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Time Varying Hydromagnetic Couette Flow with Heat Transfer of a Dusty Fluid in the Presence of Uniform Suction and Injection Considering the Hall Effect

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Abstract

The time varying Couette flow with heat transfer of a dusty viscous incompressible electrically conducting fluid under the influence of a constant pressure gradient is studied without neglecting the Hall effect. The parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below while the fluid is acted upon by an external uniform magnetic field applied perpendicular to the plates. The governing equations are solved numerically using finite differences to yield the velocity and temperature distributions for both the fluid and dust particles. It is found that both the fluid and the solid-particle phases have 2 components of velocity. The main 2 components of velocity of the fluid and dust particles, u and u_p, respectively, are found to increase with an increase in the Hall parameter m. However, the other 2 components of velocity w and w_p, which result due to the Hall effect, increase with the Hall parameter m for small m and decrease with m for large values of m. It is also found that the temperatures of both fluid and particle phases decrease with the Hall parameter m.

Key words: Two phase flow, Dusty fluids, Heat transfer, Hydromagnetic flow.

Introduction

The importance and application of solid/fluid flows and heat transfer in petroleum transport, wastewater treatment, combustion, power plant piping, corrosive particles in engine oil flow, and many others are well known in the literature. Particularly, the flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occur in magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters, and have possible applications in nuclear reactors, filtration, geothermal systems, and others. The possible presence of solid particles such as ash or soot in combustion MHD generators and plasma MHD accelerators and their effect on the performance of such devices led to studies of particulate suspensions in conducting fluids in the presence of magnetic fields. For example, in an MHD generator, coal mixed with seed is fed into a combustor. The coal and seed mixture is burned in oxygen and the combustion gas expands through a nozzle before it enters the generator section. The gas mixture flowing through the MHD channel consists of a condensable vapor (slag) and a non-condensable gas mixed with seeded coal combustion products. Both the slag and the non-condensable gas are electrically conducting (Lohrabi, 1980; Chamkha, 2000). The presence of the slag and the seeded particles significantly influences the flow and heat transfer characteristics in the MHD channel. Ignoring the effect of the slag, and considering the MHD generator start-up condi-

*On leave from: Dept. of Eng. Math. and physics, Fac. of Eng., El-Fayoum University, El-Fayoum, Egypt tion, the problem reduces to unsteady 2-phase flow in an MHD channel.

The hydrodynamic flow of dusty fluids was studied by a number of authors (Saffman, 1962; Gupta and Gupta, 1976; Prasad and Ramacharyulu, 1979; Dixit, 1980; Ghosh, 1984). Later the hydromagnetic flow of dusty fluids was studied (Singh, 1976; Mitra and Bhattacharyya, 1981; Borkakotia and Bharali, 1983; Megahed et al., 1988; Aboul-Hassan et al., 1991). In the above-mentioned work, the Hall term was ignored in applying Ohm's law, as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable under these conditions, and the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term (Crammer and Pai, 1973). The effect of the Hall current on the Hartmann flow of a clean fluid was studied by a number of authors (Sutton and Sherman, 1965; Soundalgekar et al., 1979; Soundalgekar and Uplekar, 1986; Attia, 1998, 2002). Aboul-Hassan and Attia (2002) studied the influence of the Hall current on the flow and heat transfer of a dusty conducting fluid in a rectangular channel.

In the present work, the time varying Couette flow with heat transfer of an electrically conducting, viscous, incompressible, dusty fluid is studied with consideration of the Hall current. The fluid is assumed to be incompressible and electrically conducting and the particle phase is assumed to be incompressible, electrically non-conducting dusty and pressureless. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The fluid is flowing between 2 infinite electrically insulating porous plates maintained at 2 constant but different temperatures, while the particle phase is assumed to be electrically non-conducting. The fluid is subjected to a uniform suction from above and a uniform injection from below and mass conservation is assumed. An external uniform magnetic field is applied perpendicular to the plates, while no electric field is applied and the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. The fluid is acted upon by a constant pressure gradient. The governing equations are solved numerically using finite difference approximations to obtain the velocity and temperature distributions for both the fluid and dust particles as functions of space and time up till the steady state. This numerical solution proves the physical validity of the steady state solutions since they can be obtained via a time dependent process. The effects of the magnetic field, the Hall current and the suction velocity on both the velocity and temperature fields of the fluids as well as dust particles are reported.

Description of the Problem

The dusty fluid is assumed to be flowing between 2 infinite horizontal porous plates located at the $y = \pm h$ planes, as shown in Figure 1. The upper plate is moving with a constant velocity U_o, while the lower plate is kept stationary. The plates are subjected to a uniform suction from above and a uniform injection from below. Thus the y-component of the velocity of the fluid is constant and denoted by v_o. The dust particles are assumed to be electrically non-conducting, spherical and uniformly distributed throughout the fluid and to be big enough so that they are not pumped out through the porous plates and have no y-component of velocity. The 2 plates are assumed to be electrically non-conducting and kept at 2 constant temperatures: T_1 for the lower plate and T_2 for the upper plate with $T_2 > T_1$. A uniform constant pressure gradient is applied in the x-direction. A uniform magnetic field B_o is applied in the positive y-direction. This is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number (Crammer and Pai, 1973). It is required to obtain the time varying velocity and temperature distributions for both fluid and dust particles. Due to the inclusion of the Hall current term, a z-component of the velocities of the fluid and of dust particles is expected to arise (Crammer and Pai, 1973). Since the plates are infinite in the x and z-directions, the physical quantities do not depend on the x or z-coordinates and the problem is essentially one-dimensional.

Governing Equations

The governing equations for this study are based on the conservation laws of mass, linear momentum and energy of both phases.

