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Fuzzy Optimization of Geometrical Nonlinear Space Truss Design

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Abstract

This paper presents a general algorithm for nonlinear space truss system optimization with fuzzy constraints and fuzzy parameters. The analysis of the space truss system is performed with the ANSYS program. The algorithm of multiobjective fuzzy optimization techniques was formed with ANSYS parametric dimensional language. In the formulation of the design problem, weight and minimum displacement are considered the objective functions. Three design examples are presented to demonstrate the application of the algorithm.

Key words: Nonlinear space truss, Fuzzy sets, Membership function, Multiobjective, Fuzzy optimization, ANSYS.

Introduction

As the mathematical representation of everyday language, fuzzy set theory was first introduced by Zadeh (1965). The theory and methods of fuzzy programming have been developed. The primary works of fuzzy mathematical programming have been presented by Zimmermann (1978, 1983). Since then, several papers have appeared on linear fuzzy membership (Rao, 1987; Jung et al., 1996; Eshwar et al., 2004) and nonlinear fuzzy membership function (Dhingra et al., 1992; Kim, 1994; Shih, 2003) for structural optimization, used to represent the fuzzy nature of failure. This paper deals with the application of the nonlinear fuzzy membership function model to space truss system design problems.

An optimization problem is generally recognized to be nondeterministic as well as fuzzy in nature and the nondeterministic condition is not only in the design variables, it can also be in the allowable limits. One can use the expected value and the chance constrained programming technique (Rao, 1980) to transform the stochastic problem into its deterministic form. Thus we can substitute this form in the fuzzy mathematical formulation.

Many modeling, design, control and decision-making problems can be formulated in terms of mathematical optimization. The classical framework for the optimization is the minimization (or maximization) of the objectives, given the constraints for the problems to be solved. Many design problems, however, are characterized by multiple objectives. The first note on multicriterion optimization was by Rao; since then the determination of the compromise set of a multiobjective problem has become known as fuzzy optimization (Rao, 1987; Rao et al., 1992; Chen et al., 2000).

In this paper, an algorithm is developed for the multiobjective fuzzy optimum design of space trusses that takes into account geometrical nonlinearities. The optimum design algorithm developed is obtained by coupling nonlinear analysis techniques with fuzzy sets and fuzzy parameters. In the design a multiobjective fuzzy optimal decision is used. Objective functions, volume of the minimum weight and minimum displacement are considered in the numerical examples. The volume of construction weight, displacement, geometrical property, cross-sectional a reas, membership degrees, and upper and lower limit values of the stress elements are used as constraints. This paper shows that multiobjective λ formulation of fuzzy engineering systems can be used for optimum design.

Multiobjective Fuzzy Optimization Method Developed by ANSYS Programming

Fuzzy optimization

A fuzzy nonlinear programming problem associates fuzzy input data with fuzzy membership functions. A fuzzy nonlinear programming model assumes that objectives and constraints in an imprecise and uncertain situation can be represented by fuzzy sets. The fuzzy objective function can be maximized or minimized. In fuzzy nonlinear programming the fuzziness of available resources is characterized by the membership functions over the tolerance range. In the present study objective functions are considered as fuzzy sets and inflows are considered in the form of chance constraints.

Fuzzy approach for multiobjective optimization

One can define a multiobjective fuzzy vector $f(x)$ dependent in the design variable vector $x$as follows:

$$\min f(x) = \{f_1(x), f_2(x), ..., f_k(x)\}^T \qquad (1)$$

subject to the design constraints:

$$g(\underline{x}) \underline{\leq} b_j^u, \qquad j = 1, 2, ..., m-1 \qquad (2)$$

$$g(\underline{x}) \underline{\geq} b_j^l, \qquad j = m, ..., p, \qquad (3)$$

where the wave symbols indicate that the constraints contain fuzzy information, and b_j^l and b_j^u are the allowable upper and lower limits of the jth constraint, respectively.

