Turkish J. Eng. Env. Sci.
29 (2005) , 331 – 341.
© TÜBİTAK

Steady State Response of Viscoelastically Corner Point-Supported Generally Orthotropic Rectangular Plates under the Effect of Sinusoidally Varying Moment

Turgut KOCATÜRK
Yıldız Technical University, Faculty of Civil Engineering
İstanbul-TURKEY
e-mail: kocaturk@yildiz.edu.tr
Semih SEZER, Cihan DEMİR
Yıldız Technical University, Faculty of Mechanical Engineering
İstanbul-TURKEY

Received 13.05.2005

Abstract

The vibration of generally orthotropic rectangular elastic plates having viscoelastic point supports at the corners is analyzed. Lagrange's equations are used to examine the free vibration characteristics and steady state response to a sinusoidally varying moment affecting the center of a viscoelastically point-supported, generally orthotropic elastic plate of rectangular shape. For applying the Lagrange's equations, the trial function denoting the deflection of the plate is expressed in polynomial form. By using the Lagrange's equations, the problem is reduced to the solution of a system of algebraic equations. The influence of the off-axis angle, of the mechanical properties, and of the damping of the supports to the steady state response of the viscoelastically point-supported rectangular plates is investigated numerically for a concentrated moment at the center for various values of the mechanical properties characterizing the anisotropy of the plate material, for various off-axis angles and for various damping of supports for a given stiffness of supports. The results are given for the considered frequency range of the external periodical moment. Convergence studies are performed. The validity of the results obtained is demonstrated by comparing them with the solutions of specially orthotropic plates based on the Kirchhoff-Love plate theory.

Key words: Viscoelastic point-supports, Elastic point-supports, Generally orthotropic plates, Viscoelastically point-supported plates, Steady state response.

Introduction

Free and forced vibrations of point-supported rectangular or square plates are of considerable interest to engineers designing panels at isolated points. These problems are encountered in various engineering applications from printed circuit boards in electronics to the plates used in naval and ocean engineering systems. Therefore, the vibration problems of these plates are of practical importance. Free vibration analysis of rectangular plates supported at various points and based on the Kirchhoff-Love plate theory has been performed by many researchers and is well known (for example Narita (1984), Venkateswara Rao et al. (1973), Kocatürk and İlhan (2003)). However, it appears that there are only a limited number of studies on the steady state response of viscoelastically point-supported plates.

Although there are many studies on the free vibration analysis of rectangular plates supported at various points, there are only a limited number of studies on the steady state response of point-supported rectangular plates. The steady state response to a sinusoidally varying force was determined for a viscoelastically point-supported square or rectangular plate by Yamada et al. (1985), using the generalized Galerkin method. A generalization of this study to specially orthotropic rectangular plates was investigated by Kocatürk (1998), and Kocatürk and Altıntaş (2003a, 2003b). The steady state response to sinusoidally varying moment was determined for a viscoelastically point-supported specially orthotropic square or rectangular plate by Kocatürk et al. (2004), using the Lagrange's equations with trial functions denoting the displacements of the plate.

In the present study, Lagrange's equations are used to examine the free vibration characteristics and steady state response to sinusoidally varying moment affecting the center of a viscoelastically point-supported generally orthotropic elastic plate of rectangular shape. By analyzing the steady state response of the considered problem, the peak values of the moment transmissibilities are obtained.

Analysis

Consider a viscoelastically point-supported rectangular elastic generally orthotropic plate of side lengths a and b and thickness h under effect of sinusoidally varying moment $M(t)$ at the center of the plate as shown in Figure 1, where $X_1X_2X_3$ is the principal material coordinate system, $X_1'X_2'X_3'$ is the geometric coordinate system, θ is the off-axis angle, k_i is the spring constant, c_i is the damping coefficient, and $P_i\,(X_{1i}', X_{2i}')$ is the support force of a point support at the i th support. The axes of the elastic symmetry of the plate material form an angle θ with the OX_1' and OX_2' axes. Therefore the plate is generally orthotropic. The coordinate axes OX_1' and OX_2' are oriented along the edges of the plate with the origin at O. Because the plate is generally orthotropic and the supports are viscoelastic, there are many parameters to be considered. Therefore, although it is possible to take many point supports at arbitrary points, in the numerical investigations here, for brevity, it will be considered that the plate is supported symmetrically at the 4 corner points and k_i and c_i are taken to have the same respective values at all the supports denoted by $k_i = k$ and $c_i = c$. Under the above conditions, the steady state responses of the viscoelastically corner point supported plate to a sinusoidally varying moment for various damping values will be determined using the Lagrange's equations.

For a plate undergoing sinusoidally varying moment $M(t) = Q.\, e^{i\omega t}$, where ω is the frequency, the strain energy of bending in Cartesian coordinates is given by

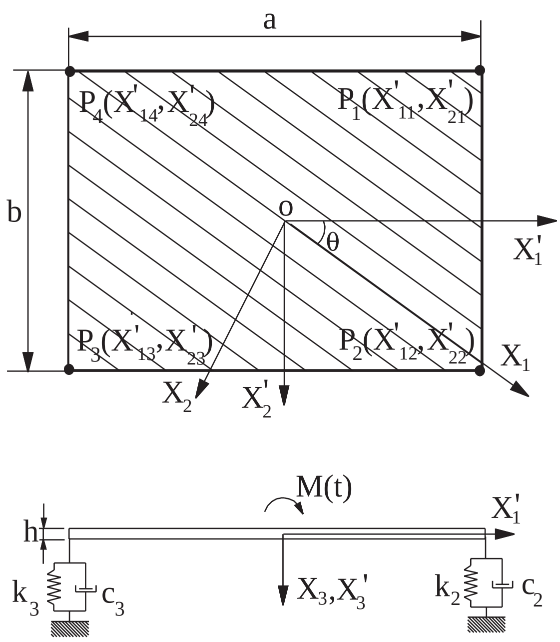

Figure 1 Viscoelastically point-supported rectangular elastic generally orthotropic plate under a sinusoidally varying moment.