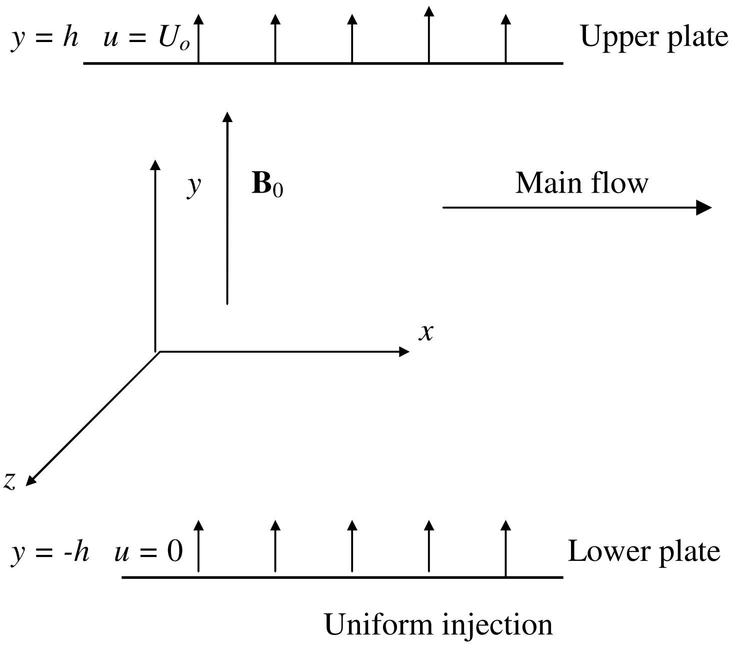

Figure 1. The geometry of the problem.

Momentum Equation

The flow of fluid is governed by the momentum equation

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} P + \mu \nabla^2 \vec{v} + \vec{J} x \vec{B}_o - KN(\vec{v} - \vec{v}_p) \quad (1)$$

where ρ is the density of clean fluid, μ is the viscosity of clean fluid, $\vec{v}$ is the velocity of the fluid, $\vec{v} = u(y,t)\vec{i} + v_o\vec{j} + w(y,t)\vec{k}$, $\vec{v}_p$is the velocity of dust particles, $\vec{v}_p = u_p(y,t)\vec{i} + w_p(y,t)\vec{k}$, $\vec{J}$ is the current density, $\vec{B}_o$ is the magnetic flux density vector, $P$is the pressure distribution, N is the number of dust particles per unit volume, K is the Stokes constant $= 6\pi\mu a$, and a is the average radius of dust particles.

The first 3 terms in the right-hand side of Eq. (1) are, respectively, the pressure gradient, viscosity, and Lorentz force terms. The last term represents the force due to the relative motion between fluid and dust particles. It is assumed that the Reynolds number of relative velocity is small. In such a case the force between dust and fluid is proportional to the relative velocity (Saffman, 1962). If the Hall term is retained, the current density $\vec{J}$ from the generalized Ohm's law is given by (Sutton and Sherman, 1965; Crammer and Pai, 1973)

$$\vec{J} = \sigma\left[\vec{E} + \vec{V} x \vec{B}_o - \beta(\vec{J} x \vec{B}_o)\right] \quad (2)$$

where σ is the electric conductivity of the fluid, and β is the Hall factor (Sutton and Sherman, 1965; Crammer and Pai, 1973). Solving Eq. (2) for $\vec{J}$ gives

$$\vec{J} x \vec{B}_o = \frac{\sigma B_o^2}{1+m^2}\left[(u+mw)\vec{i} + (w-mu)\vec{k}\right] \quad (3)$$

where $m = \sigma\beta B_o$ is the Hall parameter (Sutton and Sherman, 1965; Crammer and Pai, 1973). Thus, in terms of Eq. (3), the 2 components of Eq. (1) read

$$\rho\frac{\partial u}{\partial t} + \rho v_o \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{1+m^2}(u+mw) - KN(u-u_p) \quad (4)$$

$$\rho\frac{\partial w}{\partial t} + \rho v_o \frac{\partial w}{\partial y} = \mu\frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_o^2}{1+m^2}(w-mu) - KN(w-w_p) \quad (5)$$

The motion of the dust particles is governed by Newton's second law applied in the x and z-directions

$$m_p \frac{\partial u_p}{\partial t} = KN(u - u_p) \quad (6)$$

$$m_p \frac{\partial w_p}{\partial t} = KN(w - w_p) \quad (7)$$

where m_p is the average mass of dust particles. It is assumed that the pressure gradient is applied at t = 0 and the fluid starts its motion from rest. Thus,

$$t \leq 0 : u = u_p = w = w_p = 0 \quad (8a)$$

For $t > 0$, the no-slip condition at the plates implies that

$$t > 0, y = -h : u = u_p = w = w_p = 0 \quad (8b)$$

$$t > 0, y = h : u = U_o, u_p = w = w_p = 0 \quad (8c)$$

Energy Equation

Heat transfer takes place from the upper hot plate to the lower cold plate by conduction through the fluid. Since the hot plate is above, there is no natural convection; however, there is a forced convection due to the suction and injection. In addition to the heat transfer, there is heat generation due to both the Joule and viscous dissipations. The dust particles gain heat from the fluid by conduction through their spherical surface. Since the problem deals with a 2-phase flow, 2 energy equations are required (Schlichting, 1968; Crammer and Pai, 1973). The energy equations describing the temperature distributions for both the fluid and dust particles read

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p v_o \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \mu \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] + \frac{\sigma B_o^2}{1+m^2}(u^2 + w^2) + \frac{\rho_p c_s}{\gamma_T}(T_p - T), \quad (9)$$

$$\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma_T}(T_p - T), \quad (10)$$

where T is the temperature of the fluid, T_p is the temperature of the particles, c_p is the specific heat capacity of the fluid at constant volume, k is the thermal conductivity of the fluid, ρ_p is the mass of dust particles per unit volume of the fluid, γ_T is the temperature relaxation time, and c_s is the specific heat capacity of the particles.

