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A Sensitivity Analysis of the HCM 2000 Delay Model with the Factorial Design Method

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Abstract

The sensitivity of the Highway Capacity Manual (HCM) 2000 delay model to its parameters was investigated with the factorial design method. The study results suggest that the arrival flow, the saturation flow, and the green signal time are the main parameters that significantly affect the average control delay estimated by the delay model. Additionally, the multi-parameter interactions of the arrival flow-saturation flow and the arrival flow-green signal time have major effects on the model-estimated average control delay. The study results also demonstrate that the analysis period and the cycle length do not seem to have major effects on the estimation of the average control delay. A further factorial analysis performed to investigate the effect of parameters on the uniform delay showed that the green signal time and the cycle length appeared to significantly affect the uniform delay.

Key words: Sensitivity analysis, Factorial design method, HCM 2000 delay model, Uniform delay, Incremental delay.

Introduction

Sensitivity analysis of a model can help determine relative effects of model parameters on model results. In other words, the purpose of sensitivity testing of a model is to investigate whether a slight perturbation of the parameter values will result in a significant perturbation of the model results, that is, the internal dynamics of the model. The most commonly used sensitivity method is the change one-factor-at-a-time approach. The major weakness of this method is its inability to identify multiple factor interactions among the model parameters. As an alternative approach, the factorial design method developed by Box et al. (1978) has been successfully employed in various environmental sensitivity studies (Henderson-Sellers, 1992, 1993; Liang, 1994; Barros, 1996; Henderson-Sellers and Henderson-Sellers, 1996; Yildiz, 2001; among others). Unlike the standard change one-factor-at-a-time sensitivity approach, this method has the advantage of testing both the sensitivity of model results to changes in individual parameters and to interactions among a group of parameters.

The objective of this study is to utilize the factorial design method in the sensitivity analysis of a total delay model that estimates the difference between the actual travel time of a vehicle traversing a signalized intersection approach and the travel time of the same vehicle traversing on the intersection without impedance at the desired free flow speed. The Highway Capacity Manual (HCM) 2000 (TRB, 2000) delay model, one of the most commonly used time dependent delay models, was selected for the sensitivity study. Due to the complexity and the highly nonlinear behavior of the model, the standard change one-factor-at-a-time sensitivity method seems inadequate. Therefore, as a first attempt, the sensitivity analysis of the model was conducted with the factorial design method to identify both main parameter and multiple parameter effects of primary importance.

Control delay

Total delay, also called control or overall delay, is defined as the additional time that a driver has to spend at an intersection when compared to the time it takes to pass through the intersection without impedance at the free flow speed. This additional time is the result of the traffic signals and the effect of other traffic, and it is expressed on a per vehicle basis.

In estimating delay at signalized intersections, stochastic steady-state and deterministic delay models are used for undersaturated and oversaturated conditions, respectively. Neither model, however, deals satisfactorily with variable traffic demands. Stochastic steady-state delay models are only applicable for undersaturated conditions and predict infinite delay when the arrival flow approaches the capacity. When demand exceeds the capacity, continuous overflow delay occurs. Deterministic delay models can estimate continuous oversaturated delay, but they do not deal adequately with the effect of randomness when the arrival flow is close to the capacity, and they fail for degrees of saturation between 1.0 and 1.1. Consequently, the stochastic steady-state models work well when the degree of saturation is less than 1.0, and the deterministic oversaturation models work well when the degree of saturation is considerably greater than 1.0. There exists a discontinuity when the degree of saturation is 1.0 for which the latter models predicts zero delay, while the former models predicts infinite delay.

Time-dependent delay models, therefore, fill the gap between these 2 models and give more realistic results in estimating the delay at signalized intersections. They are derived as a mix of the steady-state and the deterministic models by using the coordinate transformation technique described by Kimber and Hollis (1978, 1979). Here, the coordinate transformation is applied to the steady-state curve to make it asymptotic to the deterministic line. Thus, time-dependent delay models predict the delay for both undersaturated and oversaturated conditions without having any discontinuity at the degree of saturation 1.0.

