Turkish J. Eng. Env. Sci.
30 (2006) , 83 – 89.

Similarity Analysis of Magnetic Field and Thermal Radiation Effects on Forced Convection Flow

Rebhi A. DAMSEH
Al-Balqa Applied University, Al-Huson University College,
Mechanical Engineering Department, P.O. Box (50), Irbid-JORDAN
e-mail: rdamseh@yahoo.com
H. M. DUWAIRI
The University of Jordan, Mechanical Engineering Department, Amman-JORDAN
M. AL-ODAT
Al-Balqa Applied University, Mechanical Engineering Department, Amman-JORDAN

Received 21.03.2005

Abstract

Magnetohydrodynamic (MHD) forced convection heat transfer from radiate surfaces in the presence of a uniform transverse magnetic field, with conductive fluid suction or injection from a porous plate is considered. A set of solutions to the non-linear equations is presented using the Keller box method. It is found that increasing the magnetic influence number decreases both the velocity boundary layer thickness and the heat transfer rates from the porous plate, also increasing the radiation-conduction parameter, Eckert numbers, dimensionless temperature ratios, which heated the fluid, and decreases heat transfer rates. Different velocity and temperature profiles, local coefficient of friction, and local Nusselt numbers for different dimensionless groups are drawn.

Key words: Magnetic field, Thermal radiation, Viscous dissipation, Forced convection.

Introduction

The study of magnetohydrodynamic (MHD) viscous flows is important to industrial, technological, and geothermal applications, such as high-temperature plasmas, cooling of nuclear reactors, liquid metal fluids, MHD accelerators, and power generation systems. Sparrow and Cess (1961) studied the effect of magnetic fields on natural convection heat transfer. Romig (1964) studied the effect of electric and magnetic fields on heat transfer to electrically conducting fluids. Garandet et al. (1992) analyzed the buoyancy-driven convection in a rectangular enclosure with a transverse magnetic field. Takhar and Ram (1994) studied the effect of magnetic fields on both natural and forced convection heat transfer problems. Hossain (1992) determined that viscous and Joule heating effects on MHD-free convection flows with variable plate temperature varies linearly with a distance from a leading edge and in the presence of a uniform transverse magnetic field; the equations governing the flow were solved numerically by applying the finite difference method along with Newton's linearization approximation. Jha (2001) studied the natural convection in unsteady Couette motion; in his work, the unsteady free convective flow of an incompressible viscous fluid between 2 vertical parallel plates, 1 of which is impulsively started, is considered.

On the other hand, radiation heat transfer effects from a porous wall on forced convection flow are very important in space technology and high temperature processes, and very little is known about the effects of radiation on the boundary layer of a radiate-MHD fluid past a body. The inclusion of radiation effects in the energy equation leads to a highly nonlinear partial differential equation. The radiation effects on free convection flow of a gas past a semi-infinite flat plate was studied by Soundalgekar et al. (1960) using the Cogley-Vincenti-Giles equilibrium model, Cogley et al. (1968). Hossain and Takhar (1996) studied the effects of radiation of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream velocity and surface temperature. In this analysis, consideration had been given to gray gases that emit and absorb, but do not scatter thermal radiation; they note that the Rosseland diffusion approximation provides one of the most straightforward simplifications of the full integro-partial differential equations. The effects of radiation on the free convection heat transfer problem in the absence of a magnetic field and viscous dissipation was studied by Hossain et al. (1999), where only the suction boundary condition is treated. More recently, Duwairi and Damseh (2004a, 2004b) studied the radiation-conduction interaction in free and mixed convection fluid flow for a vertical flat plate with the presence of a magnetic field effect.

In the present report, the specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to an isothermal porous plate, with radiation heat transfer effects accounted for. There is an interesting aspect involving MHD effects in forced convection boundary layers; induced magnetic forces modify the free stream flow and this, in turn, affects the external pressure gradient or the free stream velocity that is imposed in the boundary layer. Thus, a complete boundary-layer solution would involve a MHD solution for the inviscid free stream. As a consequence, the forced convection problem is more complicated than free convection.