The last 3 terms on the right-hand side of Eq. (9) represent the viscous dissipation, the Joule dissipation (j^2/σ), and the heat conduction between the fluid and dust particles respectively. The temperature relaxation time depends, in general, on the geometry, and since the dust particles are assumed to be spherical, the last term in Eq. (9) is equal to $4\pi aNk(T_p\text{-}T)$. Hence

$$\gamma_T = \frac{3\,\mathrm{Pr}\,\gamma_p c_s}{2c_p}$$

where γ_p is the velocity relaxation time $= 2\rho_s a^2/9\mu$, Pr is the Prandtl number $= \mu c/k$, and ρ_s is the material density of dust particles $= 3\rho_p/4\pi a^3 N$ (Saffman, 1962).

T and T_p must satisfy the initial and boundary conditions

$$t \leq 0 : T = T_p = T_1, \quad (11a)$$

$$t > 0, y = -h : T = T_p = T_1, \quad (11b)$$

$$t > 0, y = h : T = T_p = T_2. \quad (11c)$$

Equations (4)-(11) can be made dimensionless by introducing the following dimensionless variables and parameters

$$(\hat{x}, \hat{y}) = (x, y)h, \hat{t} = \frac{tU_o}{h}, (\hat{u}, \hat{w}) = \frac{(u, w)}{U_o}, (\hat{u}_p, \hat{w}_p) = \frac{(u_p, w_p)}{U_o}, \hat{P} = \frac{P}{\rho U_o^2},$$

$$\hat{T} = \frac{T - T_1}{T_2 - T_1}, \hat{T}_p = \frac{T_p - T_1}{T_2 - T_1}$$

$S = v_o/U_o$, the suction parameter,
$Re = U_o \rho h/\mu$, the Reynolds number,
$Ha = B_o h\sqrt{\sigma/\mu}$, the Hartmann number,
$Ec = U_o^2/c_p(T_2 - T_1)$, the Eckert number,
$G = m_p \mu/\rho h^2 KN$, the particle mass parameter,

$R = KNh^2/\mu$ the particle concentration parameter.

$L_o = \rho h^2/\mu\gamma_T$ the temperature relaxation time parameter.

In terms of the above dimensionless quantities Eqs. (4)-(11) read

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{1}{Re}\frac{dP}{dx} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - \frac{Ha^2}{Re(1+m^2)}(u+mw) - \frac{R}{Re}(u-u_p) \quad (12)$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} = \frac{1}{Re}\frac{\partial^2 w}{\partial y^2} - \frac{Ha^2}{Re(1+m^2)}(w-mu) - \frac{R}{Re}(w-w_p) \quad (13)$$

$$G\frac{\partial u_p}{\partial t} = u - u_p \quad (14)$$

$$G\frac{\partial w_p}{\partial t} = w - w_p \quad (15)$$

$$t \le 0 : u = u_p = w = w_p = 0 \quad (16a)$$

$$t > 0, y = -h : u = u_p = w = w_p = 0 \quad (16b)$$

$$t > 0, y = h : u = 1, u_p = w = w_p = 0 \quad (16c)$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{Re\,\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2} + \frac{Ec}{Re}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{Ha^2 Ec}{Re(1+m^2)}(u^2+w^2) + \frac{2R}{3Re\,\mathrm{Pr}}(T_p - T), \quad (17)$$

$$\frac{\partial T_p}{\partial t} = -L_o(T_p - T), \quad (18)$$

$$t \le 0 : T = T_p = 0, \quad (19a)$$

$$t > 0, y = -1 : T = T_p = 0, \quad (19b)$$

$$t > 0, y = 1 : T = T_p = 1. \quad (19c)$$

where the hats are dropped for convenience.

Numerical Solution Method

Equations (12)-(19) represent a system of partial differential equations, solved numerically using finite difference approximation. The Crank-Nicolson implicit method (Ames, 1977) is used at 2 successive time levels. Finite difference equations relating the variables are obtained by writing the equations at the mid-point of the computational cell and then replacing the different terms by their second-order central difference approximation in the y-direction. The diffusion terms are replaced by the average of the central differences at 2 successive time levels. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas-algorithm (Ames, 1977). Computations were carried out for $dP/dx = 5$, $Re = 1$, $R = 0.5$, $G = 0.8$, $L_o = 0.7$, $Pr = 1$, and $E = 0.2$.

Results and Discussion

Figures 2-4 present, respectively, the profiles of the velocity components u, u_p, w and $u_p w_p$ and temperatures T and T_p for various values of time t. The figures are plotted for Ha = 1, $m = 3$ and $S = 1$. It is observed from Figures 2a, 3a, and 4a that the velocity component u reaches the steady state faster than w, which, in turn, reaches the steady state faster than T. This is expected, since u is the source of w, while both u and w act as sources for the temperature. The same observation is clear in Figures 2b, 3b, and 4b for u_p, w_p and T_p, respectively. Comparing Figures 2a, 3a and 4a with 2b, 3b and 4b, respectively, shows that the velocity components and temperature of the fluid phase reach the steady state faster than those of the particle phase. This is because the fluid velocity is the source for the dust particles' velocity.

