Turkish J. Eng. Env. Sci.
30 (2006) , 14 – 21.

Hall Effect on the Flow of a Dusty Bingham Fluid in a Circular Pipe

Hazem Ali ATTIA
Department of Mathematics, College of Science, Al-Qasseem University,
P.O. Box 237, Buraidah 81999, Kingdom of SAUDI ARABIA
Department of Engineering Mathematics and Physics, Faculty of Engineering,
El-Fayoum University, El-Fayoum-EGYPT
e-mail: ah1113@yahoo.com

Received 24.12.2004

Abstract

In this paper, the transient flow of a dusty viscous incompressible electrically conducting non-Newtonian Bingham fluid through a circular pipe is studied taking the Hall effect into consideration. A constant pressure gradient in the axial direction and a uniform magnetic field directed perpendicular to the flow direction are applied. The particle phase is assumed to behave as a viscous fluid. A numerical solution is obtained for the governing nonlinear equations using finite differences. It is found that the magnetic field decreases the fluid and particle velocities; however, the Hall parameter leads to an increase in the average velocities of both the fluid and particle phases and, consequently, in their flow rates and the velocity gradients at the wall.

Key words: Magneto-fluid mechanics, Flow in channels, Circular pipe flow, Bingham fluid.

Introduction

The flow of a dusty and electrically conducting fluid through a circular pipe in the presence of a transverse magnetic field has important applications such as magnetohydrodynamic (MHD) generators, pumps, accelerators, and flowmeters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase by the so-called particle-phase viscosity. There have been many articles dealing with theoretical modeling and experimental measurements of the particle-phase viscosity in a dusty fluid (Soo, 1969; Grace, 1982; Gidaspow et al., 1986; Sinclair et al., 1989).

The flow of a conducting fluid in a circular pipe has been investigated by many authors (Dube et al., 1975; Ritter et al., 1977; Gadiraju et al., 1992; Chamkha, 1994). Gadiraju et al. (1992) investigated steady two-phase vertical flow in a pipe. Dube et al. (1975) and Ritter et al. (1977) reported solutions for unsteady dusty-gas flow in a circular pipe in the absence of a magnetic field and particle-phase viscous stresses. Chamkha (1994) obtained exact solutions that generalize the results reported in Dube et al. (1975) and Ritter et al. (1977) by the inclusion of the magnetic and particle-phase viscous effects. It should be noted that in the above studies the Hall effect is ignored.

A number of industrially important fluids such as molten plastics, polymers, pulps and foods exhibit non-Newtonian fluid behavior (Metzner et al. 1965; Nakayama et al., 1988). Due to the growing use of these non-Newtonian materials, in various manufacturing and processing industries, considerable efforts have been directed towards understanding their flow characteristics. It is of interest in this paper to study the influence of the magnetic field as well as the non-Newtonian fluid characteristics on the dusty fluid flow properties in situations where the particle phase is considered dense enough to include the particulate viscous stresses (Attia, 1998).

In the present study, the unsteady flow of a dusty non-Newtonian Bingham fluid through a circular pipe is investigated considering the Hall effect. The carrier fluid is assumed viscous, incompressible, and electrically conducting. The particle phase is assumed to be incompressible pressureless and electrically non-conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The governing nonlinear momentum equations for both the fluid and particle phases are solved numerically using the finite difference approximations. The effect of the Hall current, the non-Newtonian fluid characteristics and the particle-phase viscosity on the velocity of the fluid and particle phases are reported.

Governing Equations

Consider the unsteady, laminar, and axisymmetric horizontal flow of a dusty conducting non-Newtonian Bingham fluid through an infinitely long pipe of radius a driven by a constant pressure gradient. A uniform magnetic field is applied perpendicular to the flow direction. The Hall current is taken into consideration and the magnetic Reynolds number is assumed to be very small; consequently, the induced magnetic field is neglected (Sutton et al., 1965). We assume that both phases behave as viscous fluids and that the volume fraction of suspended particles is finite and constant (Chamkha, 1994).