Mathematical Analysis

Consider the problem of MHD forced convection of an electrically conducted fluid of constant thermal properties at uniform temperature. A uniform magnetic field is applied in the vertical direction, normal to the plane. The electromotive force generated by a magnetic field is known to be proportional to its speed of motion and the magnetic field strength. The electric field intensity $\boldsymbol{E}$ is defined as:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (1)$$

The magnetic flux density $(\mathbf{B})$ is expressed as:

$$\mathbf{B} = \mu_e \mathbf{H} \qquad (2)$$

where $\mathbf{H}$ is the magnetic field strength, and μ_e is the magnetic permeability. With generalized Ohm's law, the total current flow can be defined as:

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{V} \times \mathbf{B}\right) \qquad (3)$$

where σ is the electrical conductivity. From the above equations we get:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + v_m \nabla^2 \mathbf{B} \qquad (4)$$

where $v_m = 1/(\sigma\mu_e)$.

The electromagnetic force to be included in the momentum equation $(\boldsymbol{F}_m)$ is:

$$\mathbf{F_m} = \mathbf{J} \times \mathbf{B} = \sigma\left(\mathbf{V} \times \mathbf{B}\right) \times \mathbf{B} \qquad (5)$$

The analysis is carried out for the case of uniform surface temperature (T_w), which is placed in a fluid at temperature (T_∞), with free stream velocity (u_∞), and a uniform magnetic field (B_0) (independent of x). The configuration is pictured schematically in Figure 1.

The flow is steady, laminar, incompressible, and 2-dimensional. The fluid is assumed to be gray, emitting and absorbing heat, but not scattering it. The radiation is included in the governing equations by using the Rosseland diffusion approximation. The x coordinate is measured from the leading edge of the plate, and the y coordinate is measured normal to the plate. The corresponding velocities in the $x$and y directions are u and v, respectively, and $\mathrm{v}_w$is the injection or withdrawal velocity of the conductive fluid from the porous wall. Using the boundary layer approximations, the governing equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \sigma B_0^2\left(u - U_\infty\right) = \mu\frac{\partial^2 u}{\partial y^2} \qquad (7)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - \sigma B_0^2 u^2 = k\left(\frac{\partial^2 T}{\partial y^2}\right) - \frac{\partial}{\partial y}\left(-\frac{16a}{3\alpha_R}T^3\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad (8)$$

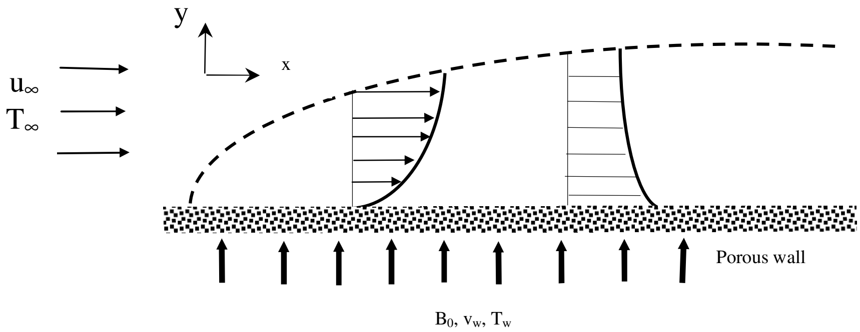

Figure 1. MHD-forced convection model from a radiate porous wall.

The governing equations include magnetic influence terms in both the momentum and energy equations, and the energy equation includes a viscous dissipation term. The radiation heat transfer fluxes for an optically thick fluid are included in the energy equation, as described by Ali et al. (1984). In the above system of partial differential equations, a is the Stefan-Boltzmann constant and α_R is the Rosseland mean absorption coefficient. The boundary conditions can be written as:

$$\begin{array}{lll} x=0, & y\rangle 0: & u=0,\ T=T_\infty \\ y=0, & x\rangle 0: & u=0,\ v=\pm v_w,\ T=T_w \\ y\to\infty, & x\rangle 0: & u=u_\infty,\ T=T_\infty \end{array} \tag{9}$$

The boundary condition $(\partial u/\partial y)_{y\to\infty}=0$, regarding the shear stress at the outer edge of the boundary layer, is employed to determine the boundary layer thickness. The minus sign velocity $(-v_w)$ corresponds to the suction or withdrawal of the conductive fluid from the porous wall, and the plus sign (v_w) corresponds to blowing or injection. In this paper, both the conductive fluid suction or injection case from the porous wall will be treated.