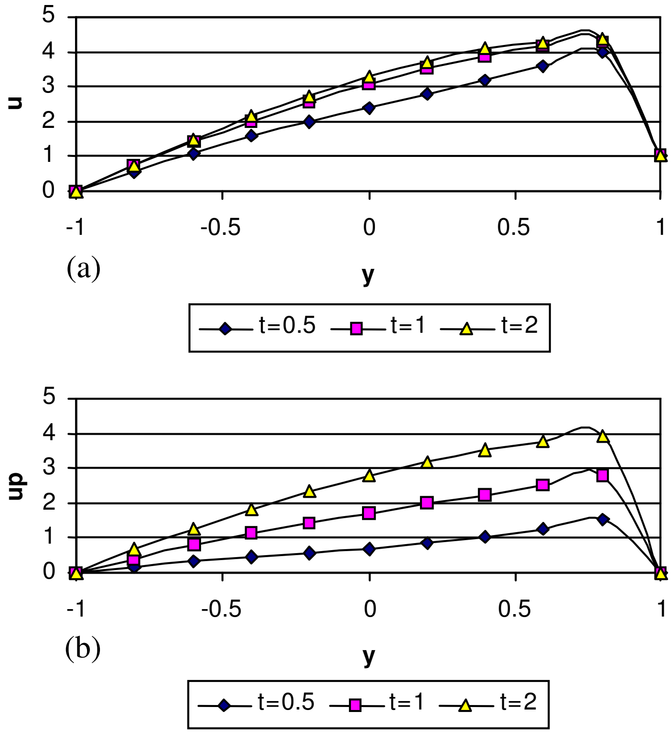

Figure 2. Time variation of the profile of: (a) u and (b) u_p. ($Ha = 1$, $m = 3$, and $S = 1$).

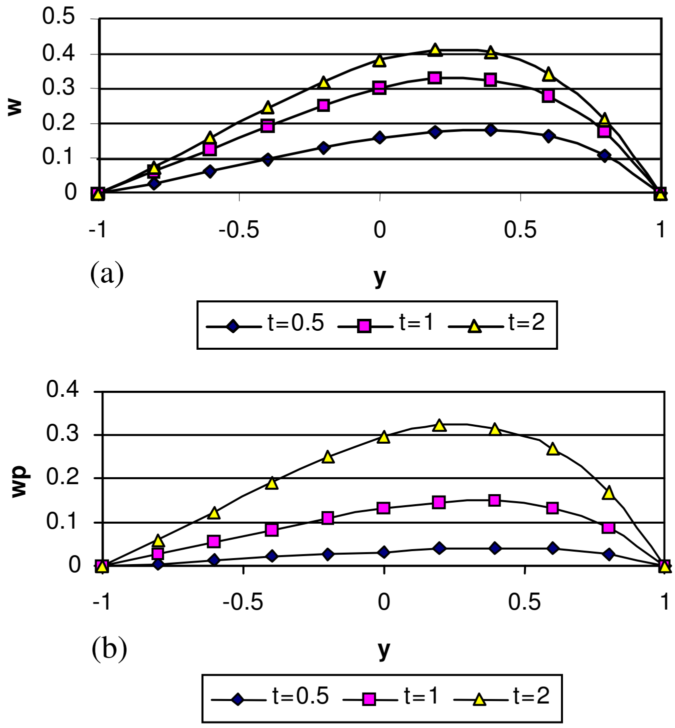

Figure 3. Time variation of the profile of: (a) w and (b) w_p. ($Ha = 1$, $m = 3$, and $S = 1$).

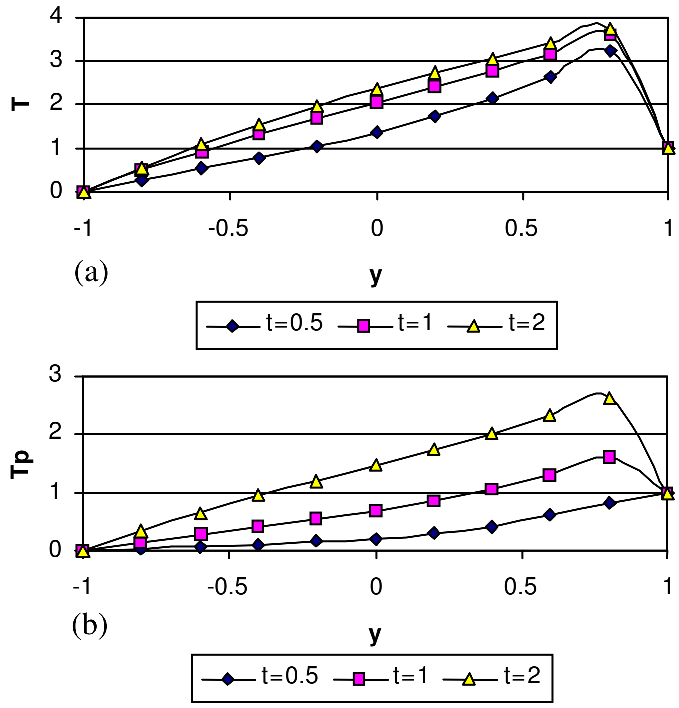

Figure 4. Time variation of the profile of: (a) T and (b) T_p. ($Ha = 1$, $m = 3$, and $S = 1$).

Figures 5-7 show the time evolution of the velocity components and temperature at the center of the channel ($y = 0$), respectively, for the fluid and particle phases for various values of the Hall parameter m and for Ha $= 1$ and $S = 0$. It is clear from Figures 5a and 5b that increasing the parameter m increases u and u_p. This is because the effective conductivity ($\sigma/(1 + m^2)$) decreases with increasing m, which reduces the magnetic damping force on u and consequently u and u_p increase. In Figures 6a and 6b, the velocity components w and w_p increase with increasing m for small values of m ($m = 0$ to 1). For large values of m (m¿ 1), the velocities decrease with m. To explain these observations, we argue that the velocity component u is the source of w, which is in turn the source for u_p. The source term is proportional to $mu/(1 + m^2)$, where u depends implicitly on m. It is clear that the source term increases or decreases with m according to whether m is small or large. Figures 7a and 7b indicate that increasing m decreases T and T_p for all t. This can be attributed to the fact that an increase in m decreases the Joule dissipation, which is proportional to ($1/(1 + m^2)$). Comparing Figures 7a and 7b ensures that the temperature of the fluid reaches the steady state faster than the temperature of the particles.