$$U=\frac{1}{2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}[\bar{D}_{11}\left(\frac{\partial^2\bar{W}}{\partial X_1'^2}\right)^2+2\bar{D}_{12}\frac{\partial^2\bar{W}}{\partial X_1'^2}\frac{\partial^2\bar{W}}{\partial X_2'^2}$$

$$+D_{22}\left(\frac{\partial^2\bar{W}}{\partial X_2'^2}\right)^2+4D_{16}\frac{\partial^2\bar{W}}{\partial X_1'^2}\frac{\partial^2\bar{W}}{\partial X_1'\partial X_2'}$$

$$+4D_{26}\frac{\partial^2\bar{W}}{\partial X_2'^2}\frac{\partial^2\bar{W}}{\partial X_1'\partial X_2'}+4D_{66}\left(\frac{\partial^2\bar{W}}{\partial X_1'\partial X_2'}\right)^2]dX_1'dX_2' \qquad (1)$$

In Eq. (1), $\bar{D}_{11}$, $\bar{D}_{12}$, $\bar{D}_{16}$, $\bar{D}_{22}$, $\bar{D}_{26}$, $\bar{D}_{66}$ are expressed as follows:

$$\bar{D}_{ij}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\bar{Q}_{ij}^{k}(z^2)dz$$

$$\begin{aligned}\bar{D}_{11} &= \frac{\bar{Q}_{11}\,h^3}{12}, \bar{D}_{16} = \frac{\bar{Q}_{16}\,h^3}{12}\\ \bar{D}_{12} &= \frac{\bar{Q}_{11}\,h^3}{12}, \bar{D}_{26} = \frac{\bar{Q}_{11}\,h^3}{12}\\ \bar{D}_{22} &= \frac{\bar{Q}_{22}\,h^3}{12}, \bar{D}_{66} = \frac{\bar{Q}_{66}\,h^3}{12}\end{aligned} \tag{2}$$

where the components of the reduced stiffness matrix $\bar{Q}_{ij}$ are defined by Choo (1990) as follows:

$$\begin{aligned}\bar{Q}_{11} &= Q_{11}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{22}s^4\\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(s^4 + c^4)\\ \bar{Q}_{22} &= Q_{11}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{22}c^4\\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})c^3s + (Q_{12} - Q_{22} + 2Q_{66})s^3c\\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})s^3c + (Q_{12} - Q_{22} + 2Q_{66})c^3s\\ \bar{Q}_{66} &= (Q_{11} - 2Q_{12} + Q_{22} - 2Q_{66})s^2c^2 + Q_{66}(c^4 + s^4)\end{aligned} \tag{3}$$

$$s = \sin\theta\,; \quad c = \cos\theta$$

where

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}} \tag{4}$$

$$Q_{66} = G_{12}$$

$$\nu_{21}E_{11} = \nu_{12}E_{22}$$

where G_{12} is the shear modulus in the X_1X_2 plane, E_{11} and E_{22} are Young's moduli in the OX_1 and OX_2 directions, respectively, and ν_{21} is the Poisson's ratio for the strain response in the X_1 direction due to an applied stress in the X_2 direction. The potential energy of the external moment is

$$F_e = -M\,(t)\,\frac{\partial W\,(0,0,t)}{\partial\, X_1} \tag{5}$$

With rotary inertia neglected, the kinetic energy of the vibrating plate is

$$T = \frac{1}{2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}} \rho\,h\,\left(\frac{\partial \overset{?}{W}}{\partial t}\right)^2 dX_1'\,dX_2' \tag{6}$$

where ρ is the mass density per unit volume and the additive strain energy and dissipation function of viscoelastic supports are

$$\begin{aligned}F_s &= \tfrac{1}{2}\sum_{i=1}^{4} kW_{Si}^2\\ D &= \tfrac{1}{2}\sum_{i=1}^{4} c(\dot{W}_{Si})^2\end{aligned} \tag{7}$$

Introducing the following non-dimensional parameters

$$x_1 = \frac{X_1'}{a}\,, \quad x_2 = \frac{X_2'}{b}\,, \alpha = \frac{b}{a}, \quad e = \frac{E_{22}}{E_{11}},\quad \bar{w}(x_1, x_2, t) = W/a \tag{8}$$

and assuming the shear modulus, G_{12}, as given by Szilard (1974) as follows;

$$G_{12} \approx \frac{E_{11}\sqrt{e}}{2\left(1 + \nu_{21}\sqrt{1/e}\right)} \tag{9}$$

the above energy expressions can be written at time t as follows:

$$U = \frac{D_{11}}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}}[\,\alpha(c^4 + 2(\nu_{12}e + 2\frac{G_{12}}{E_{11}}(1 - \nu_{12}\nu_{21}))s^2c^2 + es^4)\left(\frac{\partial^2 w}{\partial x_1^2}\right)^2 + \frac{2}{\alpha}(1 + e - 4\frac{G_{12}}{E_{11}}(1 - \nu_{12}\nu_{21}))s^2c^2$$

