Turkish J. Eng. Env. Sci.
30 (2006) , 299 – 306.

Stagnation Point Flow towards a Stretching Surface through a Porous Medium with Heat Generation

Hazem Ali ATTIA
Al-Qasseem University, Department of Mathematics, College of Science,
Buraidah-SAUDI ARABIA
e-mail: ah1113@yahoo.com

Received 25.10.2005

Abstract

An analysis is made of the steady laminar flow in a porous medium of an incompressible viscous fluid impinging on a permeable stretching surface with heat generation. A numerical solution for the governing nonlinear momentum and energy equations is obtained. The effects of the porosity of the medium, the surface stretching velocity, and the heat generation/absorption coefficient on both the flow and heat transfer are presented and discussed.

Key words: Stagnation point flow, Porous medium, Fluid mechanics, Heat transfer, Finite differences.

Introduction

The 2-dimensional flow of a fluid near a stagnation point was first examined by Hiemenz (1911), who demonstrated that the Navier-Stokes equations governing the flow can be reduced to an ordinary differential equation of third order using similarity transformation. Later the problem of stagnation point flow was extended in numerous ways to include various physical effects. The results of these studies are of great technical importance, for example in the prediction of skin-friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery. Axisymmetric 3-dimensional stagnation point flow was studied by Homann (1936). Either in the 2- or 3-dimensional case Navier-Stokes equations governing the flow are reduced to an ordinary differential equation of third order using a similarity transformation. In hydromagnetics, the problem of Hiemenz flow was chosen by Na (1979) to illustrate the solution of a third-order boundary value problem using the technique of finite differences. An approximate solution of the same problem has been provided by Ariel (1994). The effect of an externally applied uniform magnetic field on 2- or 3-dimensional stagnation point flow was studied in the presence of uniform suction or injection (Attia, 2003a, 2003b). The study of heat transfer in boundary layer flows is of importance in many engineering applications, such as the design of thrust bearings and radial diffusers, transpiration cooling, drag reduction, and thermal recovery of oil (Massoudi and Ramezan, 1990). Massoudi and Ramezan (1992) used a perturbation technique to solve for the stagnation point flow and heat transfer of a non-Newtonian fluid of second grade. Their analysis is valid only for small values of the parameter that determines the behavior of the non-Newtonian fluid. Later Massoudi and Ramezan (1992) extended the problem to nonisothermal surfaces. Garg (1994) improved the solution obtained by Massoudi and Ramezan (1992) by computing numerically the flow characteristics for any value of the non-Newtonian parameter using a pseudo-similarity solution.

Flow of an incompressible viscous fluid over a stretching surface has important applications in the polymer industry. For instance, a number of technical processes concerning polymers involve the cooling of continuous strips (or filaments) extruded from a die by drawing them through a stagnant fluid with a controlled cooling system, and in the process of drawing these strips are sometimes stretched. The quality of the final product depends on the rate of heat transfer at the stretching surface. Crane (1970) gave a similarity solution in closed analytical form for steady 2-dimensional incompressible boundary layer flow caused by the stretching of a sheet that moves in its own plane with a velocity varying linearly with the distance from a fixed point. Carragher and Crane (1982) investigated heat transfer in the above flow in the case when the temperature difference between the surface and the ambient fluid is proportional to a power of distance from the fixed point. Temperature distribution in the flow over a stretching surface subject to uniform heat flux was studied by Dutta et al. (1985). Chiam (1994) analyzed steady 2-dimensional stagnation-point flow of an incompressible viscous fluid towards a stretching surface. Temperature distribution in the steady plane stagnation-point flow of a viscous fluid towards a stretching surface was investigated by Ray Mahapatra and Gupta (2002). Steady flow of a non-Newtonian viscoelastic fluid (Rajagopal et al., 1984; Ray Mahapatra and Gupta, 2004) or micropolar fluid (Nazar et al., 2004) past a stretching sheet was investigated with zero vertical velocity at the surface.

