy *et al.* 1992). In this paper, the water surface elevation and flow rate were considered the state variables. Given initial conditions $[\delta x(0)]$, δu, and δq, Eq. (3) can be solved for variations in flow depth and flow rate as a function of time. If the system is really at equilibrium [i.e. $\delta x\ (0) = 0$ at time $t = 0$] and there is no change in the lateral withdrawal rates (disturbances), the system would continue to be at equilibrium forever and there is then no need for any control action. Conversely, in the presence of disturbances (known or random), the system would deviate from the equilibrium condition (Reddy, 1990). The actual condition of the system may be either above or below the equilibrium condition, depending upon the sign and magnitude of the disturbances. If the system deviates significantly from the equilibrium condition, the discharge rates into the laterals will be different (either more or less) from the desired values. In canal operations, however, the main objective is to keep these deviations to a minimum so that a nearly constant rate of discharge is maintained through the turnouts (Reddy *et al.*, 1992).

Gaussian Regulator with Kalman Filtering

The Linear Quadratic Gaussian (LQG) theory provides an integrated knowledge base for the development of a flexible controller (Figure 1). Since it is expensive to measure all the state variables (flow rates and flow depths) in a canal system, the number of measurements per pool must be kept to an absolute minimum. Usually the flow depths at the upstream and downstream ends of each pool are measured. The relationship between the state variables and the measured (or output) variables is

$$\delta y(k) = H\delta x(k) + \eta(k) \tag{21}$$

where $\eta(k)$ is measurement error inputs. For a steady-state Kalman filter, the observer gain matrix, L, is calculated as follows

$$L = PH^T[HPH^T + RC]^{-1} \tag{22}$$

where P is the covariance of estimation uncertainty

$$\Phi^T P\Phi - \Phi P\ H^T[RC + H^T PH]^{-1}HP\Phi^T + Q_{esti} = P \tag{23}$$

where $RC = RC^T > 0$ is a tolerance value for the RC covariance matrix, which is an identity matrix and $Q_{esti} = Q_{esti}^T \geq 0$ is a diagonal matrix. The disturbances $\delta q(k)$ and $\eta(k)$, in Eqs. (11) and (21), are assumed to be zero mean Gaussian white noise sequences with symmetric positive definite covariance matrices Q_{esti} and RC, respectively. Furthermore, sequences $\delta q(k)$ and $\eta(k)$ are assumed to be statistically independent. The system dynamic equation is used to predict the state and estimation error covariance as follows: time update equations (Tewari, 2002)

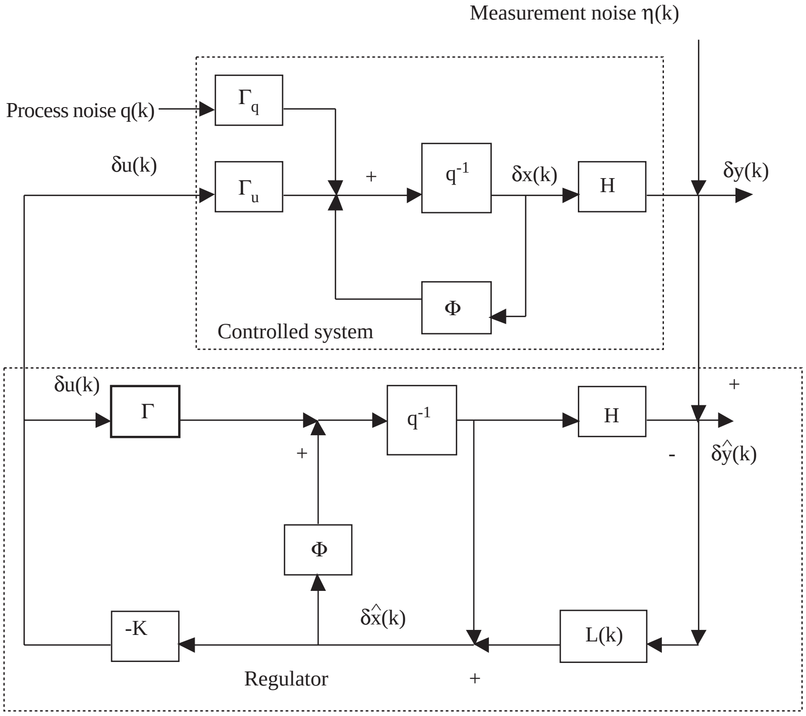

Figure 1. Linear Quadratic Gaussian (LQG) controller with a state estimator.