$$\nu_{12}e(s^4+c^4))\left(\frac{\partial^2 w}{\partial x_1^2}\right)\left(\frac{\partial^2 w}{\partial x_2^2}\right)+\frac{1}{\alpha^3}(s^4+2(\nu_{12}e+2\frac{G_{12}}{E_{11}}(1-\nu_{12}\nu_{21}))s^2c^2+ec^4)\left(\frac{\partial^2 w}{\partial x_2^2}\right)^2$$
$$+4((1-\nu_{12}e-2\frac{G_{12}}{E_{11}}(1-\nu_{12}\nu_{21}))c^3s+(\nu_{21}e-e+2\frac{G_{12}}{E_{11}}(1-\nu_{12}\nu_{21}))s^3c)\left(\frac{\partial^2 w}{\partial x_1^2}\right)\left(\frac{\partial^2 \bar{w}}{\partial x_1\partial x_2}\right)+\frac{4}{\alpha^2}((1-\nu_{12}e$$
$$-2\frac{G_{12}}{E_{11}}(1-\nu_{12}\nu_{21}))s^3c+(\nu_{12}e-e+2\frac{G_{12}}{E_{11}}(1-\nu_{12}\nu_{21}))c^3s)\left(\frac{\partial^2 w}{\partial x_2^2}\right)\left(\frac{\partial^2 \bar{w}}{\partial x_1\partial x_2}\right)+\frac{4}{\alpha}(1-\nu_{12}e+e$$
$$-2\frac{G_{12}}{E_{11}}(1-\nu_{12}\nu_{21}))s^2c^2+\frac{G_{12}}{E_{11}}(1-\nu_{12}\nu_{21})(s^4+c^4))\left(\frac{\partial^2 w}{\partial x_1\partial x_2}\right)^2]e^{iwt}dx_1dx_2 \tag{10a}$$

$$T=\frac{h.\rho.a.b.\omega^2.\alpha^2}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} w^2.e^{i\omega t}dx_1\,dx_2 \tag{10b}$$

$$F_s=\frac{a^2}{2}\sum_{i=1}^{4}k\,\bar{w}_i^2 \quad D=\frac{a^2}{2}\sum_{i=1}^{4}c(\dot{\bar{w}}_i)^2 \quad F_e=-a\,M\,(t)\;\frac{\partial\bar{w}\,(0,0,t)}{\partial x_1} \tag{10c-e}$$

It is known that some expressions satisfying geometrical boundary conditions are chosen for $\bar{w}\,(x_1,x_2,t)$ and by using the Lagrange's equations the natural boundary conditions are also satisfied. By using the Lagrange's equations, by assuming the displacement $\bar{w}\,(x_1,x_2,t)$ to be representable by a linear series of admissible functions and adjusting the coefficients in the series to satisfy the Lagrange's equations, an approximate solution is found for the displacement function. For applying the Lagrange's equations, the trial function $\bar{w}\,(x_1,x_2,t)$ is approximated by space-dependent polynomial terms $x_1^0,\,x_1^1,x_1^2,....,x_1^M$ and $x_2^0,\,x_2^1,x_2^2,....,x_2^N$, and time-dependent generalized displacement coordinates $\bar{A}_{mn}\,(t)$. Thus

$$\bar{w}\,(x_1,\,x_2,t)=\sum_{m=0}^{M}\sum_{n=0}^{N}\bar{A}_{mn}\,(t)\,x_1^m x_2^n \tag{11}$$

where $\hat{w}(x_1,x_2,t)$ is the steady state response (the transverse deflection) of the plate to sinusoidally varying moment $M(t)=Q.\;e^{i\omega t}$. Each term, x_1^m and x_2^n, must satisfy the geometrical boundary conditions. However, in the considered problem, there is no geometrical boundary condition to be satisfied. There is no need for these functions to satisfy the natural boundary conditions. However, if the natural boundary conditions are also satisfied when selecting the functions, then the rate of convergence will be high.

The function $\bar{w}\,(x_1,x_2,t)$, which is given by Eq. (11), is substituted in Eqs. (10a-e). Then the application of Lagrange's equations yields a set of linear algebraic equations. Lagrange's equations for the considered problem are given as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\,\dot{\bar{A}}_{k\,l}}\right)-\frac{\partial\,(T-U)}{\partial\,\bar{A}_{k\,l}}+\frac{\partial\,D}{\partial\,\dot{\bar{A}}_{k\,l}}+\frac{\partial F_s}{\partial\bar{A}_{k\,l}}$$
$$+\frac{\partial F_e}{\partial\,\bar{A}_{k\,l}}=0\,;\quad k,\,l=0,1,2,...,N \tag{12}$$

where the overdot stands for the partial derivative with respect to time. Introducing the following non-dimensional parameters,

$$\kappa=\frac{k\,a^3}{b\,D_{11}}\,,\quad \gamma=\frac{c\,a}{b\sqrt{\rho\,h\,D_{11}}}\,,$$
$$\lambda^2=\frac{\rho\,h\,\omega^2a^4}{D_{11}}\,,\;\; q=\frac{Q\,a}{D_{11}} \tag{13}$$

and considering that when the moment is expressed as $M(t)=Q.\;e^{i\omega t}$, then the time-dependent generalized functions can be expressed as follows:

$$\bar{A}_{mn}(t)=A_{mn}e^{i\,\omega\,t} \tag{14}$$

In Eq. (14), A_{mn} is a complex variable containing a phase angle. The dimensionless complex amplitude of the displacement of a point of the plate can be expressed as

$$w(x_1, x_2) = \sum_{m=0}^{M} \sum_{n=0}^{N} A_{mn}\, x_1^m\, x_2^n \tag{15}$$

By using Eq. (12), the following set of linear algebraic equations is obtained, which can be expressed in the following matrix form

$$[A]\{A_{mn}\} + i\lambda\gamma[B]\{A_{mn}\} - \lambda^2[C]\{A_{mn}\} = \alpha\{q\} \tag{16}$$

where $[A]$, $[B]$ and $[C]$ are coefficient matrices obtained by using Eq. (12).