Figures 8-10 show the time evolution of the velocity components and temperature at the center of the channel ($y = 0$), respectively, for the fluid and particle phases for various values of the Hartmann number Ha and for $m = 3$ and $S = 0$. Figures 8a and 8b indicate that increasing Ha decreases u and u_p as a result of increasing damping force on u. Figures 9a and 9b ensure that increasing Ha increases w and w_p for all values of Ha due to the effect of Ha in decreasing u, which decreases the damping force on w. Figures 10a and 10b show that the increasing Ha increases T and T_p as a result of increasing the Joule dissipations.

Figures 11-13 present the time evolution of the velocity components and temperature at the center of the channel ($y = 0$), respectively, for the fluid and particle phases for various values of the suction parameter S and for $Ha = 1$ and $m = 3$. Figures 11a, 11b, 12a, and 12b show that increasing the suction decreases u, w, u_p and w_p and their steady state times due to the convection of the fluid from regions in the lower half to the center, which has higher fluid speed. Figures 13a and 13b show that increasing S decreases the temperatures T and T_p at the center of the channel. This is due to the influence of convection in pumping the fluid from the cold lower half towards the center of the channel.

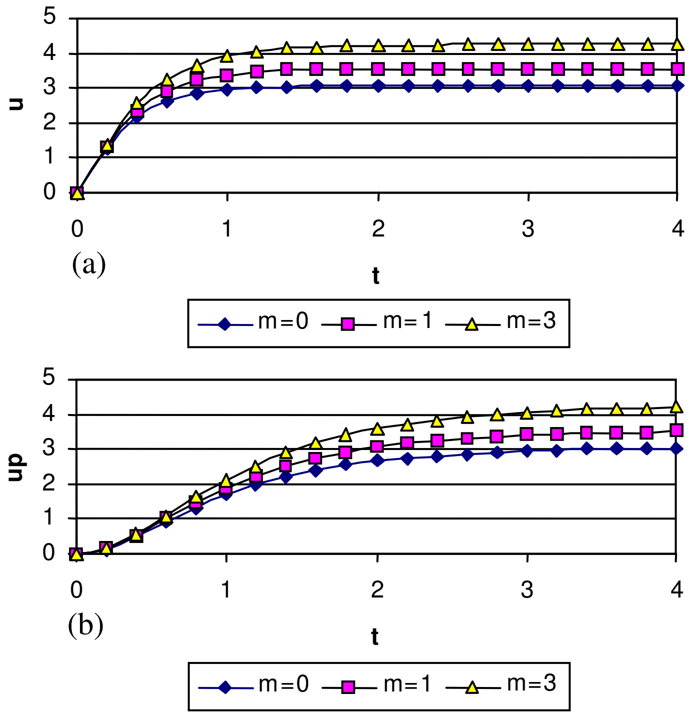

Figure 5. Effect of the parameter m on the time variation of: (a) u at $y = 0$ and (b) u_p at $y = 0$. ($Ha = 1$ and $S = 0$).

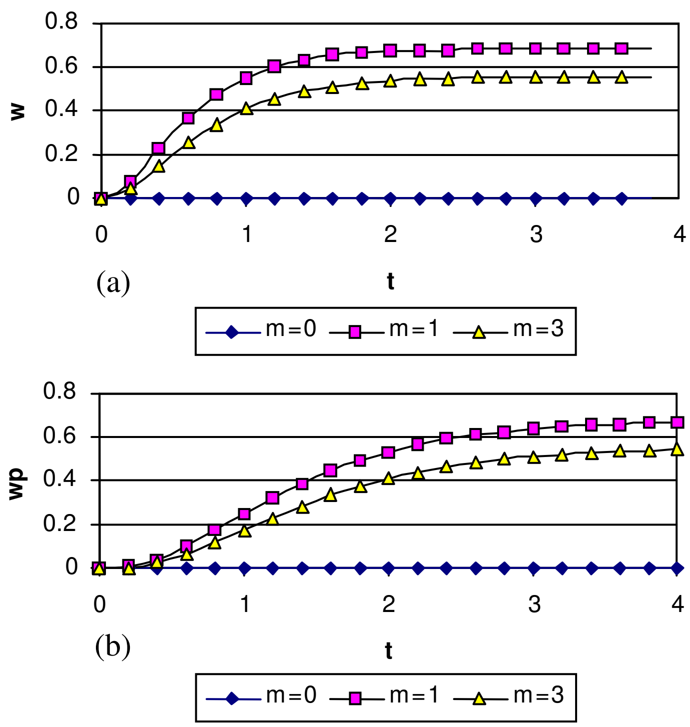

Figure 6. Effect of the parameter m on the time variation of: (a) w at $y = 0$ and (b) w_p at $y = 0$. ($Ha = 1$ and $S = 0$).

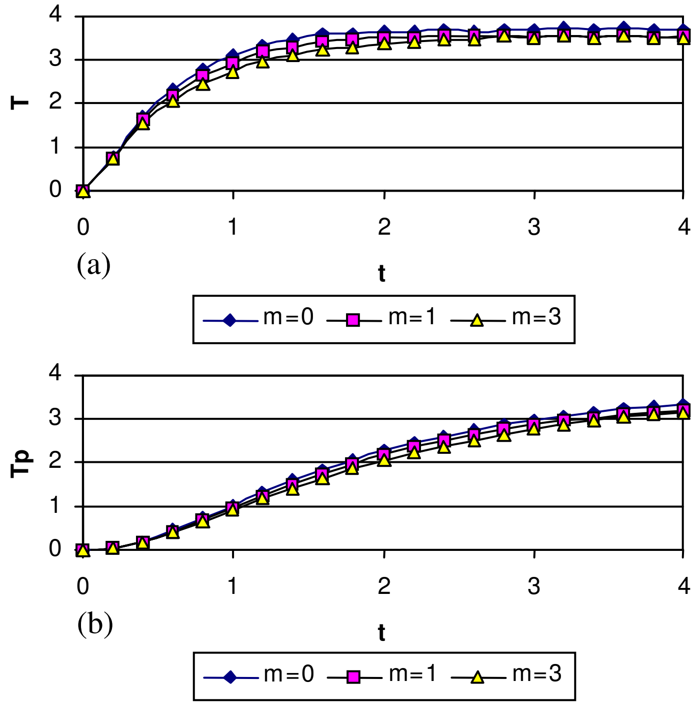

Figure 7. Effect of the parameter m on the time variation of: (a) T at $y = 0$ and (b) T_p at $y = 0$. ($Ha = 1$ and $S = 0$).