The HCM 2000 delay model

The HCM 2000 model, along with the Australian (Akcelik, 1981) and the Canadian (Teply, 1996) models, is a commonly used delay model for estimating delay at signalized intersections. General formulations of these models are similar to each other. In the HCM 2000 model, the expression of average control delay experienced by vehicles arriving in a specified time and flow period at traffic signals is given by Eq. (1):

$$d = d_1 \times (PF) + d_2 + d_3 \tag{1}$$

in which $\boldsymbol{d}$ is the average control delay per vehicle (s/veh), $\boldsymbol{d}_1$is the uniform delay term resulting from interruption of traffic flow by traffic signals at intersections, $\boldsymbol{PF}$ is the uniform delay progression adjustment factor, which accounts for effects of signal progression, $\boldsymbol{d}_2$ is the incremental delay term incorporating effects of random arrivals and oversaturated traffic conditions, and $\boldsymbol{d}_3$ is the initial queue delay term accounting for delay to all vehicles in the analysis period due to the initial queue at the start of the analysis period, taken as zero.

Uniform delay

The uniform delay term is based on deterministic queuing analysis and is predicted by the assumption that the number of vehicles arriving during each signal cycle is constant and equivalent to the average flow rate per cycle. Because of constant arrival rates, randomness in the arrivals is ignored and the discharge rate varies from zero to saturation flow according to the red and green time of the signal. The discharge rate equals the saturation flow rate only when a queue exists because of red time of the signal. On the other hand, when there is no queue, the discharge rate is equal to the arrival flow rate due to undersaturated traffic conditions, and values of degree of saturation (X) beyond 1.0 are not used in the computation of $\boldsymbol{d}_1$. The uniform delay term is expressed by Eq. (2):

$$d_1 = \frac{0.5C(1-\frac{g}{C})^2}{1-\left[\min(1,X)\frac{g}{C}\right]} \tag{2}$$

where $\boldsymbol{d}_1$is uniform delay (s/veh), $\boldsymbol{C}$ is cycle time (s), $\boldsymbol{g}$ is green time (s), $\boldsymbol{X}$ is degree of saturation indicating the ratio of arrival flow (or demand) to capacity (i.e. $\boldsymbol{v/c}$), and $\boldsymbol{g/C}$ is green ratio.

Incremental delay

The incremental delay term represents additional delay experienced by vehicles arriving during a specified flow period. Incremental delay results from both temporary and persistent oversaturation. Temporary oversaturation occurs during both undersaturated and oversaturated traffic conditions because of randomness in vehicle arrivals and temporary cycle failures. Thus, delay resulting from temporary oversaturation is called random overflow delay. The effect of the randomness in arrival flows is not important and can be neglected for low degrees of saturation because total arrivals are much less than the capacity. Conversely, for high degrees of saturation and especially when the arrival flow approaches the capacity, the effect of random variation in arrivals increases significantly.

Persistent oversaturation, on the other hand, only occurs during oversaturated traffic conditions because the arrival flow is always greater than the capacity; that is, vehicles cannot be discharged within the signal cycles. Delay resulting from persistent oversaturation is called continuous or deterministic overflow delay. The effect of the overflow delay in incremental delay increases as the duration of the analysis period (T) and the value of the degree of saturation ($\boldsymbol{X}$) increase. The expression of the incremental delay term is given in Eq. (3):

$$d_2 = 900T\left[(x-1)+\sqrt{(x-1)^2+\frac{8kIX}{cT}}\right] \qquad (3)$$

where $\boldsymbol{d}_2$ is the incremental delay to account for the effect of random and oversaturation queues, $\boldsymbol{T}$ is the duration of analysis period in hours, $\boldsymbol{k}$ is the incremental delay factor, $\boldsymbol{I}$ is the upstream filtering or metering adjustment factor, and $\boldsymbol{c}$ is capacity given as a function of saturation flow ($\boldsymbol{s}$) and green ratio (i.e. $c = s\frac{g}{C}$).