$$P_{-}(k+1) \;=\; \Phi P(k)\Phi^{T} \;+\; \Psi Q_{esti}\Psi^{T} \tag{24}$$

$$\delta\hat{x}_{-}(k+1) = \Phi\delta\hat{x}(k) + \Gamma\delta u(k) \tag{25}$$

in which $\delta\hat{x}(k)$ = estimated values of the state variables. As soon as measured values for the output variables $\delta y(k)$ are available, the time-update values are corrected using the measurement update equations as follows: measurement update equations (Tewari, 2002)

$$L(k+1) = P_{-}(k+1)H^{T}[HP_{-}(k+1)H^{T} + RC]^{-1} \tag{26}$$

$$P(k+1) = [I - L(k+1)H]P_{-}(k+1) \tag{27}$$

$$\delta\hat{x}(k+1) = \delta\hat{x}_{-}(k+1) + L(k+1)[\delta y(k+1) - H\delta\hat{x}_{-}(k+1)] \tag{28}$$

If the initial conditions and the inputs (control inputs and the disturbances) are known without error, the system dynamic equation, Eq. (2), can be used to estimate the state variables that are not measured. Since parts of the disturbances are random and usually are not measured, the canal parameters are not known very accurately, and the estimated values of the state variables would diverge from the actual values. This divergence can be minimized by utilizing the difference between measured output and the estimated output (error signal), and by constantly correcting the system model with the error signal (Reddy, 1995). Therefore, the modified state equations are given as

$$\delta\hat{x}(k+1) = \Phi\delta\hat{x}(k) + \Gamma\delta u(k) + L[\delta y(k) + H\delta\hat{x}(k)] \tag{29}$$

Gaussian Regulator with Recursive Least Squares

This estimation technique is useful in identifying time varying parameters and has been considerably discussed in the literature. The optimal estimation criterion for recursive weighted least squares is written in the following form

$$J = \sum_{j=1}^{k} \beta(k,j)\varepsilon^T(j)\Lambda^{-1}(j)\varepsilon(j) \qquad (30)$$

where $\varepsilon(j)$ = prediction error in vector form, and $\beta(k,j)$ = weighting function. The weighting function is assumed to satisfy the following relationship

$$\beta(k,j) = \lambda(k)\beta(k-1,j), 1 \le j \le k-1, \beta(k,k) = 1 \qquad (31)$$

where λ is the weighting or forgetting factor. These relations imply that

$$\beta(k,j) = \prod_{\varepsilon=j+1}^{k} \lambda(i) \qquad (32)$$

The recursive estimation algorithm can be written in a number of different equivalent forms. The following form of the recursive least square algorithm has many computational advantages (El-Hawary, 1989)

$$\delta\hat{x} = \delta\hat{x}(k-1) + K(k)r(k) \qquad (33)$$

in which $r(k)$ is the innovations sequence defined by

$$r(k) = y(k) - H(k)\delta\hat{x}(k-1) \qquad (34)$$

The gain matrix $K(k)$ is defined by

$$K(k) = P(k-1)H^T(k)\Phi^{-1}(k) \qquad (35)$$

where $\Phi(k)$ is defined by

$$\Phi(k) = \lambda(k)\Lambda(k) + H(k)P(k-1)H^T(k) \qquad (36)$$

The equivalent of the state error covariance matrix is given by

$$P(k) = \frac{1}{\lambda(k)}[1 - K(k)H(k)]P(k-1) \qquad (37)$$

The equations for the least squares method bear many similarities to the Kalman filtering equations. The differences between Kalman filtering and the least squares method are: 1) in the predictor stage of the Kalman filter, a prediction of the state based on the previous optimal estimate is obtained by (El-Hawary, 1989)

$$\delta\hat{x}_-(k) = \Phi(k-1)\delta x_+(k-1) \qquad (38)$$

whereas, in the least squares method, the transition matrix is assumed to be unity and thus

$$\delta\hat{x}_-(k) = \delta x_+(k) = \delta x(k) \qquad (39)$$

In addition, the error covariance matrix is obtained by

$$P_-(k) = \Phi(k-1)P_+(k-1)\Phi^T(k-1) + \Gamma Q_{esti}(k-1)\Gamma^T \qquad (40)$$