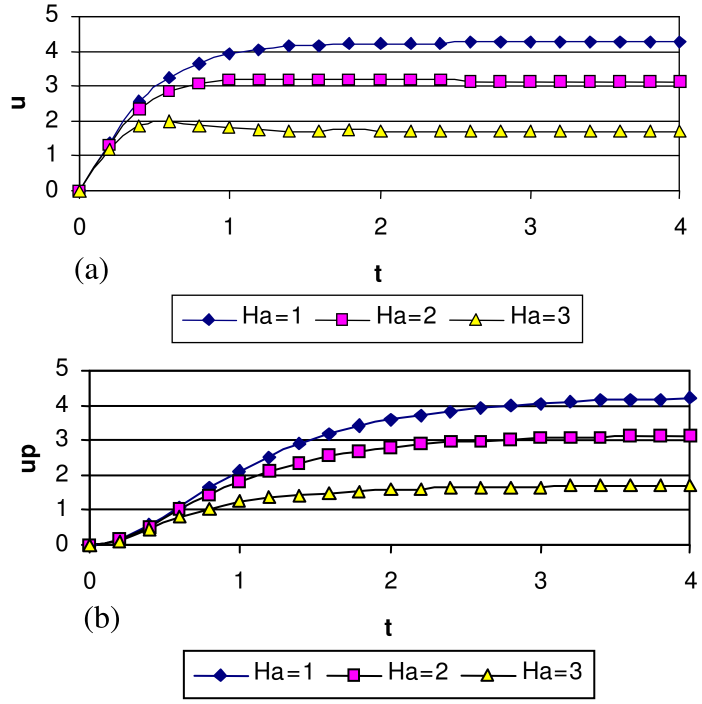

Figure 8. Effect of the parameter m on the time variation of: (a) u at $y = 0$ and (b) u_p at $y = 0$. ($m = 3$ and $S = 0$).

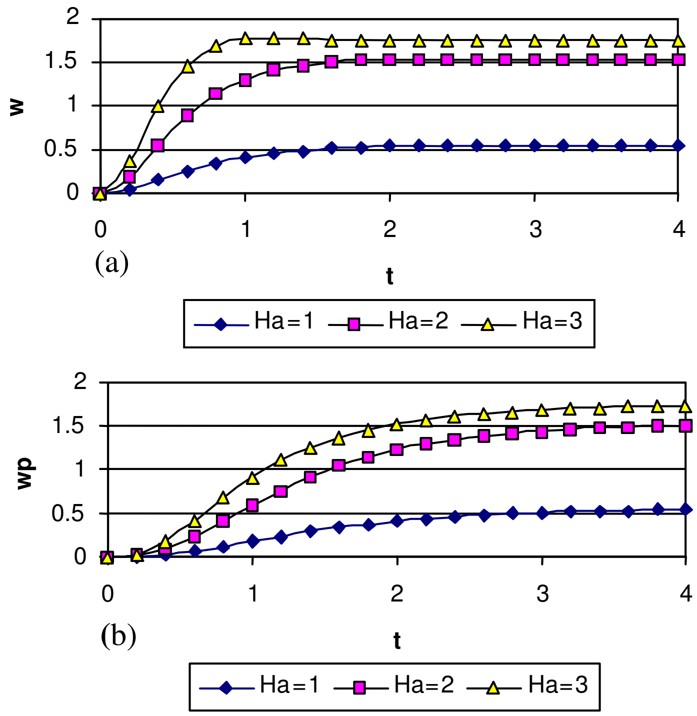

Figure 9. Effect of the parameter m on the time variation of: (a) w at $y = 0$ and (b) w_p at $y = 0$. ($m = 3$ and $S = 0$).

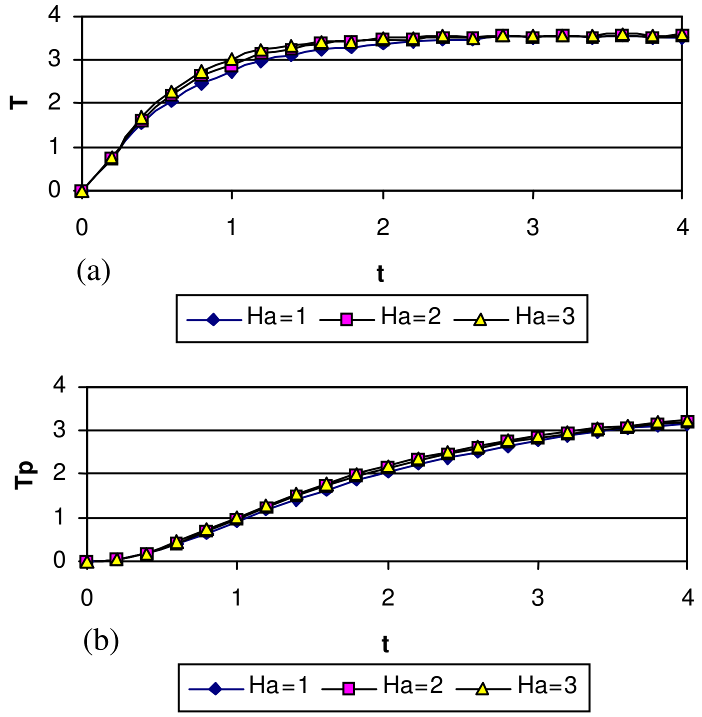

Figure 10. Effect of the parameter m on the time variation of: (a) T at $y = 0$ and (b) T_p at $y = 0$. ($m = 3$ and $S = 0$).