In the above system of equations, and in order to satisfy the continuity equation, we define the stream function as $u=\partial\psi/\partial y$, *and* $v=-\partial\psi/\partial x$. The following dimensionless variables are introduced in the transformation:

$$\eta=y\sqrt{\frac{u_\infty}{\upsilon x}}\ ,\quad \psi=\sqrt{u_\infty \upsilon x}\,f(\eta)\ ,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{10}$$

Using Eq. (10) in the governing Eqs. (6-9), the momentum and energy equations can be written as:

$$f'''+(1/2)ff''+N\,(1-f')=0 \tag{11}$$

$$\frac{1}{\Pr}\left[\left\{1+\frac{4}{3R_d}\left(1+(\theta_w-1)\theta\right)^3\right\}\theta'\right]'+(1/2)f\theta' + NEc\,(f')^2+Ec\,(f'')^2=0 \tag{12}$$

The corresponding boundary conditions can be written as:

$$\begin{array}{l} f'(0)=0,\ f(0)=\pm M,\ \theta(0)=1 \\ f'(\infty)=1,\ \theta(\infty)=0\ \ ,\ \ f''(\infty)=0 \end{array} \tag{13}$$

The corresponding dimensionless groups that appeared in the governing equations are $N=\sigma B_0^2 x/\rho u_\infty$, $Ec=u_\infty^2/cp(T_w-T_\infty)$, $\Pr=\upsilon/\alpha$, $\theta_w=T_w/T_\infty R_d=k\alpha_R/4aT_\infty^3$, and $M=(2v_w/u_\infty)\,Re_x^{1/2}$. The minus sign for M refers to suction and the plus sign refers to the injection through the porous wall. The primes denote partial differentiations with respect to η.

In the above system of equations, the radiation conduction parameter is absent from the MHD-forced convection heat transfer problem when $1/R_d\to 0$. Some of the physical quantities of practical interest include the velocity component u and $v$in the x- and y-directions, the wall shear stress, $\tau_w=\mu\,(\partial y/\partial y)_{y=0}$, and the surface heat flux, $q_w(x)=-\left\{k+(16a/3\alpha_R)T^3\right\}(\partial T/\partial y)_{y=0}=h\,(T_w-T_\infty)$. They are given by:

$$u=u_\infty f'(\eta) \tag{14}$$

$$v=-(u_\infty/2)Re_x^{-1/2}\{f(\eta)-\eta\,f'(\eta)\} \tag{15}$$

$$Cf_x Re_x^{1/2} = 2f''(0) \tag{16}$$

$$Nu_x Re_x^{-1/2} = -\left(1 + (4/3R_d)\theta_w^3\right)\theta'(\xi, 0) \tag{17}$$

The governing nonlinear differential Eqs. (11-12), with the relevant boundary condition Eq. (13), have been solved numerically using the Keller box method of Cebeci and Bradahow (1984); their numerical scheme has several desirable features that make it appropriate for solution of parabolic partial differential equations. For the sake of brevity, the detailed numerical method is not presented here. A grid independence study was carried out to examine the effect of the step size η and the edge of the boundary layer η_∞ on the solution in order to optimize them. The computations were carried out using a non-uniform grid with the first step size $\Delta\eta$ =0.005, and the variable grid parameter was chosen to be 1.01. The value of $\eta_\infty = 30$ was sufficiently large for the velocity to reach the relevant stream velocity.

Results and Discussion

In this study, the MHD-forced convection heat transfer effects on a radiate porous wall are investigated. Some important results for the MHD effects on the forced convection heat transfer problem, which is different from the previous study by Takhar and Ram (1994), are found.

It should be mentioned that the optically thick approximation should be valid for relatively low values of the radiation-conduction parameter, R_d. According to Ali et al. (1984), some values of R_d for different gases are: (1) R_d=10-30: carbon dioxide (100-650^oF) with corresponding Prandtl number range 0.76-0.6; (2) R_d=30-200: ammonia vapor (120-400^oF) with corresponding Prandtl number range 0.88-0.84; (3) R_d=30-200: water vapor (220-900^oF) with corresponding Prandtl number 1.

Figures 2 and 3 show that increasing the magnetic influence number, $N = \sigma B_0^2 x/\rho u_\infty$, decreases the velocity boundary layer thickness due to magnetic field effects on the external flow field. Additionally, the heat transfer rates from the porous plate had been decreased for either suction or injection from the porous wall.