The membership function $\mu_j(x)$ of the fuzzy allowable interval may be characterized as shown in Figure 1, where b_j^l and b_j^u are respectively the lower and upper limits of the allowable interval for the highest design level. These may even be more strict than the specification codes. d_j^l and d_j^u are the lengths of the transition stages, namely the permissible deviations or tolerances for the lower and upper limits.

$$g(\underline{x}) \ \leq \ b_j^u + d_j^u, \qquad j = 1, 2, ..., m-1 \qquad (4)$$

$$g(\underline{x}) \ \geq \ b_j^l - d_j^l, \qquad j = m, ..., p \qquad (5)$$

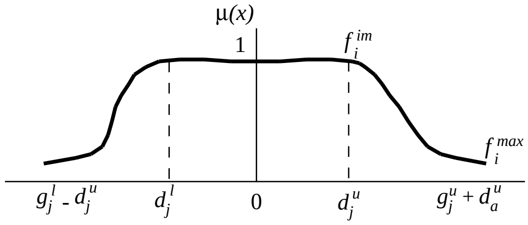

Figure 1. Nonlinear membership functions.

A membership μ_{fi} function corresponding to each design objective is constructed by the following:

$$\mu_{fi}(x) = \begin{cases} 0, & \text{if} \quad f_i(x) > f_i^{\max} \\ \lambda_i, & \text{if} \quad f_i^{\min} < f_i(x) \leq f_i^{\max} \\ 1, & \text{if} \quad f_i(x) \leq f_i^{\min} \end{cases} \qquad (6)$$

The minimum and maximum possible values of the design criteria in the continuous space are represented as $f_i^{\min}$ and $f_i^{\max}$ respectively. The maximum value of λ makes the fuzzy decision maximum.

$$\lambda_i = [f_i^{\max} - f_i(x)]/[f_i^{\max} - f_i^{\min}] \quad i = 1, 2, ..., k \qquad (7)$$

The Mathematical Models of Three Fuzzy Approaches

1-Method of the product operator

This is assumed to correspond to the logical "and". The mathematical formulation is expressed as

$$\max f(x) = \prod_{i=1}^{k} \mu_{fi}(x) \qquad (8)$$

subject to Eqs. (2) and (3). The "maximum satisfaction" can be achieved by solving this model.

2-Method of the addition operator

This is simply maximizes the sum of the membership function of design objectives. The mathematical formulation is represented as

$$\max f(x) = \sum_{i=1}^{k} \mu_{fi}(x) \tag{9}$$

and Eqs. (2) and (3).

3-Method of the min operator

The maximum value of λ_i in Eq. (7) makes the fuzzy decision maximum. The decision can be defined as the intersection of the fuzzy sets describing the constraints and the objective functions. The mathematical expression is

$$\max f(x) = \lambda_i \tag{10}$$

subject to

$$\lambda_i \le \mu_{f_i}(x), \qquad i = 1, 2..., k \tag{11}$$

The maximum degree of "overall satisfaction" can be achieved by maximizing a scalar λ_i. By using any of the above transformations, a multiobjective problem is easily converted to a single objective problem.

General objective fuzzy optimization algorithm

An algorithm that achieves optimum design of geometrical nonlinear space truss systems by the ANSYS program was written. Fuzzy sets are added to the algorithm developed and multiobjective fuzzy optimization of nonlinear space truss systems is realized.

The steps of the ANSYS based algorithm are given below:

Step 1. Geometrical nonlinear analysis is chosen by entering the initial cross-section. The elasticity modulus of the group and elements, and the coordinates of nodes of the bars are written.

Step 2. Analysis type is determined as static and the freedom degree of the nodes and external vectors at the nodes are written.

Step 3. The volume of the structure and maximum and minimum values of the displacements obtained with respect to the upper limits of the cross sections by classical optimization are written from Eq. (6). Then axial displacements are arranged at the free points or nodes of the system and maximum and minimum values of the displacements and structure volume are entered into Eq. (7); thus the equations of λ_1 and λ_2 membership functions are formed (set up). Finally, λ value conditions, which ensure the equivalency of these equations, are written.