Factorial design method

A general factorial design method tests a fixed number of possible values for each of the model parameters with specific perturbations of values (usually 2 levels: upper and lower). Unlike the standard change-one-factor-at-a-time method, this method has the advantage of testing both the sensitivity to changes in individual parameters and to interactions between groups of parameters. The method tests a fixed number of possible values for each of the model parameters, and then identifies and ranks each parameter according to some pre-established measures of model sensitivity by running the model through all possible combinations of the parameters (Box et al., 1978). For example, if there are n parameters in the model for 2 perturbation levels, then there will be 2^n combinations of the model parameters. This is illustrated in the following 3-parameter (2^3 factorial) design. Assume that parameters are called A, B, and C, and the prediction variable is called PV. The corresponding design matrix for this example is shown in Table 1.

Table 1. Factorial design matrix for single parameters.

Run	A	B	C	PV
1	-	-	-	R_1
2	+	-	-	R_2
3	-	+	-	R_3
4	+	+	-	R_4
5	-	-	+	R_5
6	+	-	+	R_6
7	-	+	+	R_7
8	+	+	+	R_8

where + and - signs represent the 2 possible values of each parameter (upper and lower levels, respectively). Within the design matrix, the effects due to each parameter and parameter interactions can be estimated as:

$$E_j = [\sum_i^n (S_{ij}R_i)]/N_j \qquad (4)$$

in which E_j represents the effect of the j^{th} factor (i.e. in the j^{th} column), n is the total number of experimental runs (i.e. $n = 8$), S_{ij} represents the sign in row i and column j, R_i represents the value of the prediction variable obtained from the i^{th} experimental run, and N_j is the number of + signs in column j.

Using Eq. (4) and the above design matrix, the effects of parameter interactions on the model results can also be estimated based on the signs of the parameter interactions using the following rule: plus times minus gives a minus, and minus times minus or plus times plus gives a plus. The corresponding design matrix for parameter interactions is given in Table 2.

Table 2. Factorial design matrix for multi-parameter interactions.

Run	A·B	A·C	B·C	A·B·C
1	+	+	+	-
2	-	-	+	+
3	-	+	-	+
4	+	-	-	-
5	+	-	-	+
6	-	+	-	-
7	-	-	+	-
8	+	+	+	+

The degree of importance of the parameters and their interactions can be determined after all the E_j values are estimated from Eq. (4). One way of identifying and ranking the parameters with major effects, as suggested by Box et al. (1978), is to plot the effects on a normal probability scale. According to this method, any outliers from the straight line on the normal probability plot could be considered to affect the model results significantly, while other effects would lead to variability in model results consistent with the result of random variation about a fixed mean, assuming that higher order interactions are negligible in a manner similar to neglecting higher order terms in a Taylor series expansion (Box et al., 1978). Another way of identifying the parameters with major effects on the model results, as suggested by Henderson-Sellers (1992, 1993), is to use an iterative method to find thresholds that are 2, 3, or 4 standard deviations from zero. Here, any effects greater than the estimated thresholds are considered to have significant effects on the model results.

Factorial design of the HCM 2000 delay model

The 2-level factorial design method was applied to the HCM Delay 2000 Model for the sensitivity analysis. Five model parameters with parameter index numbers from 1 to 5 (**1:$\boldsymbol{v}$, 2:$\boldsymbol{s}$, 3:$\boldsymbol{g}$, 4:$\boldsymbol{C}$,** and **5:$\boldsymbol{T}$**) were selected for this purpose (Table 3). Since the degree of saturation ($\boldsymbol{X}$) and the capacity ($\boldsymbol{c}$) are dependent parameters, they cannot be selected as individual parameters in the sensitivity analysis. The upper and lower levels of the selected model parameters given in Table 3 were chosen arbitrarily within their reasonable ranges. In this particular study, the progress adjustment factor ($\boldsymbol{PF}$), the incremental delay calibration factor ($\boldsymbol{k}$), and the upstream filtering adjustment factor ($\boldsymbol{l}$) were taken as 1.0, 0.5, and 1.0, respectively.

Table 3. The selected model parameters for the sensitivity analysis.