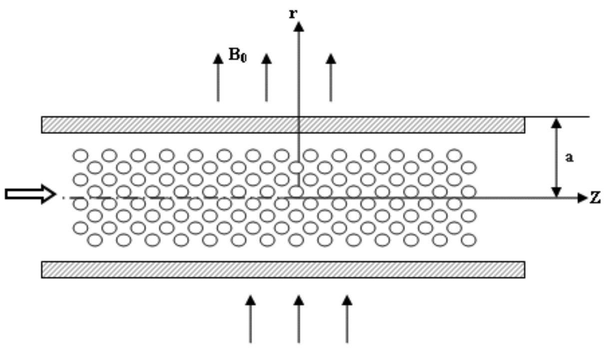


Figure 1. Sketch of the problem.

To formulate the governing equations for this investigation, the balance laws of mass and linear momentum are considered along with information about interfacial and external body forces and stress tensors for both phases. The balance laws of mass (for the fluid and particulate phases, respectively) can be written as

$$\partial_t \phi - \vec{\nabla}.((1-\phi)\vec{V}) = 0, \tag{1a}$$

$$\partial_t \phi + \vec{\nabla}.(\phi \vec{V}_p) = 0 \tag{1b}$$

where t is time, ϕ is the particulate volume fraction, $\vec{\nabla}$ is the gradient operator, $\vec{V}$ is the fluid-phase velocity vector, and $\vec{V}_p$ is the particulate-phase velocity vector. The true densities for both phases are assumed constant.

The balance laws of linear momentum (for the fluid and particulate phases, respectively) can be written as

$$\rho(1-\phi)(\partial_t \vec{V} + \vec{V}.\vec{\nabla}\vec{V}) = \vec{\nabla}.\overleftrightarrow{\sigma} - \vec{f} + \vec{b}, \tag{2a}$$

$$\rho_p \phi(\partial_t \vec{V}_p + \vec{V}_p.\vec{\nabla}\vec{V}_p) = \vec{\nabla}.\overleftrightarrow{\sigma}_p - \vec{f} + \vec{b}_p \tag{2b}$$

whereρ is the fluid-phase density, $\overleftrightarrow{\sigma}$ is the fluid-phase stress tensor,$\vec{f}$is the interphase force per unit volume associated with the relative motion between the fluid and particle phases, $\vec{b}$is the fluid-phase body force per unit volume, and $\vec{b}_p$ is the particle-phase body force per unit volume.

Along with Eqs. (1) and (2), the following constitutive equations are used

$$\overleftrightarrow{\sigma} = (1-\phi)(-P\overleftrightarrow{I} + \mu(\vec{\nabla}\vec{V} + \vec{\nabla}\vec{V}^T)), \tag{3a}$$

$$\overleftrightarrow{\sigma}_p = \phi\mu_p(\vec{\nabla}\vec{V}_p + \vec{\nabla}\vec{V}_p^T)), \tag{3b}$$

$$f = N\rho_p\phi(V - V_p), \tag{3c}$$

$$\vec{b} = -1/\rho(\vec{J} \wedge \vec{B}_o), \tag{3d}$$

$$\vec{b}_p = \vec{0} \tag{3e}$$

where P is the fluid pressure, $\overleftrightarrow{I}$ is the unit tensor, μ is the fluid dynamic viscosity, μ_p is the particle-phase dynamic viscosity, N is the inverse relaxation time (the inverse time required by a particle to reduce its velocity relative to that of the fluid by e^{-1} from its initial value (Chamkha, 1994), $\vec{J}$ is the electric current density vector, $\vec{B}_o$ is the uniform applied magnetic induction field vector, and a transposed T denotes the transpose of a second-order tensor.