Step 4. The upper and lower limits of the state variables and fuzzy dimensioning variables are written. λ_1 and λ_2, the membership degrees, are defined as constraints of the objective functions as the state variables. Equating these parameters, and finding many λ parameters, it is possible to achieve the optimum fuzzy decision.

Numerical Examples

Design of 9-bar space truss

The design of the 9-bar space truss shown in Figure 2 is considered with the objective of minimizing weight and the sum of deflection of nodes 1 and 2. The loading of the truss and the upper bounds for the displacements of the restricted joints are given in Table 1. The members of the space truss are collected in 3 groups. The minimum cross-sectional area for members is chosen as 2 cm^2. The modulus of elasticity is taken as 2.06×10^4 kN / cm^2.

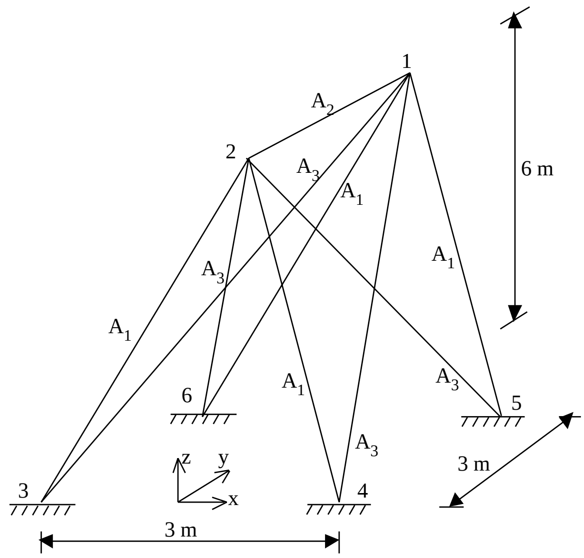

Figure 2. 9-bar space truss.

Table 1. The loading and displacement bounds for 9-bar space truss system.

Joint number	Loading (kN)			Displacement limitation (cm)	
	X	Y	Z	X	Y
1	80	0	-32	0.2	0.2
2	-80	48	-32	0.2	0.2

Objective functions;

$$\min \begin{cases} W(x) = \sum_{i=1}^{9} \rho A_i \ell_i \\ \delta(x) = \sum_{i=1}^{2} \sqrt{\delta_{ix}^2 + \delta_{iy}^2 + \delta_{iz}^2} \end{cases} \quad (12)$$

The design variables are bounded as $A_i^{(l)} \leq A_i \leq A_i^{(u)}$; $i = 1, 2, 3$ where the limiting values are taken as $A_i^{(l)} = 2.0$ cm^2, $(i = 1, 2, 3)$, $A_i^{(u)} = 10$ cm^2, $(i = 1, 2, 3)$.

From the results obtained by the classical optimization with the upper boundary values of the variables, $W^{\max} = 55,234$ cm^3, $W^{\min} = 27,760$ cm^3, $\delta^{\max} = 4.70$ cm, $\delta^{\min} = 2.40$ cm are found.

We enter these values into Eqs. (6) and (10).

$$\mu_{f_1}(x) = \begin{cases} 0. & W_i > 55,234 \\ \left(\frac{55,234 - W_i}{55,234 - 27,760}\right) & 27,760 < W_i \leq 55,234 \\ 1. & W_i \leq 27,760 \end{cases} \quad (13)$$

$$\mu_{f_2}(x) = \begin{cases} 0. & \delta_j > 4.70 \\ \left(\frac{4.70 - \delta_j}{4.70 - 2.40}\right) & 2.40 < \delta_j \leq 4.70 \\ 1. & \delta_j \leq 2.40 \end{cases} \quad (14)$$

$$\lambda_1 = [55,234 - W_i] \ / \ [55,234 - 27,760] \quad (15)$$

$$\lambda_2 = [4.70 - \delta_j] \ / \ [4.70 - 2.40] \quad (16)$$

The equations of λ_1 and λ_2 membership functions will enable us to achieve the optimum fuzzy decision by finding many λ parameters, which ensure equivalency. The results of fuzzy optimization are shown in Table 2.