For the least squares method the matrix Q_{esti} is zero and the equivalent of Eq. (16) is given by

$$P_-(k) = P_+(k-1) \qquad (41)$$

2) in the corrector stage of the Kalman filter, an updated state estimate equation is obtained

$$\delta\hat{x}_+(k) = \delta\hat{x}_-(k) + K(k)r(k) \qquad (42)$$

in which $r(k)$ is the innovation sequence and is given by

$$r(k) = y(k) - H\delta x_-(k) \qquad (43)$$

Equations (12) and (13) of the least squares method are the same as Eqs. (21) and (22). In addition, an update of the covariance matrix is obtained

$$P_+(k) = (I - K(k)H(k))P_-(k) \qquad (44)$$

This definition is similar to Eq. (16) except for the division by $\lambda(k)$. The Kalman gain matrix K is given by

$$K(k) = P_-(k)H^T(k)\Phi(k)^{-1} \qquad (45)$$

where

$$\Phi(k) = R(k) + H(k)P_-(k)H^T(k) \qquad (46)$$

Equation (24) is similar to Eq. (14), but Eq. (25) differs from Eq. (15), since $R(k)$ in Eq. (25) is replaced by $\lambda(k)\Lambda(k)$ in Eq. (15). In the least squares method, one does not need the value of the state-noise covariance matrix Q_{esti}, and the measurement error covariance matrix $RC(k)$ is replaced by the choice of the weighting function. The problem of defining the noise statistics manifested by the Q_{esti} and RC matrices of Kalman filtering can be seen from references. The approaches for defining Q_{esti} and RC matrices involve a considerable computational burden, which may be avoided by suitably selecting the weighting function to suit the practical application (El-Hawary, 1989). The choice of weighting function $\beta(k,j)$ controls the way in which each measurement is incorporated relative to other measurements. The choice should clearly be such that measurements that are relevant to current system properties are included. If one chooses to assign less weight to older measurements such as in the case of variable system parameters, then a popular choice is given by

$$\lambda(i) = \lambda \tag{47}$$

Therefore, Eq. (11) can be written as

$$\beta(k,j) = \prod_{\varepsilon=j+1}^{k} \lambda = \lambda^{k-j} \tag{48}$$

This is referred to as an Exponential Weighting into the Past (EWP), with λ being a forgetting factor that shapes the estimator's memory. Λ is chosen to be slightly less than 1. A second possible choice is such that the forgetting factor is time-varying and in this case (El-Hawary, 1989)

$$\beta(k,j) = (\alpha_k)^{k-j} \tag{49}$$

where α_k is defined by the first order discrete filter

$$\alpha_k = \lambda_0 \alpha_{k-1} + (1-\lambda_0)\alpha \tag{50}$$

Typically, α_0= 0.95, $\lambda_0 = 0.99$, and $0 < \alpha < 1$. The forgetting factor starts at α_0 and reaches a steady state value of α (El-Hawary, 1989). For a reasonably large k

$$\lambda(k) = \alpha_{k-1} = \lambda_0[\lambda(k-1) - \alpha] + \alpha \tag{51}$$

Ljung and Soderstrom (1983) take α to be unity. The convergence of the filter is influenced by the choice of the weighting function. Once the equations of the optimal state feedback and the recursive least squares method are obtained, and measured values for one or more state variables for each pool are available, the dynamics of the linear system can be simulated for any arbitrarily selected values of external disturbances. In this study, a multi-pool irrigation canal was considered. The algorithm predicts the flow rate, $Q(x,t)$, and the depth of flow, $y(x,t)$, given the initial boundary conditions. The optimal state feedback and the least squares equations were added as subroutines to this algorithm. Given the initial flow rate and the target depth at the downstream end of the each pool, the algorithm computed the backwater curve. Later on, the downstream flow requirement and the withdrawal rate into the lateral were provided as a boundary condition. The model predicted the depths and flow rates at the nodal points for the next time increment. The computed depths at the upstream and downstream ends of each pool were used with the least squares under constraints to estimate the flow depths and flow rates at some selected intermediate nodal points. These estimated values were then used in the optimal state feedback subroutine to compute the change in the upstream gate opening in order to bring the depth at the downstream end of the pool close to the target depth. When the estimated values of the state variables are used in the feedback loop, the controller equation, Eq. (16) becomes (Reddy, 1999)