If the Hall term is retained, the current density $\vec{J}$ is given by (Sutton et al., 1965)

$$\vec{J} = \varepsilon[\vec{V} \times \vec{B}_o - \beta(\vec{J} \times \vec{B}_o)] \tag{4}$$

where ε is the electric conductivity of the fluid and β is the Hall factor (Sutton et al., 1965). Equation (4) may be solved in $\vec{J}$ to obtain the electromagnetic Lorentz force in the form (Sutton et al., 1965)

$$\vec{J} \times \vec{B}_o = -\frac{\varepsilon B_o^2}{1+m^2} V\vec{k} \tag{5}$$

where m=$\varepsilon\beta B_o$ is the Hall parameter, B_o is the magnetic induction, and $\vec{k}$ is a unit vector along the z-direction. Solving Eq. (5) for $\vec{J}$ and substituting in Eqs. (1)-(3) with expanding yields

$$\begin{aligned} \rho\frac{\partial V}{\partial t} = & -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial V}{\partial r}\right) \\ & + \frac{\rho_p \phi}{1-\phi} N(V_p - V) - \frac{\varepsilon B_o^2 V}{1+m^2} \end{aligned} \tag{6}$$

$$\rho_p\frac{\partial V_p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\mu_p r\frac{\partial V_p}{\partial r}\right) + \rho_p N(V - V_p) \tag{7}$$

In the present work ϕ, ρ, ρ_p, μ_p and Nwill all be treated as constants while μ is the apparent viscosity of the Bingham fluid, which is assumed to be

$$\mu = \mu_o + \frac{\tau_o}{\left|\frac{\partial V}{\partial r}\right|}$$

where μ_o is the plastic viscosity of a Bingham fluid and τ_o is the yield stress. The last three terms on the right-hand side of Eq. (6) represent, respectively, the electromagnetic Lorentz force per unit volume, the interphase force per unit volume acting on the fluid phase and the viscous forces of the non-Newtonian Bingham fluid. It should be pointed out that the particle phase pressure is assumed negligible and that the particles are being dragged along with the fluid phase.

The initial and boundary conditions of the problem are given as

$$V(r,0) = 0, V_p(r,0) = 0, \tag{8a}$$

$$\frac{\partial V(0,t)}{\partial r} = 0, \frac{\partial V_p(0,t)}{\partial r} = 0, V(a,t) = 0, V_p(a,t) = 0 \tag{8b}$$

Equations (6)-(8) constitute a nonlinear initial-value problem that can be made dimensionless by introducing the following dimensionless variables and parameters

$$\bar{r} = \frac{r}{d}, \bar{t} = \frac{t\mu_o}{\rho d^2}, G_o = -\frac{\partial P}{\partial z}, k = \frac{\rho_p \phi}{\rho(1-\phi)}, \bar{\mu} = \frac{\mu}{\mu_o}$$

$$\bar{V}(r,t) = \frac{\mu_o V(r,t)}{G_o a^2}, \bar{V}_p(r,t) = \frac{\mu_o V_p(r,t)}{G_o a^2},$$

$\alpha = Na^2\rho/\mu_o$ is the inverse Stokes' number,
$B = \mu_p/\mu_o$ is the viscosity ratio,
$\tau_D = \tau_o/G_o a$ is the Bingham number (dimensionless yield stress),
$Ha = B_o a\sqrt{\varepsilon/\mu_o}$ is the Hartmann number (Sutton et al., 1965).

By introducing the above dimensionless variables and parameters as well as the expression of the fluid viscosity defined above, Eqs. (6)-(8) can be written as (the bars are dropped),

$$\frac{\partial V}{\partial t} = 1 + \frac{\partial^2 V}{\partial r^2} + \frac{\mu}{r}\frac{\partial V}{\partial r} + k\alpha(V_p - V) - \frac{Ha^2 V}{1+m^2} \tag{9}$$

$$\frac{\partial V_p}{\partial t} = B\left(\frac{\partial^2 V_p}{\partial r^2} + \frac{1}{r}\frac{\partial V_p}{\partial r}\right) + \alpha(V - V_p) \tag{10}$$

$$\mu = 1 + \frac{\tau_D}{\left|\frac{\partial V}{\partial r}\right|}$$

$$V(r,0) = 0, V_p(r,0) = 0, \tag{11a}$$

$$\frac{\partial V(0,t)}{\partial r} = 0, \frac{\partial V_p(0,t)}{\partial r} = 0, V(1,t) = 0, V_p(1,t) = 0 \tag{11b}$$