For free vibration analysis, when the external force and damping of the supports are zero in Eq. (18), this results in a set of linear homogeneous equations that can be expressed in the following matrix form:

$$[A]\{A_{mn}\} - \lambda^2[C]\{A_{mn}\} = \{0\} \tag{17}$$

By increasing the polynomial terms, the accuracy can be increased.

The maximum moment caused by the couple of the reaction forces of the supports is given by

$$\begin{aligned} M_{r\ \max} = {} & \tfrac{a^2}{2}(k + i\,c\,\omega) \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn}\, x_{11}^m\, x_{21}^n + \tfrac{a^2}{2}(k + i\,c\,\omega) \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn}\, x_{12}^m\, x_{22}^n \\ & - \tfrac{a^2}{2}(k + i\,c\,\omega) \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn}\, x_{13}^m\, x_{23}^n - \tfrac{a^2}{2}(k + i\,c\,\omega) \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn}\, x_{14}^m\, x_{24}^n \end{aligned} \tag{18}$$

and therefore the moment transmissibility is determined by

$$\begin{aligned} T_M = \sum_{j=1}^{4} M_{r\ \max}/Q = {} & \tfrac{1}{2(\alpha q)}(\kappa + i\gamma\,\lambda) \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn}\, x_{11}^m\, x_{21}^n + \tfrac{1}{2(\alpha q)}(\kappa + i\gamma\,\lambda) \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn}\, x_{12}^m\, x_{22}^n \\ & - \tfrac{1}{2(\alpha q)}(\kappa + i\gamma\,\lambda) \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn}\, x_{13}^m\, x_{23}^n - \tfrac{1}{2(\alpha q)}(\kappa + i\gamma\,\lambda) \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn}\, x_{14}^m\, x_{24}^n \end{aligned} \tag{19}$$

The number of unknown coefficients is $(M+1) \times (N+1)$. Again, the number of equations that can be written for each A_{mn} coefficient by using Eq. (12) is $(M+1) \times (N+1)$, which is given in matrix form by Eq. (16). Therefore, the total number of these equations is equal to the total number of unknown displacements and these unknowns can be determined by solving the equations above.

The eigenvalues (characteristic values) λ are found from the condition that the determinant of the system of equations given by Eq. (17) must vanish.

Numerical Results

The steady state response to sinusoidally varying moment $M(t) = Q.\ e^{i\omega t}$ acting at the center of a generally orthotropic square plate, viscoelastically point-supported at the 4 corners, is calculated numerically. Because of the structural symmetry and symmetry of the external force, only symmetrical vibrations arose in the studies by Kocatürk and İlhan (2003), Yamada et al. (1985) and Kocatürk (1998); therefore it was possible to reduce the number of polynomial terms. However, because the principal material coordinate system $X_1X_2X_3$ does not coincide with the geometric coordinate system, $X_1'X_2'X_3'$, in the case of a generally orthotropic viscoelastically point-supported plate, there is no symmetry property to reduce the number of polynomial terms.

A short investigation of the free vibration of an elastically point-supported generally orthotropic plate is made for $\theta = 0$ for comparing the obtained results with the existing results of the elastically point-supported specially orthotropic plate. The natural frequencies of the elastically point-supported generally orthotropic plate are determined by calculating the eigenvalues λ of the frequency Eq. (19).

In Table 1, the calculated frequency parameters

λ, which are in the considered frequency range of the external force, are compared with those given by Kocatürk and İlhan (2003). Furthermore, the convergence is tested in the table by taking various numbers of terms as given in Table 1. It is seen that the present converged values show excellent agreement with those of Kocatürk and İlhan (2003).

It is shown that the convergence with respect to the number of the polynomial terms is excellent in the considered cases. As observed from Table 1, the frequency parameter decreases as the number of polynomial terms increases.

In Table 2, the eigenfrequencies are determined for various off-axis angles for $\kappa = 100, e = 0.6,\ 0.8$ and 1.0 and $\nu_{21} = 0.3$.

From here on, in the calculation of the results of the present study, 8×8 terms of the polynomial series are used, giving a determinant of size 64×64.

The moment transmissibilities are determined for various damping parameters γ and off-axis angles θ for $\kappa = 100$ by using Eq. (19). In all of the numerical calculations, ν_{21} is taken as 0.3.

Figures 2a, b, c and d show the moment transmissibilities in the considered range of the external moment for various values of the off-axis angles for $\kappa = 100$, $e = 0.6$ for $\gamma = 0$, $\gamma = 1$, $\gamma = 5$ and $\gamma = 10$, respectively.

Table 1. Comparison of the obtained results with the existing results and convergence study of frequency parameters λ for corner point supported square plates, $\nu_{21} = 0.3$, $\alpha = 1$, $e = 0.6$.