$$\delta u(k) = -K\delta\hat{x}(k) - K_l \delta x_l(k) \tag{52}$$

Equation (52) computes the desired change in gate opening as a function of the estimated (instead of measured) deviations in the state variables. Based upon this gate opening, the new flow rate into the pool at the upstream end was calculated and used as the boundary condition at the upstream end of the each pool. This process was repeated during the entire simulation period.

Results and Analysis

To explore and compare the performance of the Kalman estimator and recursive least squares method, an LQG regulation problem for a discrete-time multi-pool irrigation canal was simulated. The data used in the simulation study are demonstrated in Table 1. These data were first used to calculate the steady state values, which in turn were used to compute the initial gate openings and the elements of the Φ, Γ, and H matrices using a sampling interval of 30 s. After computing steady state values, the control algorithm formulates an LQG controller with a Kalman filter and recursive least squares, respectively. As a first part of the LQG controller, a full-state feedback controller (assuming all state variables are available) was designed to regulate the 6-pool canal system using a constant-level control approach. The system response was simulated using the controller in the feedback loop. In the derivation of the feedback gain matrix K, the control cost weighting matrix, R, of dimension 6, was set equal to 100, whereas the state cost weighting matrix, Qx, was set equal to an identity matrix of dimension 85. The matrix dimension 85 comes from the system dimension. Since the irrigation canal is divided into 49 nodes and each node has a set of 2 equations, the dimension of the system should be equal to 98. However, the system has 7 gates and 6 turnouts; therefore, the system matrix dimensions numbered 85. The cost weighting matrix and the control cost matrix must be symmetric and positive definite (i.e. all eigenvalues of R and Qx must be positive real numbers). A priori, we do not know what values of Qx and R will produce the desired effect. In the absence of a well-defined procedure for selecting the elements of these matrices, these values are selected based upon trial and error. At first, both Qx and R as identity matrices were selected. By doing so, it was specified that all state variables and control inputs were equally important in the objective function, i.e. it was equally important to bring all the deviations in the state variables (water surface elevations and flow rate) and the deviations in the control inputs to zero while minimizing their overshoots. Note that the existence of a unique, positive definite solution to the algebraic Riccati equation (Eq. 18) is guaranteed if Qx and R are positive semi-definite and positive definite, respectively, and the system is controllable. To test whether the system was controllable, the system controllability matrix was calculated and it was found that the system was controllable. In the derivation of the feedback gain matrix, R was set equal to 100, whereas Qx was set equal to an identity matrix of dimension 85 (the dimensions of the system). After defining Qx and R matrices, the optimal feedback gain matrix, K, was calculated. Since measurement of all the state variables was expensive, the control algorithm first estimated state variables using a Kalman filter. Next, the recursive least squares method was employed to estimate the values for the state variables in the algorithm. The Kalman filter for the system used the control input $\delta u(k)$, generated by the optimal state feedback, measured water depths $\delta y(k)$ for each pool, the disturbances noise $\delta q(k)$, and measurement noise, $\eta(k)$. In the design of the Kalman filter, in lieu of actual field data on withdrawal rates from the turnouts, the random disturbances were assumed to have some prespecified levels of variance. The variances of the disturbances, Q_{esti}, were $w_1 = 1^2$, $w_2 = 1.3^2$, $w_3 = 0.7^2$, $w_4 = 1.4^2$, $w_5 = 1.3^2$, and $w_6 = 1.2^2$. The actual time series of the demands was not used in the design of the filter; only the variance of the time-series was required in the design of the filter. Usually the sensors used to measure flow depths in an open-channel are reasonably accurate to a fraction of a centimeter; therefore, the variance of the measurement error is usually very small. A value of 0.0005 was used for the variance of the measurement matrix (RC), and this was an identity matrix. Using the given initial values, the system response was simulated for 250 time increments or 7500 s. After designing the LQG controller using a Kalman filter, the algorithm designed a recursive least squares based on a defined weighting factor (β). The