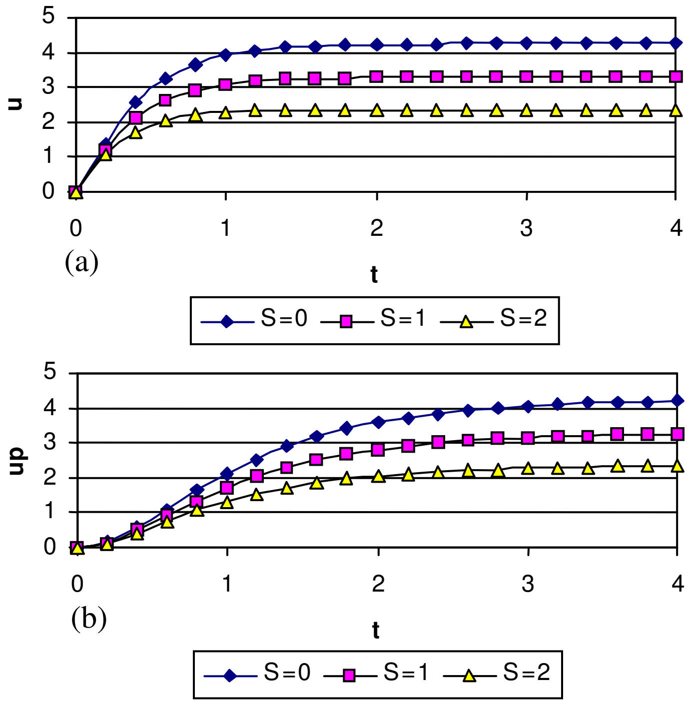

Figure 11. Effect of the parameter m on the time variation of: (a) u at $y = 0$ and (b) u_p at $y = 0$. ($m = 3$ and $Ha = 1$).

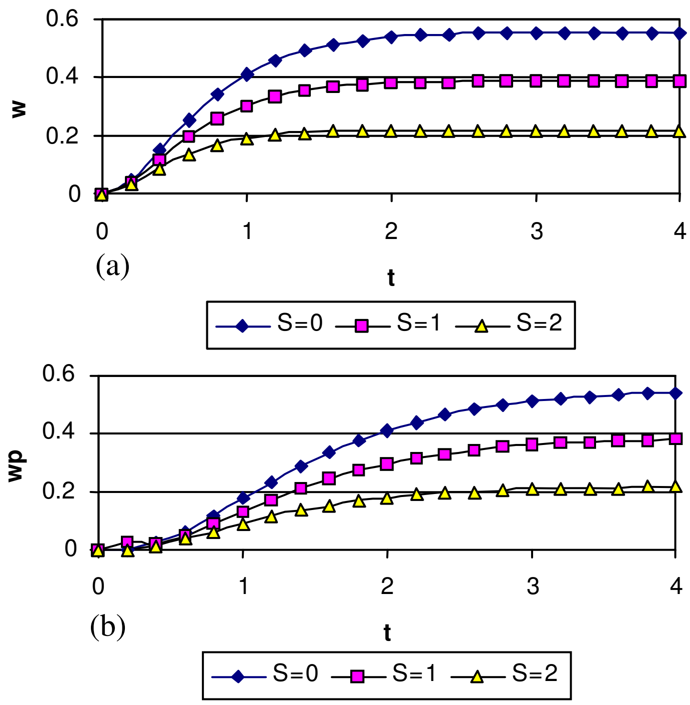

Figure 12. Effect of the parameter m on the time variation of: (a) w at $y = 0$ and (b) w_p at $y = 0$. ($m = 3$ and $Ha = 1$).

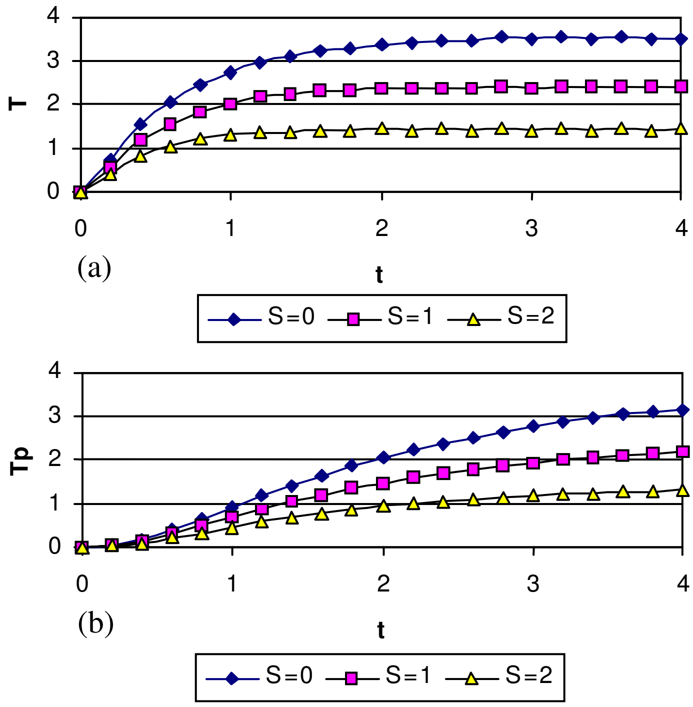

Figure 13. Effect of the parameter m on the time variation of: (a) T at $y = 0$ and (b) T_p at $y = 0$. ($m = 3$ and $Ha = 1$).

Conclusions

The time varying Couette flow with heat transfer of a dusty conducting fluid under the influence of an applied uniform magnetic field was studied, considering the Hall effect in the presence of uniform suction and injection. The effect of the magnetic field, the Hall parameter, and the suction and injection velocity on the velocity and temperature distributions for both the fluid and particle phases was investigated. It was found that both the fluid and the solid-particle phases have 2 components of velocity. The main 2 components of velocity of the fluid and dust particles u and u_p, respectively, increased with an increase in the Hall parameter m. However, the other 2 components of velocity w and w_p, which result due to the Hall effect, increase with the Hall parameter m for small m and decrease with m for large values of m. It was also found that the temperatures of both fluid and particle phases decrease with the Hall parameter m.