Parameter Index No.	Parameter Name	Symbol	Lower Level	Upper Level
1	Arrival flow (veh/h)	$\boldsymbol{v}$	250	750
2	Saturation flow (veh/h)	$\boldsymbol{s}$	1000	2000
3	Green time (s)	$\boldsymbol{g}$	30	90
4	Cycle time (s)	$\boldsymbol{C}$	120	180
5	Duration of analysis period (h)	$\boldsymbol{T}$	0.5	1.0

For the given number of parameters and perturbation levels, the design matrix for the main parameters is shown in Table 4.

Table 4. The design matrix for the main model parameters.

	Model Parameters				
Run	**1**	**2**	**3**	**4**	**5**
1	-	-	-	-	-
2	+	-	-	-	-
3	-	+	-	-	-
4	+	+	-	-	-
5	-	-	+	-	-
6	+	-	+	-	-
7	-	+	+	-	-
8	+	+	+	-	-
9	-	-	-	+	-
10	+	-	-	+	-
11	-	+	-	+	-
12	+	+	-	+	-
13	-	-	+	+	-
14	+	-	+	+	-
15	-	+	+	+	-
16	+	+	+	+	-
17	-	-	-	-	+
18	+	-	-	-	+
19	-	+	-	-	+
20	+	+	-	-	+
21	-	-	+	-	+
22	+	-	+	-	+
23	-	+	+	-	+
24	+	+	+	-	+
25	-	-	-	+	+
26	+	-	-	+	+
27	-	+	-	+	+
28	+	+	-	+	+
29	-	-	+	+	+
30	+	-	+	+	+
31	-	+	+	+	+
32	+	+	+	+	+

The corresponding computation matrix for the multiple parameter interactions was obtained using the design matrix (Table 5).

Table 5. The computation matrix for the multiple parameter interactions.

	Multiple Parameter Interactions																									
Run	**12**	**13**	**14**	**15**	**23**	**24**	**25**	**34**	**35**	**45**	**123**	**124**	**125**	**134**	**135**	**145**	**234**	**235**	**245**	**345**	**1234**	**1245**	**2345**	**1235**	**1345**	**12345**
1	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	-
2	-	-	-	-	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	+	-	-	+
3	-	+	+	+	-	-	-	+	+	+	+	+	+	-	-	-	+	+	+	-	-	-	-	-	+	+
4	+	-	-	-	-	-	-	+	+	+	-	-	-	+	+	+	+	+	+	-	+	+	-	+	-	-
5	+	-	+	+	-	+	+	-	-	+	+	-	-	+	+	-	+	+	-	+	-	+	-	-	-	+
6	-	+	-	-	-	+	+	-	-	+	-	+	+	-	-	+	+	+	-	+	+	-	-	+	+	-
7	-	-	+	+	+	-	-	-	-	+	-	+	+	+	+	-	-	-	+	+	+	-	+	+	-	-
8	+	+	-	-	+	-	-	-	-	+	+	-	-	-	-	+	-	-	+	+	-	+	+	-	+	+
9	+	+	-	+	+	-	+	-	+	-	-	+	-	+	-	+	+	-	+	+	-	-	-	+	-	+
10	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	+	+	+	-	-	+	-
11	-	+	-	+	-	+	-	-	+	-	+	-	+	+	-	+	-	+	-	+	+	+	+	-	-	-
12	+	-	+	-	-	+	-	-	+	-	-	+	-	-	+	-	-	+	-	+	-	-	+	+	+	+
13	+	-	-	+	-	-	+	+	-	-	+	+	-	-	+	+	-	+	+	-	+	-	+	-	+	-
14	-	+	+	-	-	-	+	+	-	-	-	-	+	+	-	-	-	+	+	-	-	+	+	+	-	+
15	-	-	-	+	+	+	-	+	-	-	-	-	+	-	+	+	+	-	-	-	-	+	-	+	+	+
16	+	+	+	-	+	+	-	+	-	-	+	+	-	+	-	-	+	-	-	-	+	-	-	-	-	-
17	+	+	+	-	+	+	-	+	-	-	-	-	+	-	+	+	-	+	+	+	+	-	-	-	-	+
18	-	-	-	+	+	+	-	+	-	-	+	+	-	+	-	-	-	+	+	+	-	+	-	+	+	-
19	-	+	+	-	-	-	+	+	-	-	+	+	-	-	+	+	+	-	-	+	-	+	+	+	-	-
20	+	-	-	+	-	-	+	+	-	-	-	-	+	+	-	-	+	-	-	+	+	-	+	-	+	+
21	+	-	+	-	-	+	-	-	+	-	+	-	+	+	-	+	+	-	+	-	-	-	+	+	+	-
22	-	+	-	+	-	+	-	-	+	-	-	+	-	-	+	-	+	-	+	-	+	+	+	-	-	+
23	-	-	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	-	+	+	-	-	+	+
24	+	+	-	+	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-	-	-	-	-	+	-	-
25	+	+	-	-	+	-	-	-	-	+	-	+	+	+	+	-	+	+	-	-	-	+	+	-	+	-
26	-	-	+	+	+	-	-	-	-	+	+	-	-	-	-	+	+	+	-	-	+	-	+	+	-	+
27	-	+	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	-	+	-	+	-	-	+	+	+
28	+	-	+	+	-	+	+	-	-	+	-	+	+	-	-	+	-	-	+	-	-	+	-	-	-	-
29	+	-	-	-	-	-	-	+	+	+	+	+	+	-	-	-	-	-	-	+	+	+	-	+	-	+
30	-	+	+	+	-	-	-	+	+	+	-	-	-	+	+	+	-	-	-	+	-	-	-	-	+	-
31	-	-	-	-	+	+	+	+	+	+	-	-	-	-	-	-	+	+	+	+	-	-	+	-	-	-
32	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Sensitivity Results and Discussion