analysis was started by evaluating the system stability. All the eigenvalues of the feedback matrix were positive and had values less than 1. The system was also found to be both controllable and observable. In the derivation of the control matrix elements, Γ, it was assumed that both the upstream and downstream gates of each reach could be manipulated to control the system dynamics. The last pool's downstream-end gate position was frozen at the original steady state value, and only the upstream gates of the given canal were controlled to maintain the system at the equilibrium condition. The effect of variations in the opening of the downstream gate must be taken into account through real-time feedback of the actual depths immediately upstream and downstream of the downstream gate (node N). Figure 3 demonstrates the variations in flow depths for each pool and for all 3 techniques. The variations in flow depths for recursive least squares were compared to the variations in flow depths computed using optimal state feedback as well as a steady-state Kalman filter. Since pool 1 is the first pool of the irrigation canal, with an increase in flow rate into the lateral (turnout) or downstream demand, the depth of flow at the downstream end of pool 1 decreased rapidly and approached a maximum deviation of -0.157 m for least squares, -0.15 m for optimal state feedback and -0.13 m for the Kalman filter at approximately 2000 s from the beginning of the disturbance period. By the end of the simulation, the system returned very close to the original equilibrium condition for all 3 techniques. In pool 2, in the first 1700 s of the simulation, the flow depth decreased dramatically and reached a maximum deviation of -0.144 m for optimal state feedback, -0.141 m for least squares and -0.118 m for the Kalman filter. The variations in flow depth in pool 3 reached -0.162 m for least squares and -0.16 m for optimal state feedback and -0.127 for the Kalman filter after around 1500 s of the simulations period. Pool 4 and pool 5 have less fluctuation in comparison to the other pools. Pool 6 has the highest variations in flow depth and the depth of flow at the downstream end decreased rapidly and approached a maximum deviation of -0.32 m for optimal state feedback, -0.28 m for least squares and -0.25 for the Kalman filter at around 2000 s of the simulation period. The rapid decreases in the downstream depth of flow in each pool resulted in an attendant sudden increase in the gate opening at the upstream end of each reach to release more water into the pool. However, because of the wave travel time, the depth of flow at the downstream end did not start to rise until around 1700 s. In all the pools considered, the maximum deviation in depth of flow occurred at the first and last pools of the canal for all 3 design techniques. To meet the downstream target depth, the last pool had the highest fluctuations. The fluctuations in the first pool were due to releasing more water into the downstream pools and meeting the demand at the downstream end. Figure 4 demonstrates the incremental gate openings for each design technique (optimal state feedback, Kalman filter and least squares) and for each gate in the canal. The deviation in the gate openings for recursive least squares was compared with the deviation in gate opening computed using optimal state feedback as well as the steady-state Kalman filter. At the beginning, gate 1 had sharp peaks for all 3 design techniques. Since optimal state feedback has the best stability properties, the state feedback curves will be the target loop. At gate 1, incremental gate opening for recursive least squares was closer to optimal state feedback (target loop function) than were those for the Kalman filter. After 6000 s, gate 1 reached an equilibrium position for all 3 techniques. At gates 2, 3, 4, 5 and 6, incremental gate opening values for the Kalman filter were far away from the optimal state feedback values in comparison to the least squares values. At the end of the simulation, the variations in the gate openings (for all gates) approached a constant value, indicating that a new equilibrium condition was established. It is obvious from the figures that the variations in flow depth and in gate openings for recursive least squares are closer to the target loop function (optimal state feedback) than are those for the Kalman estimator. In other words, the recursive least squares filter has better stability properties than the Kalman filter in the control of irrigation canals. Moreover, in the computation of estimation, one does not need to find the disturbance covariance matrix, Q_{esti}, and measurement covariance matrix, RC. Therefore, a computational burden may be avoided by selecting the appropriate weighting function.