Of special interest are the fluid-phase volume flow rate, the particle phase volume flow rate, and the fluid phase skin-friction coefficient. They are given, respectively, by the following relations (Chamkha, 1994):

$$Q = 2\pi \int_0^1 rV(r,t)dr,\ Q_p = 2\pi \int_0^1 rV_p(r,t)dr,$$
$$C = -\frac{\partial V(1,t)}{\partial r},\ C_p = -Bk\frac{\partial V_p(1,t)}{\partial r} \qquad (12)$$

where the expressions for Q andQ_p are derived from the integral of the velocity of fluid phase and particle phase, respectively, over the cross-sectional area of the pipe. The quantities C andC_p are related to the velocity gradient at the wall of both phases. It should be mentioned that the above expressions for flow rates do not depend on the volume fractions because of the constant volume fraction assumption employed in this problem (Chamkha, 1994, 1995). There are more elaborate constitutive theories that predict nonuniform particulate volume fraction distributions that will eventually be needed for modeling more complex multiphase problems (Chamkha, 1995). The particle phase with a constant viscosity is a step in that direction.

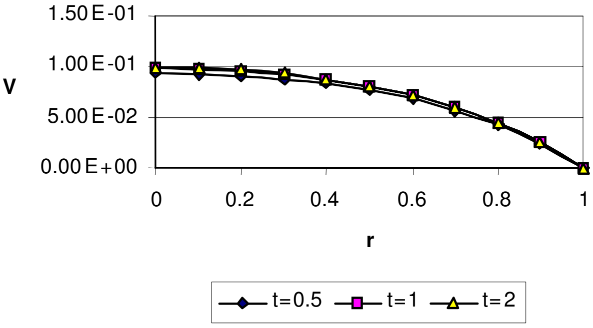
Results and Discussion

Equations (9) and (10) represent a coupled system of non-linear partial differential equations that are solved numerically under the initial and boundary conditions (11) using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. The computational domain is divided into meshes each of dimension $\Delta t$and Δr in time and space, respectively. Then the Crank-Nicolson implicit method is used at two successive time levels (Mitchell et al., 1980; Evans et al., 2000). An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for the next time step and the iterations are continued untill convergence, within a prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas algorithm (Mitchell et al., 1980; Evans et al., 2000). Computations have been performed for $\alpha = 1$ and $k = 10$. Grid-independence studies show that the computational domain $0 < t < \infty$ and $0 < r < 1$ can be divided into intervals with step sizes $\Delta t = 0.0001$ and $\Delta r = 0.005$ for time and space, respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns V, V_p, $\partial V/\partial r$, and $\partial V_p/\partial r$ for the last two approximations differ from unity by less than 10^{-6} for all values of r in 0 ¡ r¡ 1 at every time step. It should be mentioned that the results obtained herein reduce to those reported by Dube et al. (1975) and Chamkha (1994) for the cases of non-magnetic, inviscid particle-phase (B= 0), and Newtonian fluid. These comparisons lend confidence to the accuracy and correctness of the solutions.

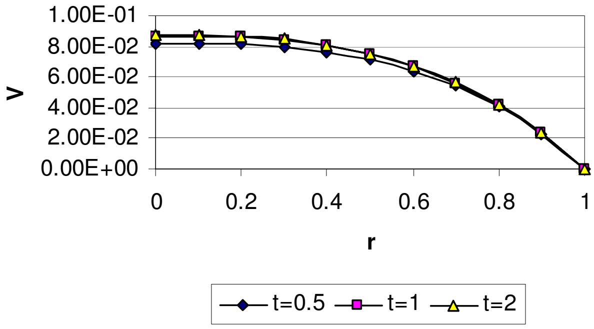
The imposing of a magnetic field normal to the flow direction gives rise to a drag-like or resistive force and it has the tendency to slow down or suppress the movement of the fluid in the pipe, which, in turn, reduces the motion of the suspended particle phase. This is translated into reductions in the average velocities of both the fluid and particle phases and, consequently, in their flow rates. In addition, the reduced motion of the particulate suspension in the pipe as a result of increasing the strength of the magnetic field causes lower velocity gradients at the wall. This has the direct effect of reducing the skin-friction coefficients of both phases. Including the Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity. Therefore, the Hall parameter leads to an increase in the average velocities of both the fluid and particle phases and, consequently, in their flow rates and the velocity gradients at the wall.