An analysis is made in this paper of the steady laminar flow in a porous medium of an incompressible viscous fluid impinging on a permeable stretching surface with heat generation. The wall and stream temperatures are assumed to be constants. In the analysis of the flow in the porous media the differential equation governing the fluid motion is based on Darcy's law, which accounts for the drag exerted by the porous medium (Joseph et al., 1982; Ingham and Pop, 2002; Khaled and Vafai, 2003). A numerical solution is obtained for the governing momentum and energy equations using finite difference approximations taking into account the asymptotic boundary conditions. The numerical solution computes the flow and heat characteristics for the whole range of the porosity parameter, the surface stretching velocity, the heat generation/absorption coefficient and Prandtl number.

Formulation of the Problem

Consider the steady 2-dimensional stagnation point flow in a porous medium of a viscous incompressible fluid near a stagnation point at a surface coinciding with the plane $y = 0$, the flow being in a region $y > 0$ where the space above the plane sheet is filled with the porous medium as shown in Figure 1. As pointed out by Joseph et al. (1982), the self-consistent non-linear Navier-Stokes equation that would govern the flow in a surrounding porous medium is given by (Wu et al., 2005)

$$\rho(\vec{u}.\vec{\nabla})\vec{u} + \vec{\nabla}P = \mu\nabla^2\vec{u} - \frac{\mu}{K}\vec{u} - \frac{c\rho}{\sqrt{K}}|\vec{u}|\vec{u} \qquad (1)$$

where the last 2 terms on the right-hand side of Eq. (1) describe the non-linear Darcy-Forchheimer resistance of the surrounding porous medium. Hereμ is the fluid viscosity, K is the Darcy permeability, $\vec{u}$ is the local velocity, ρ is the density of the fluid, and c is the Forchheimer constant, which has been experimentally measured for different porous media. In the present paper, we shall limit our consideration to flows where the non-linear Forchheimer term is neglected but the linear Darcy term describing the distributed body force exerted by the fibers in the porous medium is retained (Wu et al., 2005).

Two equal and opposing forces are applied along the x-axis so that the surface is stretched, keeping the origin fixed. The potential flow that arrives from the y-axis and impinges on a flat wall placed at $y = 0$, divides into 2 streams on the wall and leaves in both directions. The viscous flow must adhere to the wall, whereas the potential flow slides along it. (u,v) are the components for the potential flow of velocity at any point (x,y) for the viscous flow, whereas (U,V) are the velocity components for the potential flow. The velocity distribution in the frictionless flow in the neighborhood of the stagnation point is given by

$$U(x) = ax, V(y) = -ay$$

where the constant $a(> 0)$ is proportional to the free stream velocity far away from the stretching surface. The continuity and momentum equations for the 2-dimensional steady state flows, using the usual boundary layer approximations (Nazar et al., 2004), reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (2)$$

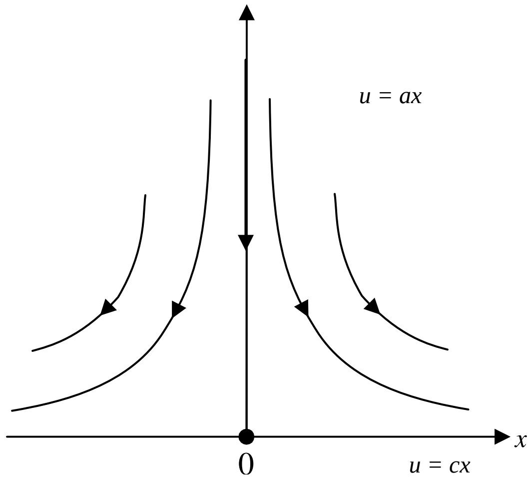

Figure 1. Physical model and coordinate system.

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=U\frac{dU}{dx}+\mu\left(\frac{\partial^2 u}{\partial y^2}\right)+\frac{\mu}{K}(U(x)-u)=0, \quad (3)$$

The boundary conditions for the above flow situation are

$$y=0: u=cx, v=0, \quad (4a)$$

$$y\to\infty: u\to ax \quad (4b)$$

where c is a positive constant.

The boundary layer equations (2) and (3) admit a similarity solution

$$u(x,y)=cxf'(\eta), v=-\sqrt{c\nu}f(\eta), \eta=\sqrt{\frac{c}{\nu}}y \quad (5)$$

where $\nu=\mu/\rho$ is the kinematic viscosity of the fluid and the prime denotes differentiation with respect to η. Using Eq. (4), we find that Eq. (2) is identically satisfied, and Eqs. (3) and (4) lead to

$$f'^2-f'''-ff''-C^2-M(C-f')=0 \quad (6)$$

$$f(0)=0, f'(0)=1, f'(\infty)=C, \quad (7)$$

where $M=\nu/cK$ is the porosity parameter and $C = a/c$ is the stretching parameter.