	Determinant size	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
	36x36	5.8397	12.1384	12.9754	16.6897	29.4600	37.2302
	49x49	5.8392	12.1387	12.9749	16.5800	29.4600	36.2777
Present study	64x64	5.8392	12.1384	12.9748	16.5799	29.4516	36.2726
$\kappa = 100$	81x81	5.8392	12.1384	12.9748	16.5799	29.4516	36.2726
$\theta = 0$	100x100	5.8392	12.1384	12.9748	16.5799	29.4516	36.2726
	121x121	5.8392	12.1384	12.9748	16.5799	29.4516	36.2725
	144x144	5.8392	12.1384	12.9748	16.5799	29.4516	36.2725
Present study $\kappa = 100$ $\theta = \pi/2$	144x144	5.8392	12.1384	12.9748	16.5799	29.4516	36.2725
Kocatürk and İlhan (2003)	5.8392	12.1383	12.9748	16.5799	29.4513	36.2726	

Table 2. The eigenfrequencies for various off-axis angles for $\kappa = 100$, $e = 0.6, 0.8, 1.0$ and $\nu_{21} = 0.3$.

e = 0.6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
$\theta = 0$	5.8392	12.1383	12.9750	16.5800	29.4520	36.2780	38.2251	43.5528
$\theta = \frac{\pi}{12}$	5.8751	11.9090	13.1940	16.5210	29.2380	36.5270	38.5761	43.0370
$\theta = \frac{\pi}{6}$	5.9477	11.6050	13.4770	16.4050	28.8420	36.9950	39.5062	42.0185
$\theta = \frac{\pi}{4}$	5.9846	11.4850	13.5850	16.3470	28.6560	37.2180	40.5727	40.9281
e = 0.8	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
$\theta = 0$	6.3633	13.4810	13.8418	18.1990	31.7380	38.6590	43.5281	45.8028
$\theta = \frac{\pi}{12}$	6.3708	13.3670	13.9541	18.1880	31.6900	38.7120	43.6797	45.6438
$\theta = \frac{\pi}{6}$	6.3854	13.2190	14.0979	18.1660	31.5950	38.8170	44.0897	45.2207
$\theta = \frac{\pi}{4}$	6.3928	13.1620	14.1538	18.1550	31.5480	38.8680	44.5863	44.7184
e = 1	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
$\theta = 0$	6.72357	-	-	19.5963	33.3695	40.6246	69.2786	80.6074

Figures 3a, b, c and d show the moment transmissibilities in the considered range of the external moment for various values of the off-axis angles for $\kappa = 100$, $e = 0.8$ for $\gamma = 0$, $\gamma = 1$, $\gamma = 5$ and $\gamma = 10$, respectively.

Figure 4 shows the moment transmissibilities in the considered range of the external moment for various values of γ for $\kappa = 100$, $e = 1.0$.

It is observed from Figures 2 and 3 that when θ is different from zero the third and seventh modes appear. The frequencies of the second and third modes and also the sixth and seventh modes are very close to each other. When $\theta = 0$, namely when the plate is specially orthotropic as in the study by Kocatürk et al. (2004), the third and seventh modes do not appear. This situation can also be observed in Table 3.

Figures 2 and 3 show that within the frequency range of the figures, when the E_2/E_1 ratio is different from unity and $\gamma = 0$, 4 resonant peaks appear, and antiresonant peaks or lowest values appear between adjacent frequencies. By choosing appropriate damping parameters, resonant peaks of the moment transmissibilities and of the displacements disappear and the related peak quantities become small. The damping of the supports is especially effective on the third and seventh modes. Within a certain range of the frequencies, the moment transmissibilities are less than unity, which indicates the possibility of vibration isolation.

It can be deduced from Table 2 and Figures 2 and 3 that when e is different from unity, in the case when the supports are elastic, i.e. $\gamma = 0$, then there are 2 resonant peaks for off-axis angle $\theta = 0$ and 4 resonant peaks for off-axis angle $\theta \neq 0$. It is seen from Tables 2 and 3 and Figures 2 and 3 that these resonant peaks correspond to the second and eighth modes of the considered system in the case $\theta = 0$, and to the second, third, seventh and eighth modes of the considered system in the case $\theta \neq 0$. It means that these modes of the system are excited for the related off-axis angles within the frequency range of the external moment. It is seen from Figures 2b, c and d (except at $\theta = \pi/12$ when $\gamma = 1$ and 10) and 3b, c and d that, in the case of viscoelastic supports,