Figures 2 and 3 present the time evolution of the profiles of the velocity of the fluid V and dust particles V_p, respectively, for various values of the Bingham number τ_D and for m= 0, Ha = 0.5 and B= 0.5. Both V and V_p increase with time and V reaches the steady state faster than V_p for all values of τ_D. It is clear from Figures 2 and 3 that increasing τ_D, which decreases the driving force for V, decreases V and, consequently, decreases V_p while its effect on their steady-state times can be neglected.

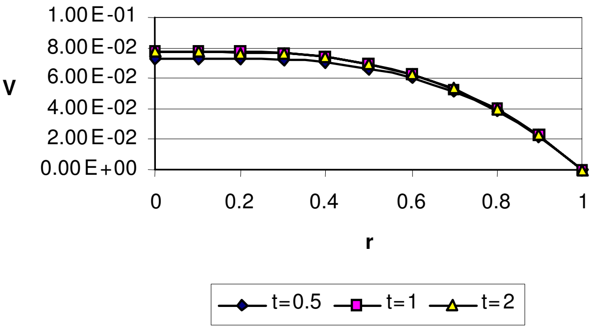
Figures 4 and 5 present the time evolution of the profiles of the velocity of the fluid V and dust particles V_p, respectively, for various values of the Bingham number τ_D and for m= 1, Ha = 0.5 and B= 0.5. It is seen in the figures that increasing m increases V and, in turn, V_p due to the decrease in the effective conductivity $(\sigma/(1 + m^2))$ which reduces the damping magnetic force on V.



(a) τ_D=0.0

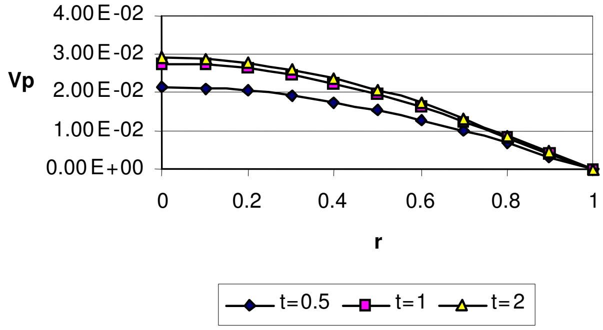


(b) τ_D=0.025

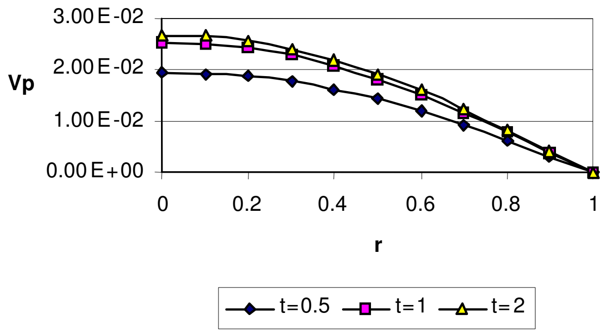


(c) τ_D=0.05

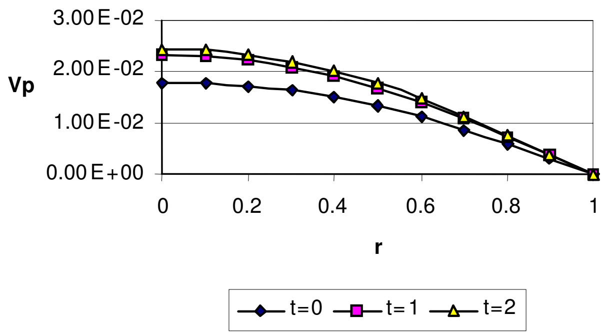
Figure 2. Time development of V for various values of $\tau_D (m = 0)$.