The governing boundary layer equation of energy, neglecting the dissipation and assuming constant thermal conductivity (Wu et al., 2005), with temperature dependent heat generation or absorption is

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty) \quad (8)$$

where c_p is the specific heat capacity at constant pressure of the fluid, k is the thermal conductivity of the fluid, T_∞ the constant temperature of the fluid far away from the sheet, Q is the volumetric rate of heat generation/absorption, and T is the temperature profile. A similarity solution exists if the wall and stream temperatures, T_w and T_∞, are constants—a realistic approximation in typical stagnation point heat transfer problems (White, 1991).

The thermal boundary conditions are

$$y=0: T=T_w, \quad (9a)$$

$$y\to\infty: T\to T_\infty, \quad (9b)$$

By introducing the non-dimensional variable

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},$$

and using Eq. (4), we find that Eqs. (7) and (8) reduce to

$$\theta'' + \Pr f\theta' + \Pr B\theta = 0 \qquad (10)$$

$$\theta(0) = 1, \theta(\infty) = 0, \qquad (11)$$

where $\Pr = \mu c_p/k$ is the Prandtl number and $B = Q/c\rho c_p$ is the dimensionless heat generation/absorption coefficient.

The flow Eqs. (6) and (7) are decoupled from the energy Eqs. (10) and (11), and need to be solved before the latter can be solved. The flow Eq. (6) constitutes a non-linear, non-homogeneous boundary value problem (BVP). In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and natural choice. The boundary value problem given by Eqs. (6) and (7) may be viewed as a prototype for numerous other situations similarly characterized by a boundary value problem having a third-order differential equation with an asymptotic boundary condition at infinity. Therefore, its numerical solution merits attention from a practical point of view. The flow Eqs. (6) and (7) are solved numerically using finite difference approximations. A quasi-linearization technique is first applied to replace the non-linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence. Then the Crank-Nicolson method is used to replace the different terms by their second-order central difference approximations. An iterative scheme is used to solve the quasi-linearized system of difference equations. The solution for the Newtonian case is chosen as an initial guess and the iterations are continued until convergence within prescribed accuracy. Finally, the resulting block tri-diagonal system was solved using a generalized Thomas algorithm.

The energy Eq. (10) is a linear second-order ordinary differential equation with variable coefficient, $f(\eta)$, which is known from the solution of the flow Eqs. (6) and (7) and the Prandtl number Pr is assumed constant. Equation (10) is solved numerically under the boundary condition (11) using central differences for the derivatives and Thomas' algorithm for the solution of the set of discritized equations. The resulting system of equations has to be solved in the infinite domain $0 < \eta < \infty$. A finite domain in the η-direction can be used instead with η chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. Grid-independence studies show that the computational domain $0 < \eta < \eta_\infty$ can be divided into intervals each of uniform step size 0.02. This reduces the number of points between $0 < \eta < \eta_\infty$ without sacrificing accuracy. The value $\eta_\infty = 10$ was found to be adequate for all the ranges of parameters studied here. Convergence is assumed when the ratio of every one of f, f', f'', orf''' for the last 2 approximations differed from unity by less than 10^{-5} at all values of η in $0 < \eta < \eta_\infty$. It should be mentioned that the results obtained herein reduce to those reported by Chiam (1994) when $M = B = 0$, and the results of the 2 papers are in close agreement, which ensures the validity of the presented solution.

Results and Discussion

Figures 2 and 3 present the velocity profiles of f and $f\prime$, respectively, for various values of C and M. The figures show that increasing the parameter C increases both f and f'. The effect of M on both f and $f\prime$ depends on C. For $C < 1$, increasing M increases f and $f\prime$ while for $C > 1$ increasing M decreases them. The figures indicate also that the effect of C on f and f' is more pronounced for smaller values of M. Moreover, increasing C decreases the velocity boundary layer thickness. Figure 4 presents the profile of temperature θ for various values of C and M and for $Pr = 0.7$ and $B = 0.1$. It is clear that increasing C decreases θ and its effect on θ becomes more apparent for smaller values of M. The figure indicates that the thermal boundary layer thickness decreases when C increases. Increasing M decreases θ for all C and its effect is clearer for smaller C.