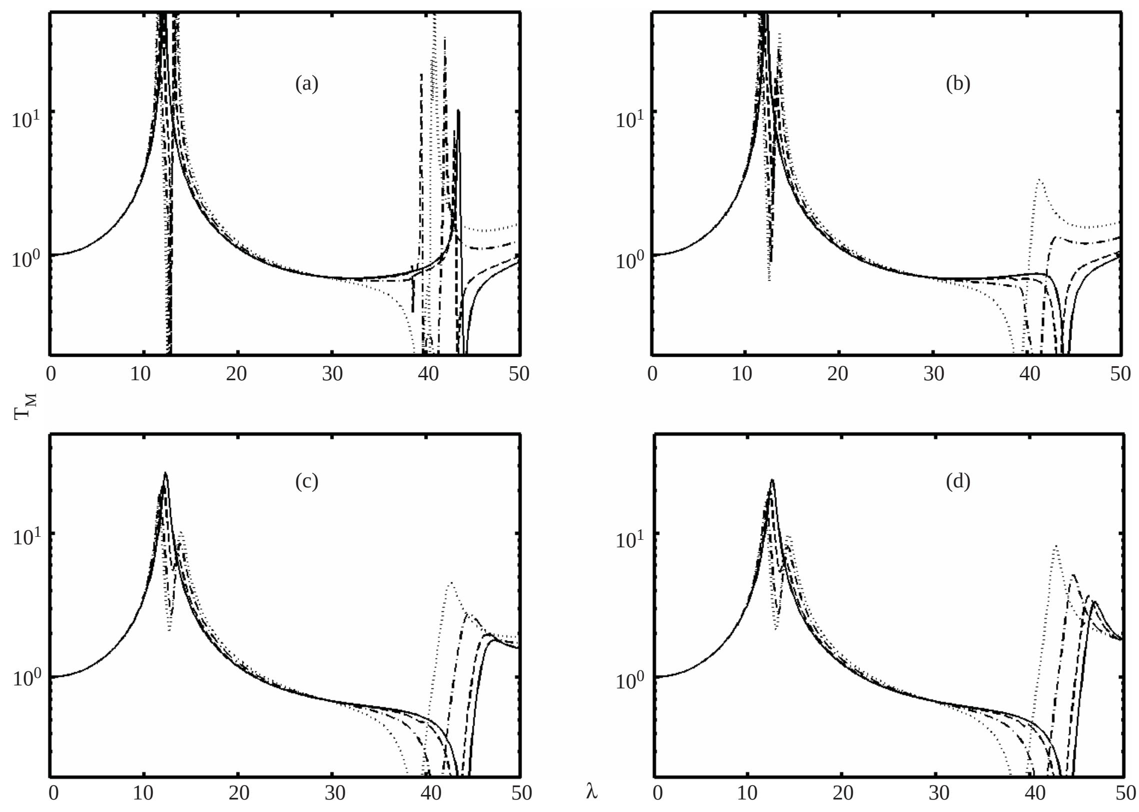

Figure 2. The moment transmissibilities for various values of θ for $e = 0.6$, $\kappa = 100$ for (a) $\gamma = 0$, (b)$\gamma = 1$, (c) $\gamma = 5$, (d) $\gamma = 10$, $\theta = 0$ ——, $\theta = \pi/12$, $\theta = \pi/6 - \cdot - \cdot -$, $\theta = \pi/4 \cdots\cdots$.

Table 3. The frequencies at which the peak values of the moment transmissibilities for the second, third, seventh and eighth modes occur for various values of off-axis angles θ, of damping ratio γ, of orthotrophy ratio e, for $\kappa = 100$.

$\kappa = 100, \nu 21 = 0.3$, e=1					
Modes		$\theta = 0$	$\theta = \pi/12$	$\theta = \pi/6$	$\theta = \pi/4$
λ_2	$\gamma = 0$	14.52	-	-	-
	$\gamma = 1$	14.54	-	-	-
	$\gamma = 5$	14.90	-	-	-
	$\gamma = 10$	15.35	-	-	-
λ_8	$\gamma = 0$	47.60	-	-	-
	$\gamma = 1$	48.67	-	-	-
	$\gamma = 5$	50.28	-	-	-
	$\gamma = 10$	50.36	-	-	-
$\kappa = 100, \nu 21 = 0.3$, e=0.8					
Modes		$\theta = 0$	$\theta = \pi/12$	$\theta = \pi/6$	$\theta = \pi/4$
λ_2	$\gamma = 0$	13.48	13.37	13.22	13.16
	$\gamma = 1$	13.49	13.37	13.22	13.16
	$\gamma = 5$	13.76	13.62	13.43	13.35
	$\gamma = 10$	14.10	13.95	13.75	13.66
λ_3	$\gamma = 0$	-	13.95	14.10	14.15
	$\gamma = 1$	-	14.01	14.14	14.19
	$\gamma = 5$	-	-	14.48	14.57
	$\gamma = 10$	-	-	14.90	14.99
λ_7	$\gamma = 0$	-	43.68	44.09	44.59
	$\gamma = 1$	-	-	-	-
	$\gamma = 5$	-	-	-	-
	$\gamma = 10$	-	-	-	-
λ_8	$\gamma = 0$	45.80	45.64	45.22	44.72
	$\gamma = 1$	47.41	47.11	46.25	45.39
	$\gamma = 5$	48.91	48.63	47.87	46.90
	$\gamma = 10$	48.92	48.66	47.94	47.00
$\kappa = 100, \nu 21 = 0.3$, e=0.6					
Modes		$\theta = 0$	$\theta = \pi/12$	$\theta = \pi/6$	$\theta = \pi/4$
λ_2	$\gamma = 0$	12.14	11.91	11.60	11.48
	$\gamma = 1$	12.15	11.92	11.61	11.49
	$\gamma = 5$	12.31	12.05	11.72	11.59
	$\gamma = 10$	12.54	12.27	11.92	11.78
λ_3	$\gamma = 0$	-	13.19	13.48	13.59
	$\gamma = 1$	-	13.23	13.50	13.61
	$\gamma = 5$	-	13.56	13.85	13.94
	$\gamma = 10$	-	13.89	14.20	14.29
λ_7	$\gamma = 0$	-	38.57	39.51	40.57
	$\gamma = 1$	-	38.11	-	-
	$\gamma = 5$	-	-	-	-
	$\gamma = 10$	-	39.72	-	-
λ_8	$\gamma = 0$	43.45	43.04	42.02	40.93
	$\gamma = 1$	41.01	39.82	43.17	41.32
	$\gamma = 5$	47.24	46.45	44.60	42.61
	$\gamma = 10$	46.92	46.26	44.63	42.75