A total of 32 runs were conducted for the given factorial design. The results for average control delay and parameter effects for main and multiple parameter interactions are given in Table 6.

Table 6. Results of 32 runs and parameter effects.

Runs	Average Control Delay (s/veh)	Parameter Index No.	Parameter Effects
1	204.2	**1**	1225.0
2	1385.2	**2**	-827.9
3	22.8	**3**	-1275.4
4	257.8	**4**	565.7
5	5.3	**5**	467.5
6	169.4	**12**	-627.1
7	2.4	**13**	-1030.1
8	4.2	**14**	382.6
9	832.1	**15**	406.9
10	3224.3	**23**	637.3
11	109.4	**24**	-280.2
12	963.2	**25**	-270.5
13	66.6	**34**	-380.0
14	797.6	**35**	-408.8
15	28.0	**45**	179.6
16	70.4	**123**	441.1
17	658.7	**124**	-130.3
18	5435.3	**125**	-210.0
19	22.9	**134**	-221.5
20	933.2	**135**	-348.2
21	5.4	**145**	127.4
22	620.0	**234**	122.4
23	2.4	**235**	211.8
24	4.2	**245**	-95.2
25	3082.9	**345**	-125.8
26	12,674.4	**1234**	-24.7
27	166.3	**1245**	-43.0
28	3663.3	**2345**	41.3
29	77.4	**1235**	151.4
30	3047.7	**1345**	-73.5
31	28.0	**12345**	-10.8
32	103.2		

In order to determine the main and multiple parameter interactions with major effects on the HCM 2000 Delay model results, the parameter effects were plotted on a standard normal probability scale as suggested by Box et al. (1978). The outliers marked on Figure 1 are $\boldsymbol{v}$, $\boldsymbol{s}$, and $\boldsymbol{g}$ as main parameters, and $\boldsymbol{v}$-$\boldsymbol{s}$ and $\boldsymbol{v}$-$\boldsymbol{g}$ as 2-parameter interactions.

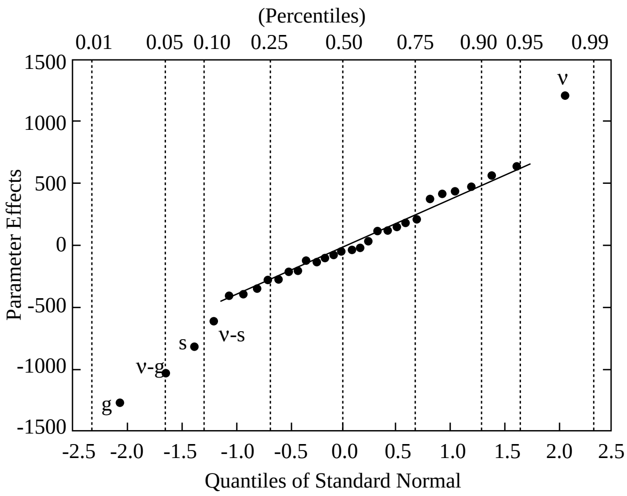

Figure 1. Parameter effects plotted on a normal probability scale.