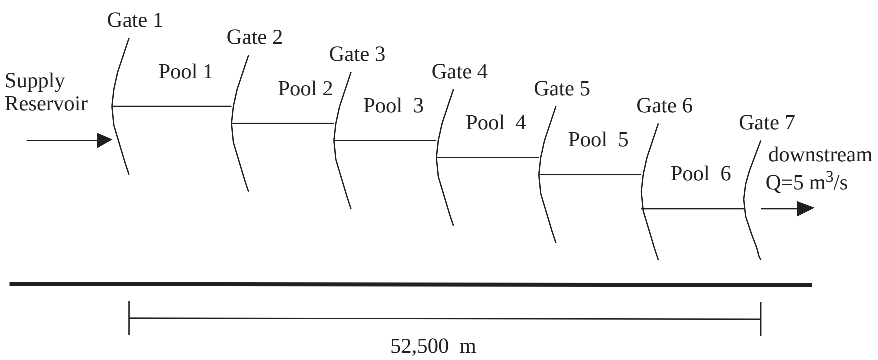

Figure 2. Schematic of multi-pool irrigation canal.

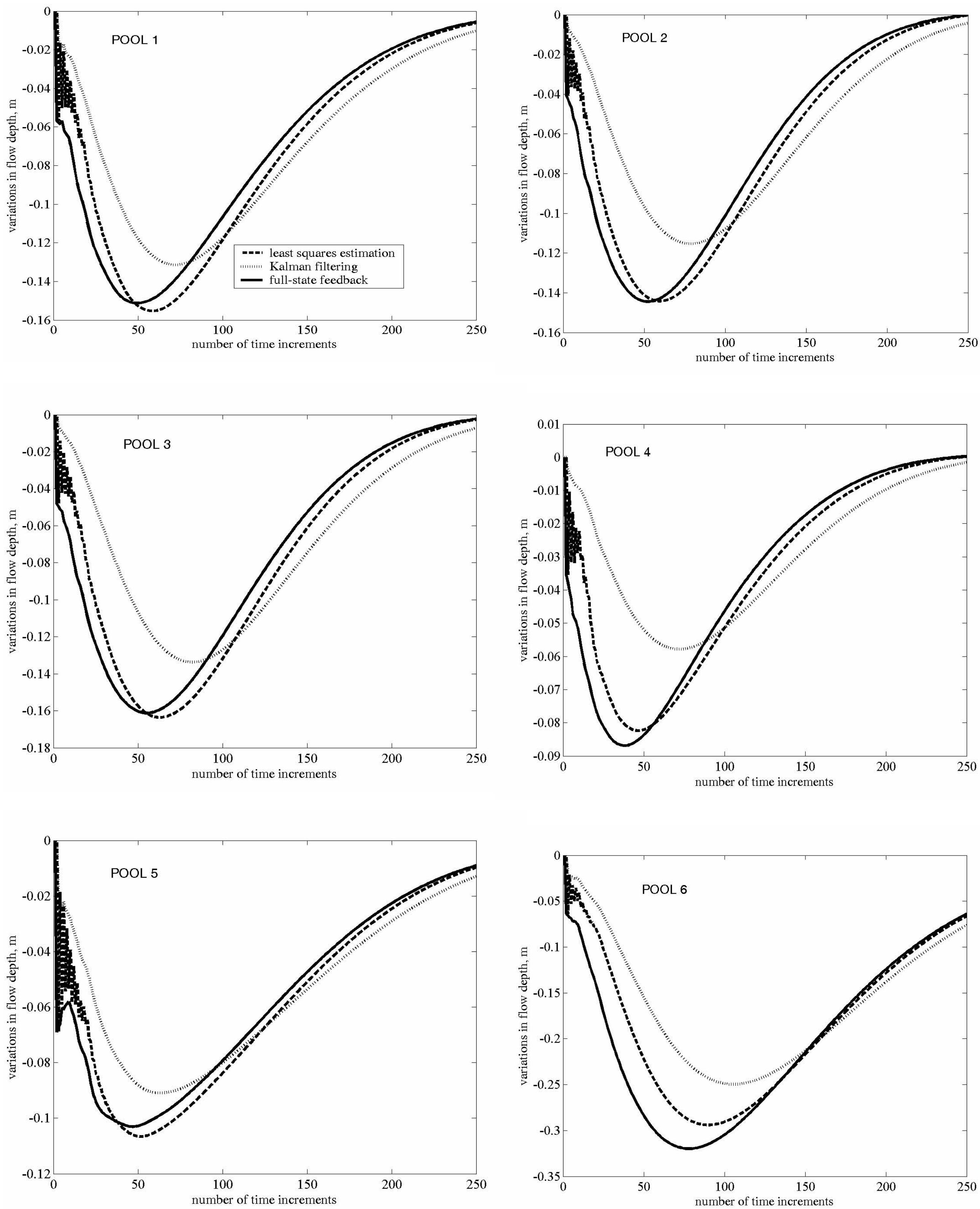

Figure 3. Comparison of flow depth variations for a full-state feedback regulator using Kalman filtering and a regulator using recursive least squares.