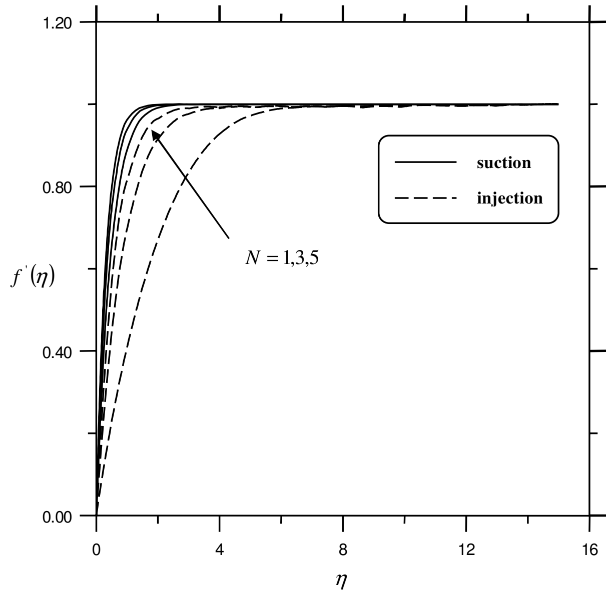

Figure 2. Dimensionless velocity profile for different Nat $M = \pm 3, \theta_w = 2, \Pr = 1, Rd = 1, Ec = 0.01$.

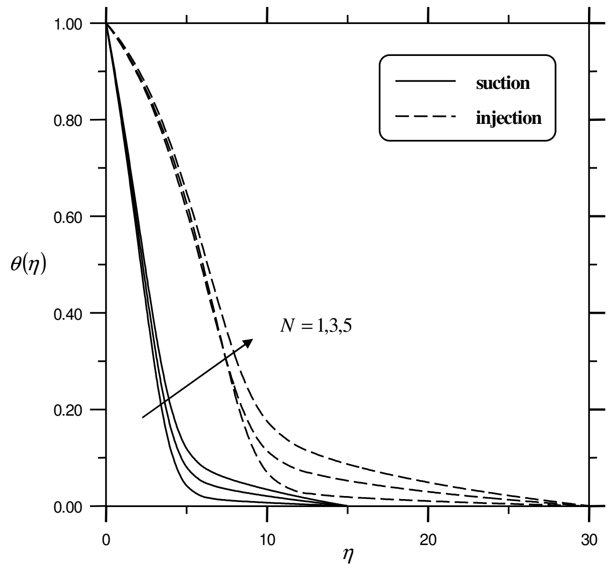

Figure 3. Dimensionless temperature profile for different Nat $M = \pm 3, \theta_w = 2, \Pr = 1, Rd = 1, Ec = 0.01$.

The radiation-conduction parameter, $R_d = k\alpha_R/4aT_\infty^3$, had no effects on the velocity boundary layer for the MHD-forced convection heat transfer problem under consideration. Figure 4 shows the dimensionless temperature profiles for different values of the radiation-conduction parameter. Figure 5 shows that increasing the Eckert number increased

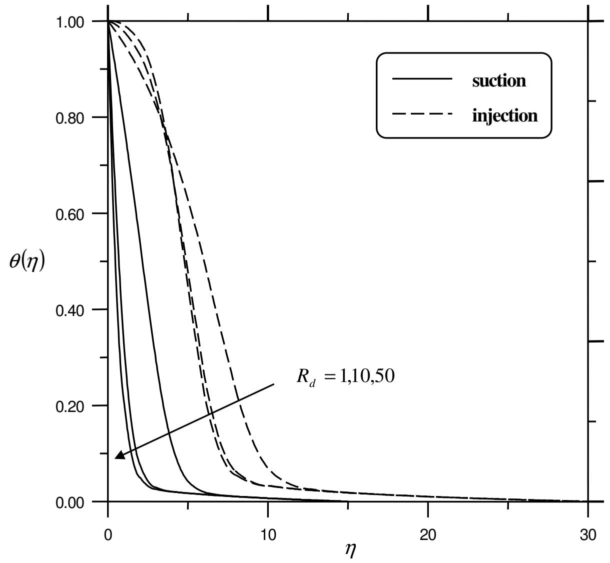

Figure 4. Dimensionless temperature profile for different R_d at $M = \pm 3, \theta_w = 2, \Pr = 1, N = 1, Ec = 0.01$.