A flow diagram of the λ formulation approach in fuzzy multiobjective optimization is described in Figure 3.

Design of 120-bar space truss

The second example is a 120-bar nonlinear space truss whose members are collected in 7 groups as shown in Figure 4. Angle sections are adopted for members. The loading of the truss and the upper bounds for the displacements of the restricted joints are given in Table 3. The modulus of elasticity and the minimum member cross-sectional area are taken as 2.06×10^4 kN/cm^2 and 2 cm^2, respectively. The result of optimum design is shown in Table 4.

$$\min \begin{cases} W(x) = \sum_{i=1}^{120} \rho A_i \ell_i \\ \delta(x) = \sqrt{\delta_{1x}^2 + \delta_{1y}^2 + \delta_{1z}^2} \end{cases} \quad (17)$$

Table 2. Multiobjective fuzzy optimization solutions of 9-bar space truss system.

	Fuzzy parameter	Areas (cm^2)			Displacement (cm)				minδ	min W
	λ^*	A_1	A_2	A_3	δ_{x1}	δ_{y1}	δ_{x2}	δ_{y2}	(cm)	(cm^3)
Linear	0.5951	7.99	2.04	6.72	1.59	0.41	1.59	0.60	3.33	38,846
Nonlinear	0.5970	7.95	2.05	6.74	1.58	0.42	1.58	0.59	3.37	38,808