Using the iterative approach suggested by Henderson-Sellers (1993, 1996), the identified parameters were then classified into 2 categories: primary importance and secondary importance. More specifically, the importance of these parameters was ranked based on the absolute value of their effects at the 4-, 3-, and 2-standard deviations (i.e. 4σ, 3σ, and 2σ) thresholds as shown in Table 7.

Table 7. Importance of identified parameters based on thresholds of $|4\sigma|$, $|3\sigma|$ and $|2\sigma|$.

Outliers	Primary Importance	Secondary Importance	
	$\|4\sigma\|$	$\|3\sigma\|$	$\|2\sigma\|$
$\boldsymbol{v}$	√		
$\boldsymbol{s}$		√	
g	√		
$\boldsymbol{v}$-$\boldsymbol{s}$			√
$\boldsymbol{v}$-$\boldsymbol{g}$		√	

Referring to the cumulative queuing polygon (Figure 2), the sensitivity results are consistent with the fact that the average delay per vehicle at signalized intersections is minimized when the arrival flow ($\boldsymbol{v}$) is less than the capacity of the intersection ($\boldsymbol{c}$). In this case, vehicles are mainly subjected to uniform delay and the amount of delay becomes equal to the effective red signal time or less. On the other hand, as the arrival flow exceeds the capacity, vehicles need to wait for a few signal cycles to be discharged and this causes an increase in the average delay per vehicle.

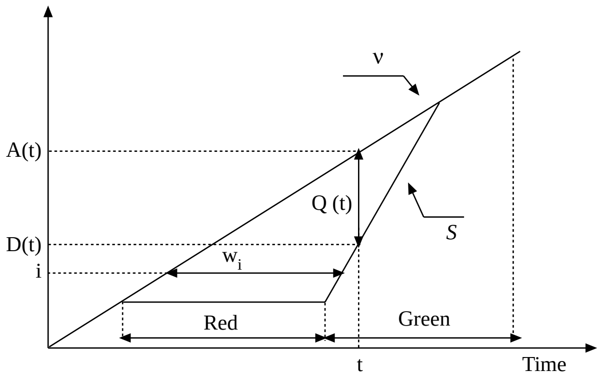

Figure 2. Cumulative queuing polygon.

In addition to the arrival flow, the saturation flow (***s***) is also a significant parameter of average delay. As queued vehicles at a signalized intersection discharge at a relatively higher rate, the effect of the queue will diminish and the average delay will decrease. On the other hand, as the arrival flow approaches the saturation flow, or vehicles discharge at a relatively lower rate, the average delay increases accordingly.

As it is known, the capacity of a signalized intersection is linearly dependent upon the saturation flow as well as the allocation of the green time (***g***) in a signal cycle. Therefore, if the green time increases, the number of vehicles to be discharged also increases and, in turn, the average delay per vehicle decreases.

The results of sensitivity analysis indicate that only 2 parameter interactions of ***v-s*** and ***v-g*** have significant effects on model results. Not surprisingly, this is due to their respective individual main parameter effects.

The study results also suggest that the remaining main parameters (i.e. the cycle length [***C***] and the analysis period [***T***]) do not have major effects on the average delay as much as the arrival flow, the saturation flow, and the green time.

A further factorial analysis was performed to investigate the effect of parameters on the uniform delay. The results showed that the green time and the cycle length appeared to be significant parameters on the uniform delay.

Using the factorial design method, a sensitivity testing of the HCM 2000 delay model to parameters was performed in this study. The evaluation of the sensitivity results show that the arrival flow, the saturation flow, and the green time are the main parameters with significant effects on the average control delay. Additionally, ***v-s*** and ***v-g*** are multiple parameters having major effects on the average control delay.

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