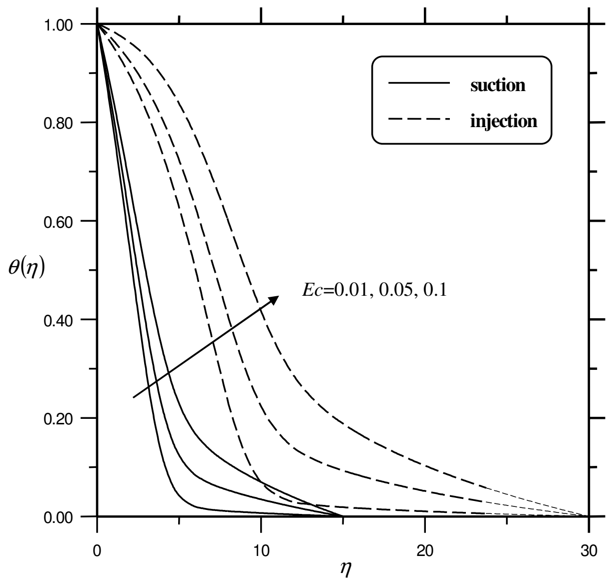

Figure 5. Dimensionless temperature profile for different Ec at $M = \pm 3, \theta_w = 2, \Pr = 1, N = 1, R_d = 1$.

the conductive fluid temperature inside the boundary layer and decreased the heat transfer rates form the porous wall. Figure 6 shows the variation of the coefficient of friction against the magnetic field parameter for both suction and injection cases, where increasing this parameter increased sheer stresses near the porous wall. Figure 7 shows that increasing the temperature ratio, $\theta_w = T_w/T_\infty$, also increased the temperatures inside the boundary layer for the suction and injection cases and consequently decreased the heat transfer rates.

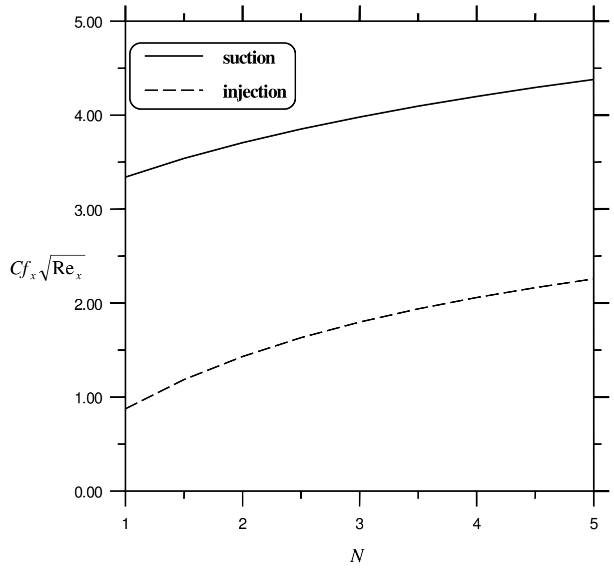

Figure 6. Coefficient of friction against N at $\theta_w = 2$, $\Pr = 1, Ec = 0.01, M = \pm 3, R_d = 1$.

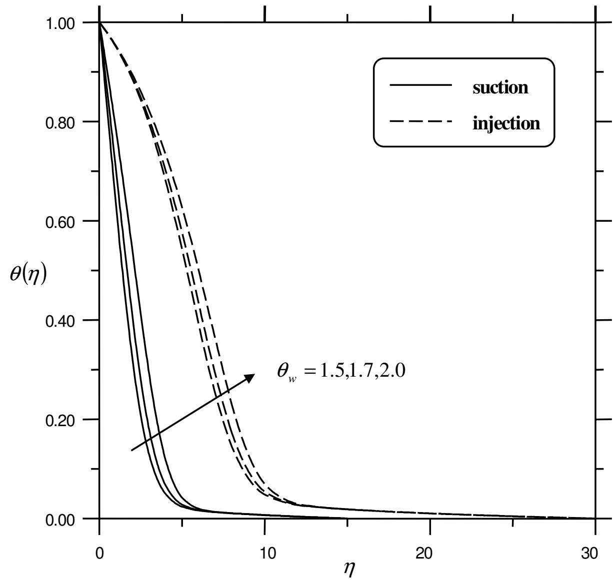

Figure 7. Dimensionless temperature profile for different θ_w at $M = \pm 3, R_d = 1, \Pr = 1, N = 1, Ec = 0.01$.