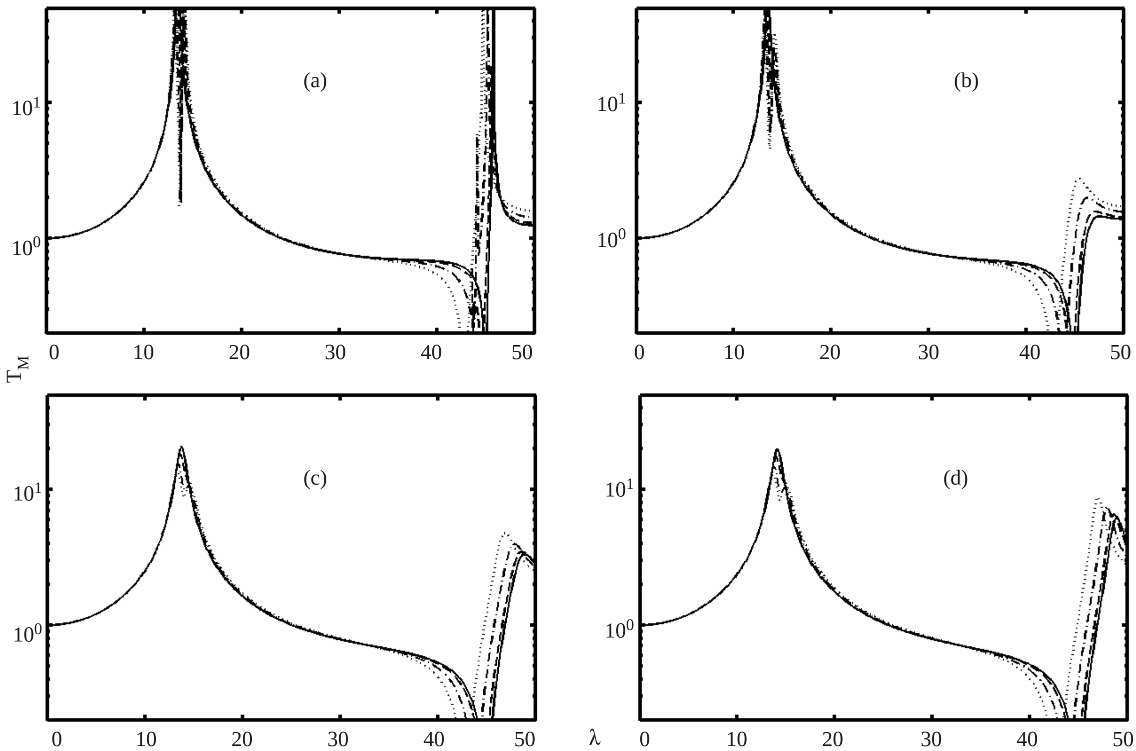

Figure 3. The moment transmissibilities for various values of θ for $e = 0.8$, $\kappa = 100$ for (a) $\gamma = 0$, (b) $\gamma = 1$, (c) $\gamma = 5$, (d) $\gamma = 10$, $\theta = 0$ ——, $\theta = \pi/12$ – – – –, $\theta = \pi/6$ – · – ·–, $\theta = \pi/4 \cdots\cdots$.

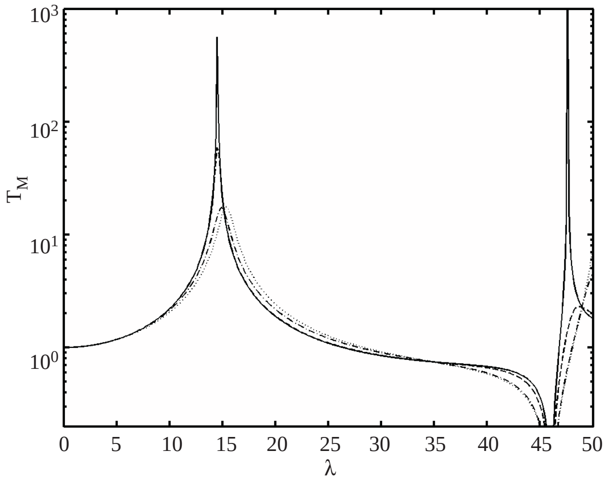

Figure 4. The moment transmissibilities for $e = 1.0$, $\kappa = 100$, for $\gamma = 0$ ——, $\gamma = 1$ – – – –, $\gamma = 5$ – · – ·–, $\gamma = 10 \cdots\cdots$.

the peak of the seventh mode disappears, and as can be seen from Figures 2a, b, c and d and 3a, b, c and d when $\theta = 0$ the third and seventh peaks disappear, and the third peak disappears when $\gamma = 5$ and 10 at $\theta = \pi/12$. It can be deduced from this that by choosing appropriate damping parameters the peaks of the third and seventh modes can be eliminated. By choosing appropriate damping parameter γ especially the third and seventh peaks of the moment transmissibilities disappear.