(a) τ_D=0.0

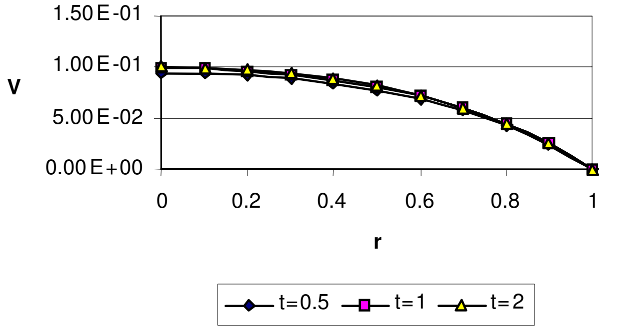


(b) τ_D = 0.025

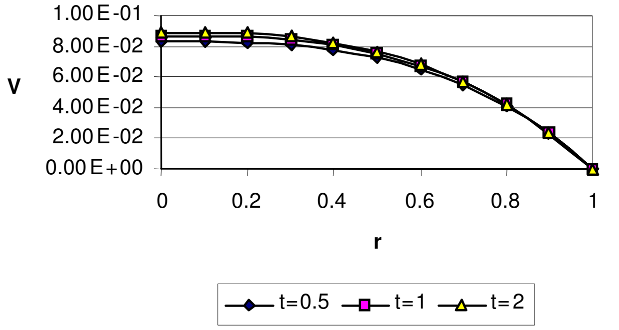


(c) τ_D = 0.05

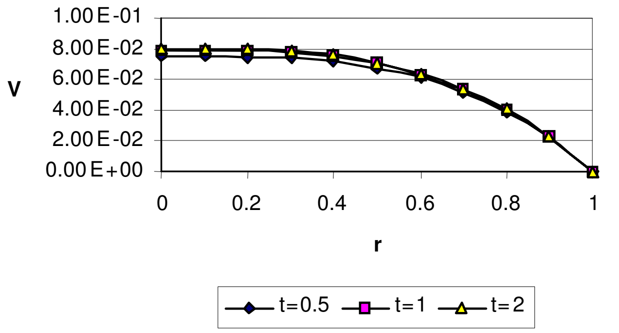
Figure 3. Time development of V_p for various values of $\tau_D (m = 0)$.