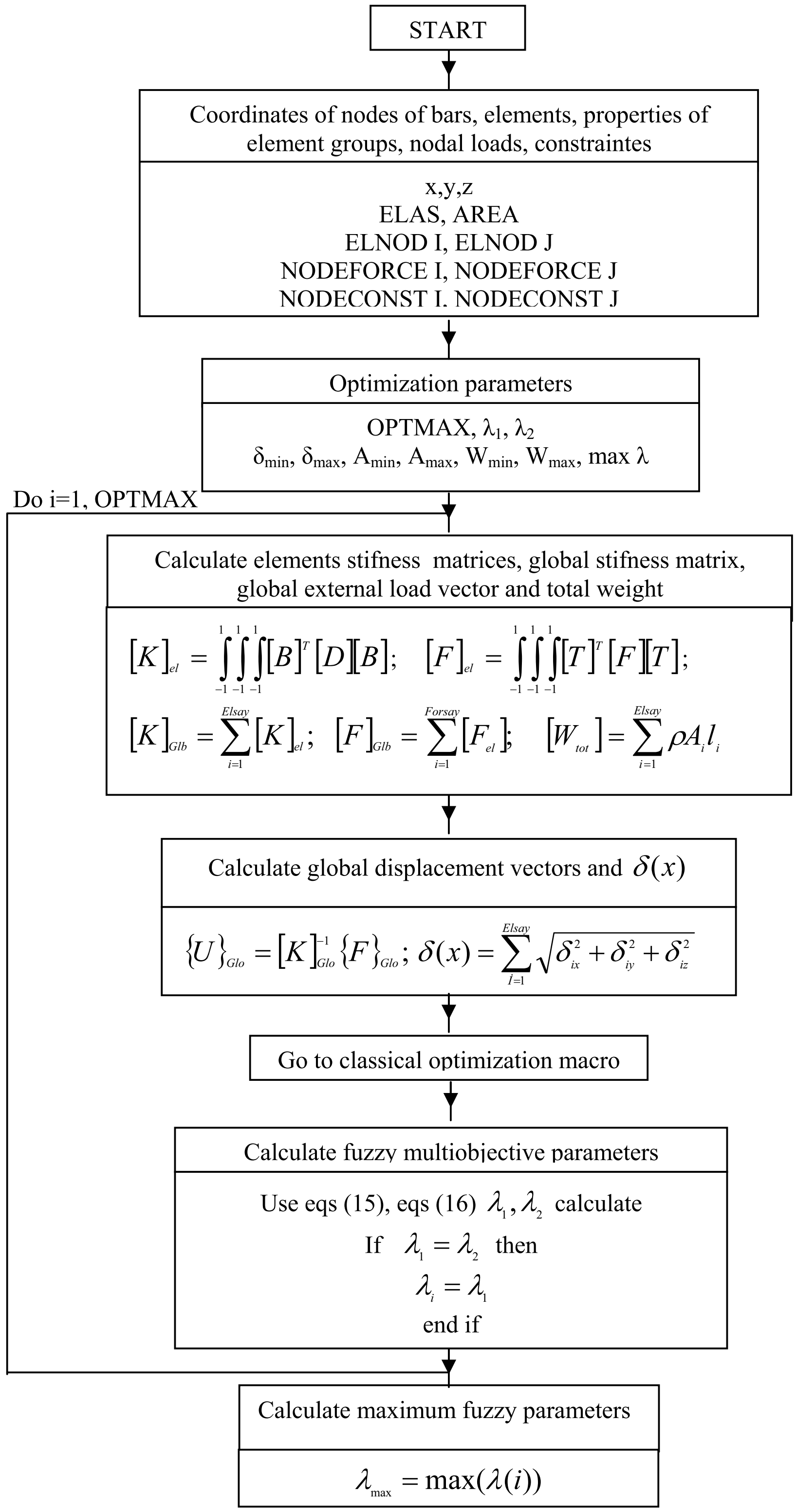

Figure 3. Flow diagram of a λ-formulation for a multiobjective fuzzy optimization problem.

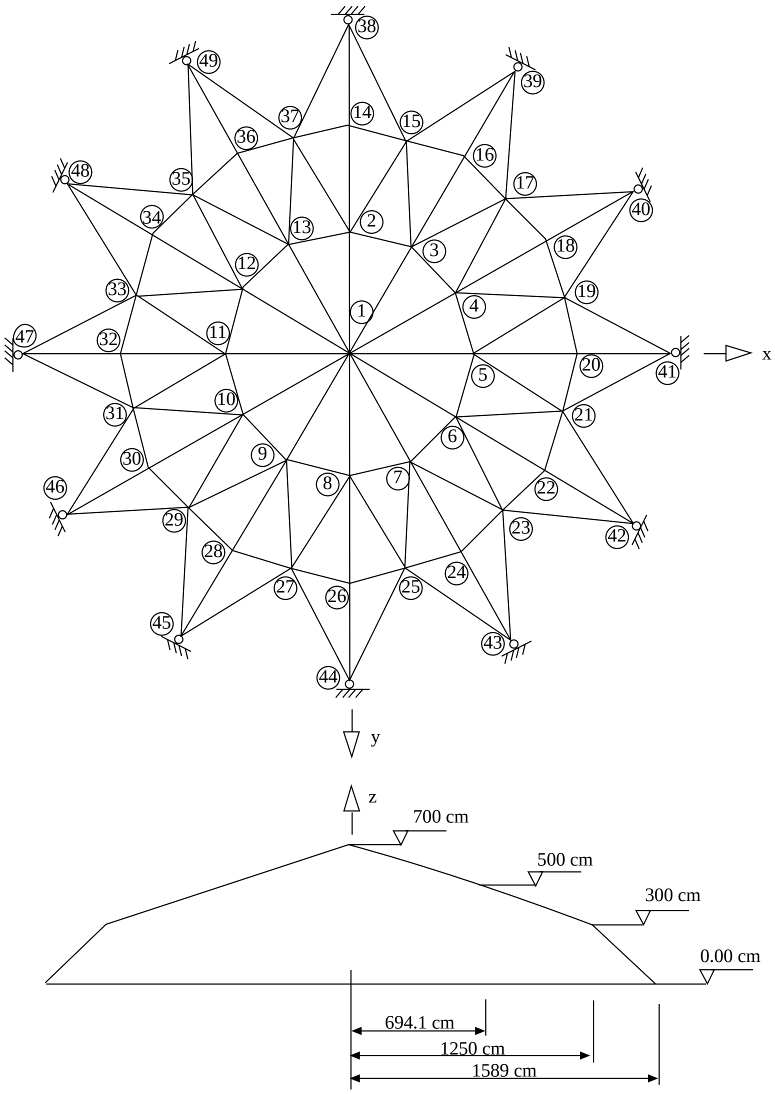

Figure 4. 120-bar space truss.