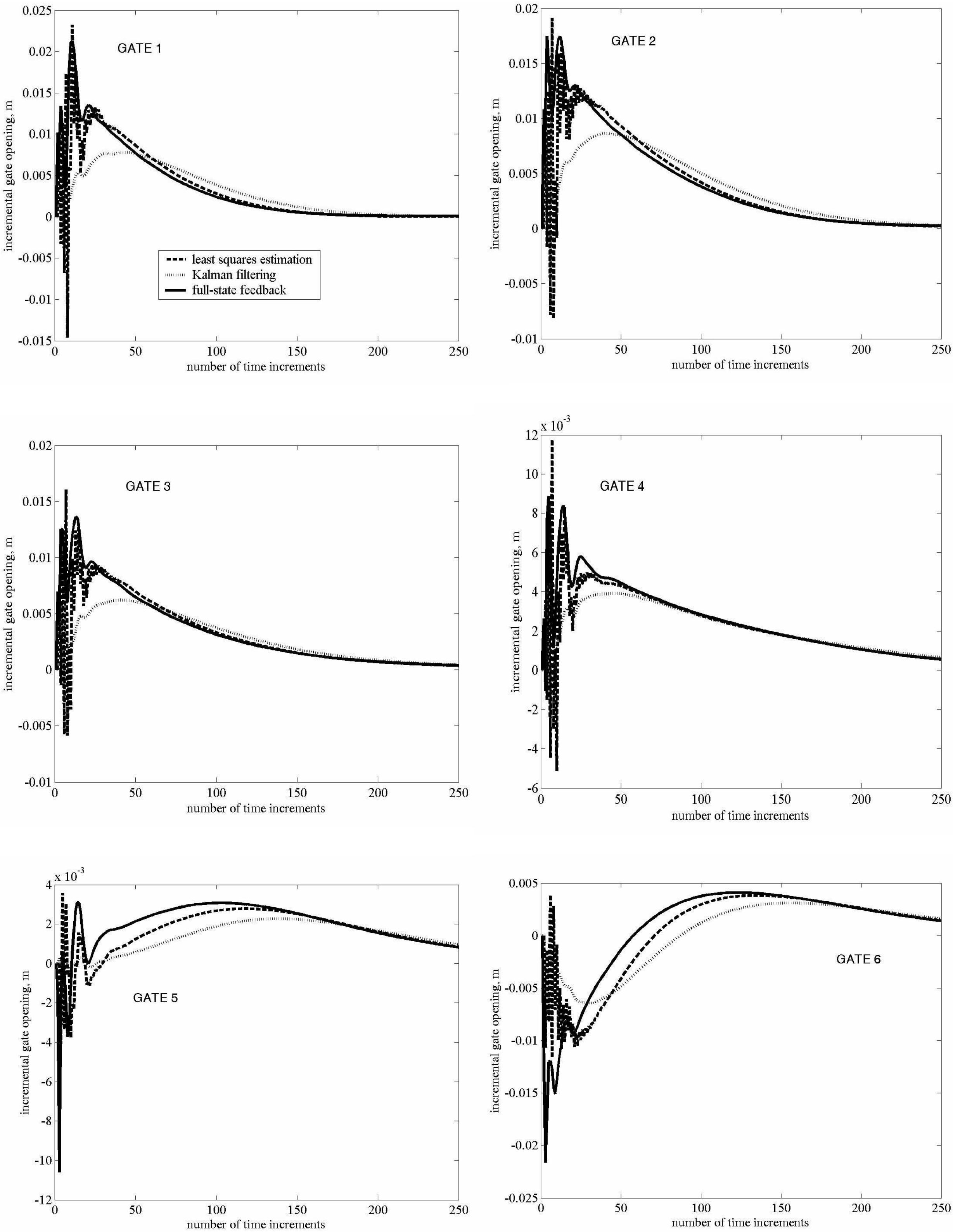

Figure 4. Comparison of incremental gate openings for a full-state feedback regulator using Kalman filtering and a regulator using recursive least squares.