Figures 8 and 9 show the local Nusselt number variations against magnetic field numbers for different values of the radiation-conduction parameter and

Eckert numbers, respectively. Increasing these parameters decreased the Nusselt numbers because of excessive heating of the conducting fluid. Note that the local Nusselt numbers are greater for the suction case of the porous wall than that of the injection case.

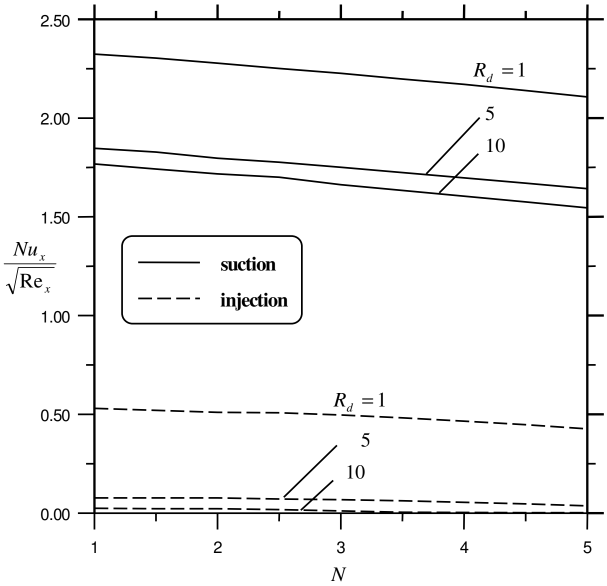

Figure 8. Local Nusselt number against $N$for different R_d at $M = \pm 3, \theta_w = 2, \mathrm{Pr} = 1, Ec = 0.01$.

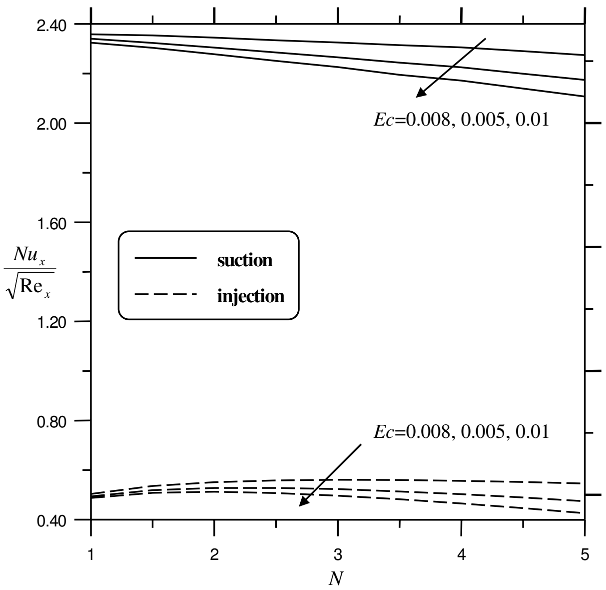

Figure 9. Local Nusselt number against $N$for different Ec at $M = \pm 3, \theta_w = 2, \mathrm{Pr} = 1, R_d = 1$.

Conclusions

In this paper, the effect of the magnetic field on the forced convection flow from a porous wall, where radiation occurs, is studied. Increasing the radiation parameter leads to decreased heat transfer rates for both the conductive fluid suction and injection, while increasing the magnetic field strength increased the velocity inside the boundary layer and decreased the heat transfer rates from the radiate porous wall. Including the viscous dissipation effects in the energy equation was found to decrease the local Nusselt numbers for both suction and injection cases. The effect of the temperature ratio was found to increase the temperature inside the boundary layer and decrease the heart transfer rate.