Table 3. The loading and displacement bounds for 120-bar space truss system.

Joint number	(kN)			Displacement limitation (cm)
	X	Y	Z	Z
1	0	0	60	1
2	0	0	30	1
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
14	0	0	30	1
15	0	0	10	1
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
37	0	0	10	1

Table 4. Multiobjective fuzzy optimization solutions of 120-bar space truss system.

	Fuzzy parameter	Areas (cm^2)							minδ	min W
	λ^*	A_1	A_2	A_3	A_4	A_5	A_6	A_7	(cm)	(cm^3)
Linear	0.4354	36.17	50.00	27.81	34.99	28.40	40.15	34.87	0.52	2,175,715
Nonlinear	0.7244	34.44	26.68	40.11	32.70	39.73	33.44	32.73	0.33	2,134,888

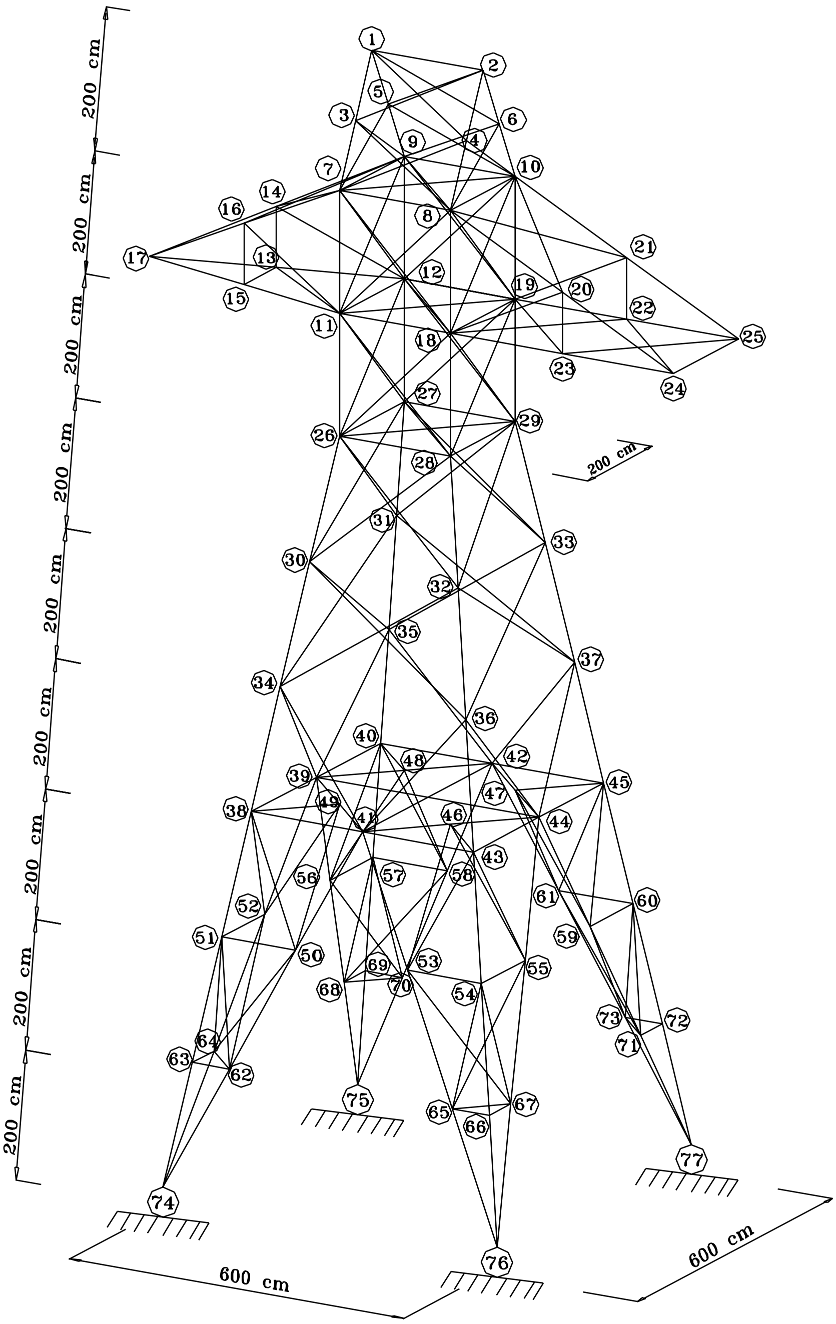

Figure 5. 244-bar space truss.

Design of 244-bar transmission tower

The design of a 244-bar transmission tower, shown in Figure 5, is considered as the last example. The members of this nonlinear space truss are combined in 32 groups. The modulus of elasticity is considered to be 2.06×10^4 kN/cm^2. The loading and bounds imposed on the displacements are given in Table 5. The minimum cross-sectional area for members is chosen as 2 cm^2. The results of fuzzy optimization are shown in Table 6.

$$\min\begin{cases} W(x)=\sum\limits_{i=1}^{244}\rho A_i\ell_i \\ \delta(x)=\sum\limits_{i=1}^{2}\sqrt{\delta_{ix}^2+\delta_{iy}^2+\delta_{iz}^2} \end{cases} \tag{18}$$

Table 5. The loading and displacement bounds for 244-bar space truss system.

Joint number	Loading (kN)		Displacement limitation (cm)	
	X	Z	X	Z
1	-10	-30	4.5	1.5
2	10	-30	4.5	1.5
17	35	-90	3	1.5
24	175	-45	3	1.5
25	175	-45	3	1.5

Table 6. Multiobjective fuzzy optimization solutions of 244-bar space truss system.

Design Variables	Linear	Nonlinear
A_1	10.07	2.04