Table 1. Data used in the simulation study.

parameter	pool 1	pool 2	pool 3	pool 4	pool 5	pool 6
length, m	9000	9000	9000	9000	9000	9000
width, m	5	5	5	5	5	5
bottom slope	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
side slope	1.5	1.5	1.5	1.5	1.5	1.5
initial lateral flow depth, m^3/s	2.5	2.5	2.5	2.5	2.5	2.5
initial downstream depth, m	4	3.2	2.86	2.5	2.12	1.81
parameter	**gate 1**	**gate 2**	**gate 3**	**gate 4**	**gate 5**	**gate 6**
width, m	5	5	5	5	5	5
discharge coefficient	0.8	0.8	0.8	0.8	0.8	0.8
initial opening, m	1.13	1.37	1.16	0.97	0.85	0.63
disturbances, m^3/s	2.5	2.5	2.5	2.5	2.5	2.5

Conclusions

In this paper, recursive least squares estimation has been employed in the control of multi-pool irrigation canals to estimate the state variables (flow depth and flow rate) at intermediate nodes based on the measured variables. The performance of the recursive least squares was compared to the performance of the optimal state feedback and the Kalman filter in terms of variations in the depths of flow and the upstream gate opening. Since the full-state feedback (assuming all state variables are measured) has the best robustness and stability properties, it was chosen as a target loop function. The results obtained from simulations indicate that the least squares algorithm provides good stability and performance in the control of irrigation canals. The algorithm is simpler than Kalman filtering in terms of the knowledge of covariance matrices required and provides an attractive alternative to Kalman filtering. Overall, the performance of the recursive least squares technique for constant-level control was better than the performance of the Kalman filter.

References

Durdu, O.F., Robust Control of Irrigation Canals, PhD Dissertation, Colorado State University, Civil Engineering Department, Hydraulic Division, Fort Collins, CO, USA, 2003.

Durdu, O.F., "Estimation of State Variables for Controlled Irrigation Canals via a Singular Value Based Kalman Filter", Fresenius Environmental Bulletin, 13(11), 2004.

El-Hawary, F., "A Comparison of Recursive Weighted Least Squares Estimation and Kalman Filtering for Source Dynamic Motion Evaluation", OCEANS '89. Proceedings, 4, 1082-1086, September 18-21, 1989.

Kwakernaak, H. and Sivan, R., Linear Optimal Control Systems. Wiley Publishers, New York, 1972.

Ljung, L. and Soderstrom, T., "Theory and Practice of Recursive Identification", MIT Press, Cambridge, MA, 1983.

Malaterre, P.O., "Multivariable Predictive Control of Irrigation Canals. Design and Evaluation on a 2-pool Model", International Workshop on Regulation of Irrigation Canals, Morocco, 230-238, 1997.

Reddy, J.M., "Local Optimal Control of Irrigation Canals", Journal of Irrigation and Drainage Engineering, 116, 616-631, 1990.

Reddy, J.M., Dia, A. and Oussou, A., "Design of Control Algorithm for Operation of Irrigation Canals", Journal of Irrigation and Drainage Engineering, 118, 852-867, 1992.

Reddy, J.M., "Kalman Filtering in the Control of Irrigation Canals", Int. J. Appl. Math. Modeling, 19(4), 201–209, 1995.

Reddy, J.M., "Simulation of Feedback Controlled Irrigation Canals", Proceedings USCID Workshop, Modernization of Irrigation Water Delivery Systems, 605-617, 1999.

Reddy, J.M. and Jacquot R.G., “Stochastic Optimal and Suboptimal Control of Irrigation Canals”, J. Water Resources Pln. and Mgt., 125, 369-378, 1999.

Seatzu, C., “Design of Decentralized Constant-Volume Controllers for Open-Channels by Solving a Least Squares Problem”, International Journal of Systems Science, 31(6), 759-770, June, 2000.

Seatzu, C. and Usai G., “A Decentralized Volume Variations Observer for Open-Channels”, Applied Mathematical Modelling, 26(10), 35-61, October 2002.

Tewari, A., Modern Control Design with Matlab and Simulink. Wiley Publishers, New York, 2002.