Nomenclature

$\bar{N}u$	average Nusselt numbers, $\bar{h}L/k$
Nu_x	local Nusselt number, hx/k
Re_x	local Reynolds number, $u_\infty x/\upsilon$
T	temperature
u_∞	free stream velocity
u, v	velocity components in x-and y-directions
a	Stefan-Boltzmann constant
B_0	magnetic field flux density, Wb/m^2
Cf_x	local skin friction factor
c_p	specific heat capacity
$\mathbf{E}$	electric field intensity
Ec	Eckert number, $u_\infty^2/cp(T_w - T_\infty)$
f	dimensionless stream function
$\mathbf{H}$	magnetic field strength
k	thermal conductivity
L	length of the plate
M	porous wall section or injection parameter, $(2v_w/u_\infty)\, Re_x^{1/2}$
N	magnetic influence number, $\sigma B_0^2 x/\rho u_\infty$
p	pressure
Pr	Prandtl number
$q_w(x)$	local surface heat flux
R_d	radiation-conduction parameter, $k\alpha_R/4aT_\infty^3$
T_∞	free stream temperature
T_w	wall temperature
v_w	porous wall suction or injection velocity
x, y	axial and normal coordinates

Greek symbols

α thermal diffusivity
α_R Rosseland mean absorption coefficient
β coefficient of thermal expansion, $-1/\rho\,(\partial\rho/\partial T)_p$
η pseudo-similarity variable
θ dimensionless temperature
θ_w ratio of surface temperature to the ambient temperature, T_w/T_∞
μ dynamic viscosity
υ kinematic viscosity
ρ fluid density
σ electrical conductivity
τ_w local wall shear stress
μ_e magnetic permeability
ψ dimensional stream function

References

Ali, M.M., Chen T.S. and Armaly B.F., "Natural Convection Radiation Interaction in Boundary Layer Flow over Horizontal Surfaces", AIAA Journal, 22, 1797-1803, 1984.

Cebeci, T. and Bradshaw, P., Physical and Computational Aspects of Convective Heat Transfer, Springer, New York, 1984.

Cogely, A.C., Vinceti W.G. and Giles S.E., "Differential Approximation for Radiative Heat Transfer in a Non–Gray Gas Near Equilibrium", AIAA Journal, 6, 551-553, 1968.

Duwairi, H.M. and Damseh R.A., "Magnetohydrodynamic Natural Convection Heat Transfer from Radiate Vertical Porous Surfaces", Heat Mass Transfer, 40, 787-792, 2004a.

Duwairi, H.M. and Damseh R.A., "MHD-Buoyancy Aiding and Opposing Flows with Viscous Dissipation Effects from Radiate Vertical Surfaces", The Canadian Journal of Chemical Engineering, 82, 613-618, 2004b.

Garandet, J.P., Alboussiere T. and Moreau R., "Buoyancy Driven Convection in a Rectangular Enclosure with a Transverse Magnetic Field", Int. J. Heat Mass Transfer, 35, 741-749, 1992.

Hossain, M.A., Alim M.A. and Rees D.A., "The Effect of Radiation on Free Convection from a Porous Vertical Plate", Int. J. Heat Mass Transfer, 42, 181-191, 1999.

Hossain, M.A., "Viscous and Joule Heating Effects on MHD-Free Convection Flow with Variable Plate Temperature", Int. J. Heat Mass Transfer, 35, 3485-3487, 1992.

Hossain, M.A. and Takhar H.S., "Radiation Effect on Mixed Convection along a Vertical Plate with Uniform Surface Temperature", Heat and Mass Transfer, 31, 243-248, 1996.

Jha, B.K., "Natural Convection in Unsteady MHD Couette Flow", Heat and Mass Transfer, 37, 329-331, 2001.

Romig, M., "The Influence of Electric and Magnetic Field on Heat Transfer to Electrically Conducting Fluids", Adv. Heat Transfer, 1, 268-352, 1964.

Soundalgekar, V.M., Takhar H.S. and Vighnesam N.V., "The Combined Free and Forced Convection Flow Past a Semi-Infinite Plate with Variable Surface Temperature", Nuclear Engineering and Design, 110, 95-98, 1960.

Sparrow, E.M. and Cess R.D., "Effect of Magnetic Field on Free Convection Heat Transfer", Int. J. Heat Mass Transfer, 3, 267-27, 1961.

Takhar, H.S. and Ram P.C., "Magnetohydrodynamic Free Convection Flow of Water at $4\,C^0$ through a Porous Wall", Int. Comm. In Heat and Mass Transfer, 21, 371-376, 1994.