A_2	10.45	3.07
A_3	10.65	2.40
A_4	10.24	2.04
A_5	10.88	7.55
A_6	13.20	5.83
A_7	10.68	4.17
A_8	35.60	36.44
A_9	37.36	66.70
A_{10}	10.38	2.06
A_{11}	13.74	45.60
A_{12}	10.26	2.13
A_{13}	10.43	2.01
A_{14}	12.30	9.95
A_{15}	10.30	2.07
A_{16}	9.57	2.12
A_{17}	10.03	2.01
A_{18}	9.30	62.57
A_{19}	10.07	2.04
A_{20}	100	100
A_{21}	21.24	19.83
A_{22}	13.37	4.43
A_{23}	10.54	2.28
A_{24}	10.19	2.03
A_{25}	10.61	2.65
A_{26}	10.27	2.05
A_{27}	10.38	2.07
A_{28}	10.64	2.11
A_{29}	10.54	3.05
A_{30}	10.70	2.12
A_{31}	10.57	4.47
A_{32}	10.13	2.05
max λ	0.7871	0.7943
$\min\delta(x)=\sum\limits_{i=1}^{2}\sqrt{\delta_{ix}^2+\delta_{iy}^2+\delta_{iz}^2}\ (cm)$	2.88	2.86
$\min W(x)=\sum\limits_{i=1}^{32}\rho A_i\ell_i\,(cm^3)$	1,709,688	1,679,200

Conclusions

The multiobjective optimization of fuzzy engineering systems is considered using λ formulation. An algorithm was developed by ANSYS programming to solve nonlinear space truss systems with fuzzy optimization. A comparative study of the 3 procedures is performed using 9-bar, 120-bar and 244-bar space truss optimization problems.

Optimization with fuzzy sets was seen to be faster for obtaining results and to require a smaller software. Objective functions were also added to the systems as constraints. Fuzzy set theory was seen to be suitable for the modeling of unstable and complex structures of the design problem. A design problem performing a single objective optimization was transformed into a multiobjective optimization problem using fuzzy sets. A certainty assumption must be provided for a problem able to be solved by classical optimization, yet this is not necessary for fuzzy optimization.

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