It is seen from Figure 4 that when $e = 1$ it is obvious that there is no significance of the off-axis angle and in this case there are only 2 resonant peaks in the considered frequency range of the external moment. These resonant peaks are only for the second and eighth modes. In Figure 4, the solid lines represent the response curve of a plate with undamped elastic point supports($\gamma_s = 0$), and the dotted lines a plate for $\gamma_s = 10$. The intersection points of these 2 lines are fixed points, through which all the response curves pass, regardless of the damping parameters. This result was also determined by Yamada et al. (1985) for isotropic viscoelastically point-supported plates. By choosing a suitable value for the damping parameter γ, it is possible to reduce the peak values of the moment transmissibilities to the values of the moment transmissibilities corresponding to the intersection points shown in Figure 4. The existence of such points is useful for an optimum design of a system by choosing an appropriate damping parameter. This subject was studied in detail by Kocatürk and

Altıntaş (2003a), by Kocatürk and Altıntaş (2003b) for viscoelastically point-supported rectangular specially orthotropic plates under a concentrated external force and by Kocatürk et al. (2004) for viscoelastically point-supported specially orthotropic plates under concentrated external moment. It is seen from Figures 2, 3 and 4 that the peak values of the moment transmissibilities occur at different values of λ while changing the damping parameter γ. However, the frequency parameter λ remains between the frequency parameters λ obtained for $\gamma = 0$ and $\gamma = \infty$. In Table 3, the frequencies at which the peak values of the moment transmissibilities for the second, third, seventh and eighth modes occur are determined by using Eq. (19) for various values of the off-axis angles θ, of damping ratio γ, of orthotrophy ratio e, for $\kappa = 100$. When $\theta = 0$ in Table 3, the results obtained for $\kappa = 100$, $\gamma = 0, 1, 5, 10, e = 1, 0.8, 0.6$ are identical with the related results given in Table 3 of the study by Kocatürk et al. (2004).

In the isotropic case, for $\gamma = 5$, $\kappa = 100$, $\nu_{21} = 0.3$, the frequency for the peak value of the moment transmissibility shown in Table 3 is 14.90 in the present study; the same value was obtained by Kocatürk et al. (2004). When E_2/E_1 is unity, the fourth and fifth vibration modes do not arise in the plate. Therefore, in the case of the fourth and fifth modes, the peak values of the moment transmissibilities do not occur for the isotropic case and the lines for these modes are not shown in Table 3. Furthermore, it is an expected result that when E_2/E_1 is unity the off-axis angle θ does not affect the eigenvalues, mode shapes or peak values of the moment transmissibilities. Therefore, the peak values of the moment transmissibilities remain unchanged with respect to variation of the off-axis angle θ in Table 3. The hypen sign (-) in Table 3 shows that there is no resonant peak for the considered parameters.

Conclusions

By using Lagrange's equations, the natural frequencies in the considered frequency range of the external moment for the elastically point-supported generally orthotropic square plates and the steady state response of a viscoelastically point-supported generally orthotropic square plate to sinusoidally varying moment were studied and compared with the existing results. To use Lagrange's equations with the trial function in the polynomial form is a very good way of studying the structural behavior of plates with point supports.

By the application of the above-mentioned solution technique, the first 8 values of the natural frequencies are determined, and the converge characteristics of the frequency parameters are investigated numerically for specially orthotropic square plates elastically supported at 4 points at the corners and compared with the existing results. It is seen that the rate of convergence is very high.

The response curves to sinusoidally varying moment acting at the center are determined numerically for generally orthotropic square plates viscoelastically supported at 4 points at the corners. The effect of the off-axis angle of the plate, orthotropy of the plate, viscosity of the supports for a given stiffness of the supports on the frequency parameters and response curves is investigated and shown in the figures and tables. It is seen that because of the off-axis angle the third mode is excited within the frequency range of the external load. A small off-axis angle causes excitation of the third mode. Therefore, this must be considered in the design of such systems.

All of the obtained results are very accurate and may be useful for designing mechanical systems under external dynamic loads.

References

Choo, V.K.S., Fundamentals of Composite Materials, Dover, Delaware: Knowen Academic Press, 1990.

Kocatürk T., "Determination of the Steady State Response of Viscoelastically Point-Supported Rectangular Anisotropic (Orthotropic) Plates", J. Sound Vib., 213, 665-672, 1998.

Kocatürk, T. and Altıntaş, G., "Determination of the Steady State Response of Viscoelastically Point-Supported Rectangular Specially Orthotropic Plates by an Energy Based Finite Difference Method", J. Sound Vib., 267, 1143-1156, 2003 a.

Kocatürk, T. and Altıntaş, G., "Determination of the Steady-State Response of Viscoelastically Point-Supported Rectangular Orthotropic Plates", Mechanics of Composite Materials, 39, 681-696, 2003 b.

Kocatürk, T., Demir, C., Sezer, S. and İlhan, N., "Determination of the Steady State Response of Viscoelastically Corner Point-Supported Rectangular Specially Orthotropic Plates under the Effect of Sinusoidally Varying Moment", J. Sound Vib.,275, 317-330, 2004.

Kocatürk, T. and İlhan, N., "Free Vibration Analysis of Elastically Point Supported Orthotropic Plates", Journal of Yıldız Technical University, 2003-1, 104-115, 2003.

Narita, Y., "Note on Vibrations of Point Supported Rectangular Plates", J. Sound Vib., 93, 593-597, 1984.

Szilard, R., Theory and Analysis of Plates, Prentice-Hall, New Jersey 1974.

Venkateswara Rao, G., Raju, I.S. and Amba-Rao, C.L., "Vibrations of Point Supported Plates", J. Sound Vib., 29, 387-391, 1973.

Yamada, G., Irie, T. and Takahashi, M., "Determination of the Steady State Response of Viscoelastically Point-Supported Rectangular Plates", J. Sound Vib., 102, 285-295,1985.