Nomenclature

a — the average radius of dust particles,
B_o — magnetic induction,
c_p — specific heat at constant pressure,
c_s — the specific heat capacity of the particles,
Ec — Eckert number,
G — the particle mass parameter,
h — half of the separation between the 2 plates,
Ha — Hartmann number,
J — current density,
K — thermal conductivity,
K — the Stokes constant,
L_o — the temperature relaxation time parameter.
M — the Hall parameter,
N — the number of dust particles per unit volume,
P — pressure distribution,
Pr — Prandtl number,
R — the particle concentration parameter,
Re — the Reynolds number,
S — the suction parameter,
T — temperature of the fluid,
T_p — temperature of the particles,
T_1 — temperature of the lower plate,
T_2 — temperature of the upper plate,
U — velocity component of the fluid in the x-direction,
u_p — velocity component of the particles in the x-direction,

U_o velocity of the upper plate,
v_o suction velocity,
w velocity component of the fluid in the z-direction,
w_p velocity component of the particles in the z-direction,
x axial direction,
y distance in the vertical direction,
μ viscosity of the fluid,
ρ density of the fluid,
σ electrical conductivity of the fluid,
β the Hall factor,
ρ_p the mass of dust particles per unit volume of the fluid,
ρ_s the material density of dust particles,
γ_T the temperature relaxation time,
γ_p the velocity relaxation time.

References

Aboul-Hassan, A.L., Sharaf El-Din, H., and Megahed, A.A., "Temperature Due to the Motion of One of Them", First International Conference of Engineering Mathematics and Physics, Cairo, 723-735, 1991.

Aboul-Hassan, A.L. and Attia, H.A., "Hydromagnetic Flow of a Dusty Fluid in a Rectangular Channel with Hall Current and Heat Transfer", Can. J. Phys,. 80, 579-589, 2002.

Ames, W.F., Numerical Solutions of Partial Differential Equations, Second Ed., Academic Press, New York, 1977.

Attia, H.A., "Hall Current Effects on the Velocity and Temperature Fields of an Unsteady Hartmann Flow", Can. J. Phys., 76, 739-746, 1998.

Attia, H.A., "Transient Hartmann Flow with Heat Transfer Consideration the Ion Slip", Physica Scripta, 66, 470-475, 2002.

Borkakotia, K. and Bharali, A., "Hydromagnetic Flow and Heat Transfer between Two Horizontal Plates, the Lower Plate being a Stretching Sheet", Quarterly of Applied Mathematics, II, 461-472, 1983.

Chamkha, A.J., "Unsteady Laminar Hydromagnetic Fluid-Particle Flow and Heat Transfer in Channels and Circular Pipes", International Journal of Heat and Fluid Flow, 21, 740-746, 2000.

Crammer, K.R. and Pai, S., Magnetofluid Dynamics for Engineers and Applied Physicists, McGraw-Hill, New York, 1973.

Dixit, L.A., "Unsteady Flow of a Dusty Viscous Fluid through Rectangular Ducts", Indian Journal of Theoretical Physics, 28, 129-136, 1980.

Gupta, R.K., and Gupta, S.C., "Flow of a Dusty Gas through a Channel with Arbitrary Time Varying Pressure Gradient", Journal of Applied Mathematics and Physics, 27, 119-128, 1976.

Ghosh, A.K. and Mitra, D.K., "Flow of a Dusty Fluid through Horizontal Pipes", Rev. Roum. Phys., 29, 631-645, 1984.

Lohrabi, J., "Investigation of Magnetohydrodynamic Heat Transfer in Two-Phase Flow", Ph.D. Thesis, Tennessee Technological University, P.I., 1980.

Mitra, P. and Bhattacharyya, P., "Unsteady Hydromagnetic Laminar Flow of a Conducting Dusty Fluid between Two Parallel Plates Started Impulsively from Rest", Acta Mechanica, 39, 171-183, 1981.

Megahed, A.A., Aboul-Hassan, A.L., and Sharaf El-Din, H., "Effect of Joule and Viscous Dissipation on Temperature Distributions through Electrically Conducting Dusty Fluid", Fifth Miami International Symposium on Multi-Phase Transport and Particulate Phenomena; Miami Beach, Florida, USA, 3, 111-132, 1988.

Prasad, V.R., and Ramacharyulu, N.C.P., "Unsteady Flow of a Dusty Incompressible Fluid between Two Parallel Plates under an Impulsive Pressure Gradient", Def. Sci. Journal, 30, 125-137, 1979.

Saffman, P.G., "On the Stability of a Laminar Flow of a Dusty Gas", Journal of Fluid Mechanics, 13, 120-134, 1962.

Schlichting, H., Boundary Layer Theory, McGraw-Hill, New York, 1968.

Singh, K.K., "Unsteady Flow of a Conducting Dusty Fluid through a Rectangular Channel with Time Dependent Pressure Gradient", Indian Journal of Pure and Applied Mathematics, 8, 1124-1133, 1976.

Soundalgekar, V.M., Vighnesam, N.V., and Takhar, H.S., "Hall and Ion-Slip Effects in MHD Couette Flow with Heat Transfer", IEEE Transactions on Plasma Science, 7, 178-182, 1979.

Soundalgekar, V.M. and Uplekar, A.G., "Hall Effects in MHD Couette Flow with Heat Transfer", IEEE Transactions on Plasma Science, 14, 579-583, 1986.

Spiegel, M.R., Theory and Problems of Laplace Transforms, McGraw-Hill, New York, 1986.

Sutton, G.W. and Sherman, A., Engineering Magnetohydrodynamics, McGraw-Hill, New York, 1965.