(a) τ_D =0.0

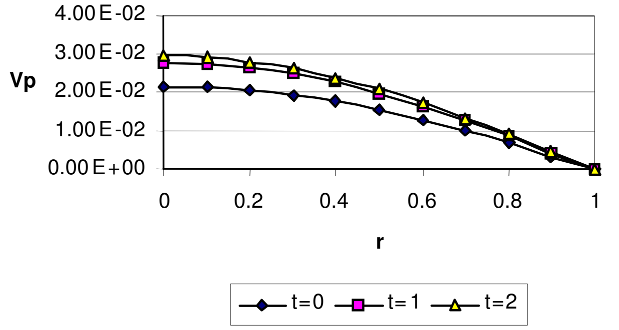


(b) τ_D =0.025

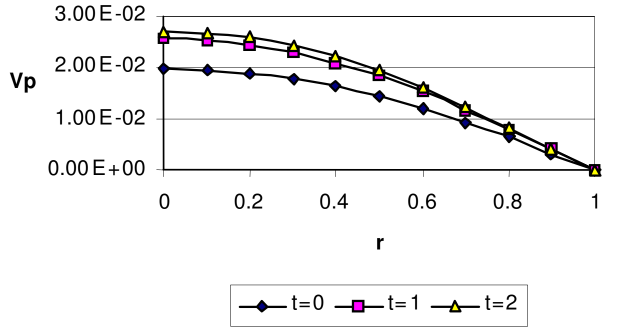


(c) τ_D =0.05

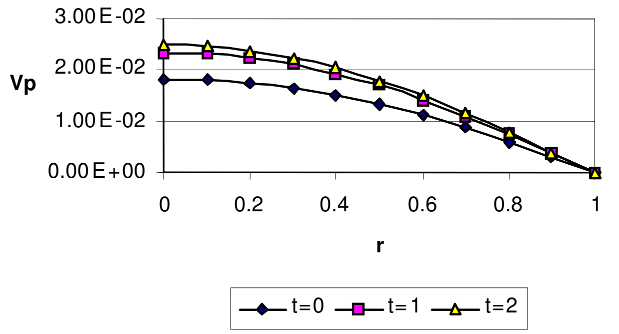
Figure 4. Time development of V for various values of $\tau_D (m = 1)$.



(a) τ_D =0.0



(b) τ_D =0.025



(c) τ_D =0.05

Figure 5. Time development of V_p for various values of $\tau_D (m = 1)$.

Table 1 presents the steady-state values of the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p for various values of the parameters τ_D and m for $Ha = 0.5$ and $B = 0.5$. It is clear that increasing the parameter m increases Q, Q_p, C, and C_p for all values of τ_D. This comes from the fact that increasing m increases the velocities and their gradients which increases the average velocities of both the fluid and particle phases and, consequently, increases their flow rates and skin-friction coefficients of both phases. It is also shown that increasing τ_D decreases Q, Q_p, C, and C_p for all values of m as a result of decreasing the velocities of both phases.

Table 2 presents the steady-state values of the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p for various values of the parameters m and B for $Ha = 0.5$ and $\tau_D = 0$. It is clear that increasing m increases Q, Q_p, C, and C_p for all values of B and its effect becomes more pronounced for smaller values of B. Increasing the parameter B decreases the quantities Q, Q_p, and C, but increases C_p for all values of m. This can be attributed to the fact that increasing B increases viscosity and therefore the flow rates of both phases as well as the fluid phase wall friction decrease considerably. However, since $C_p$is defined as directly proportional to B, it increases as B increases at all times.

Table 1. The steady-state values of Q, Q_p, C, C_p for various values of m and τ_D ($Ha = 0.5$ and $B = 0.5$).

$\tau_D = 0$	$m = 0$	$m = 1$	$m = 2$
Q	0.1764	0.1779	0.1789
Q_p	0.0426	0.0430	0.0433
C	0.2818	0.2834	0.2844
C_p	0.2111	0.2129	0.2140

$\tau_D = 0.025$	$m = 0$	$m = 1$	$m = 2$
Q	0.1649	0.1663	0.1673
Q_p	0.0396	0.0400	0.0402
C	0.2704	0.2719	0.2729
C_p	0.1975	0.1995	0.2005

$\tau_D = 0.05$	$m = 0$	$m = 1$	$m = 2$
Q	0.1525	0.1535	0.1564
Q_p	0.0364	0.0369	0.0372
C	0.2583	0.2598	0.2612
C_p	0.1834	0.1859	0.1868

Table 2. The steady-state values of Q, Q_p, C, C_p for various values m and B ($Ha = 0.5$ and $\tau_D = 0$).

$B = 0$	$m = 0$	$m = 1$	$m = 2$
Q	0.3032	0.3075	0.3101
Q_p	0.2582	0.2615	0.2635
C	0.4125	0.4167	0.4193
C_p	0	0	0

$B = 0.5$	$m = 0$	$m = 1$	$m = 2$
Q	0.1764	0.1779	0.1789
Q_p	0.0426	0.0430	0.0433
C	0.2818	0.2834	0.2844
C_p	0.2111	0.2129	0.2140

$B = 1$	$m = 0$	$m = 1$	$m = 2$
Q	0.1640	0.1654	0.1662
Q_p	0.0226	0.0228	0.0229
C	0.2702	0.2716	0.2724
C_p	0.2231	0.2249	0.2260

Conclusion

The transient MHD flow of a particulate suspension in an electrically conducting non-Newtonian Bingham fluid in a circular pipe is studied considering the Hall effect. The governing nonlinear partial differential equations are solved numerically using finite differences. The effect of the magnetic field parameter Ha, the Hall parameter, the non-Newtonian fluid characteristics (Bingham number τ_D), and the particle-phase viscosity B on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle phases is studied. It is shown that increasing the magnetic field decreases the fluid and particle velocities, while increasing the Hall parameter increases both velocities. It is found that increasing the parameter m increases Q, Q_p, C, and C_p for all values of τ_D. The effect of the Hall parameter on the quantities Q, Q_p, C, and C_p becomes more pronounced for smaller values of B.

Nomenclature

a	pipe radius,
B_o	magnetic induction,
C	fluid-phase skin-friction coefficient,
C_p	particle-phase skin-friction coefficient,
Ha	Hartmann number,

m	Hall parameter,	B	viscosity ratio,
N	momentum transfer coefficient,	ϕ	particle-phase volume fraction,
P	pressure gradient,	k	particle loading,
Q	fluid-phase volumetric flow rate,	μ	fluid-phase viscosity,
Q_p	fluid-phase volumetric flow rate,	μ_p	particle-phase viscosity,
r	distance in the radial direction,	ρ	fluid-phase density,
t	time,	ρ_p	fluid-phase density,
V	fluid-phase velocity,	ε	fluid electrical conductivity,
V_p	particle-phase velocity,	τ_o	yield stress
z	axial direction,		

References

Attia, H.A., "Unsteady Fow of a Dusty Conducting Non-Newtonian Fluid through a Pipe", Can. J. Phys., 81, 789-795, 2003.

Chamkha, A.J., "Unsteady Flow of a Dusty Conducting Fluid through a Pipe", Mechanics Research Communications, 21, 281-286, 1994.

Chamkha, A.J., "Hydromagnetic Two-phase Flow in a Channel", International Journal of Engineering Science, 33, 437-446, 1995.

Dube, S.N. and Sharma, C.L., "A Note on Unsteady Flow of a Dusty Viscous Liquid in a Circular Pipe", J. Phys. Soc. Japan, 38, 298-310, 1975.

Evans, G.A., Blackledge, J.M. and Yardley, P.D., "Numerical Methods for Partial Differential Equations", Springer Verlag, New York, 2000.

Gadiraju, M., Peddieson, J. and Munukutla, S., "Exact Solutions for Two-Phase Vertical Pipe Flow", Mechanics Research Communications, 19, 7-13, 1992.

Gidaspow, D., "Hydrodynamics of Fluidization and Heat Transfer: Super Computer Modeling", Appl. Mech. Rev., 39, 1-23, 1986.

Grace, J.R., "Fluidized-Bed Hydrodynamic, Handbook of Multiphase Systems", G. Hetsoroni, Ed., Ch. 8.1, McGraw-Hill, New York., 1982.

Metzner, A.B.", Heat Transfer in Non-Newtonian Fluid", Adv. Heat Transfer, 2, 357-397, 1965.

Mitchell, A.R. and Griffiths, D.F., "The Finite Difference Method in Partial Differential Equations", John Wiley & Sons, New York, 1980.

Nakayama, A. and Koyama, H., "An Analysis for Friction and Heat Transfer Characteristics of Power-Law Non-Newtonian Fluid Flows Past Bodies of Arbitrary Geometrical Configuration", Warme-und Stoffubertragung, 22, 29-37, 1988.

Ritter, J.M. and Peddieson, J., "Transient Two-Phase Flows in Channels and Circular Pipes", Proc. 1977 the Sixth Canadian Congress of Applied Mechanics, 1977.

Sinclair, J.L. and Jackson, R., "Gas-Particle Flow in a Vertical Pipe with Particle-Particle Interactions", AICHE J., 35, 1473-1486, 1989.

Soo, S.L., "Pipe Flow of Suspensions", Appl. Sci. Res., 21, 68-84, 1969.

Sutton, G.W. and Sherman, A., "Engineering Magnetohydrodynamics", McGraw-Hill, New York, 1965.