Figure 5 presents the temperature profiles for various values of C and Pr and for $M = 1$ and $B = 0.1$. This figure clearly demonstrates the effect of the Prandtl number on the thermal boundary layer thickness. Increasing Pr decreases the thermal boundary layer thickness for all C. Increasing C decreases θ and its effect is more apparent for smaller Pr. Figure 6 presents the temperature profiles for various values of C and B and for $M = 0.5$ and $Pr = 0.7$. Increasing B increases the temperature θ and the boundary layer thickness. The effect of B on θ is more pronounced for smaller C. However, the effect of C on θ is more apparent for higher B.

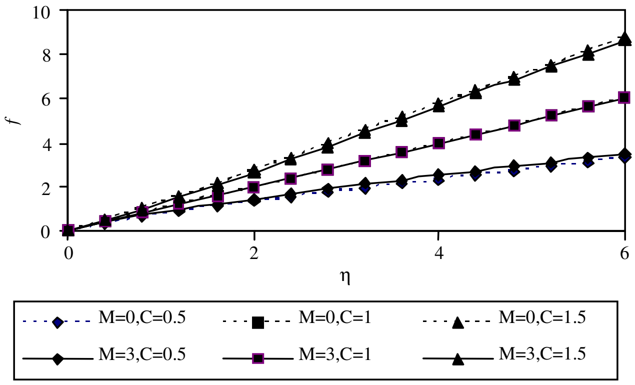

Figure 2. Effect of the parameters C and M on the profile of f.

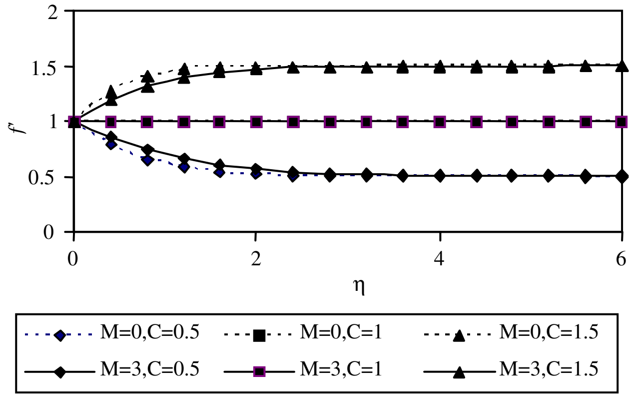

Figure 3. Effect of the parameters C and M on the profile of f'.

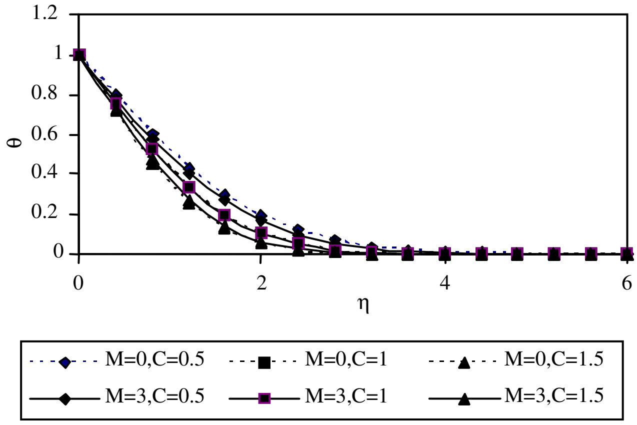

Figure 4. Effect of the parameters C and Pr on the profile of θ (M= 1, B= 0.1).

Tables 1 and 2 present the variation of the dimensionless wall shear stress $f''(0)$ and the dimensionless heat transfer rate at the wall $\theta'(0)$, respectively, for various values of C and M and for Pr = 0.7 and B = 0. Increasing C increases $f''(0)$ for all M. For small M, increasing M decreases the magnitude of $f''(0)$ but increasing M more increases it. It is of interest to see the reversal of the sign of $f''(0)$ for $C < 1$ for all M. Table 2 shows that increasing C or M increases $-\theta'(0)$ for all M. The effect of C on $f''(0)$ and $-\theta'(0)$ is more pronounced for smaller M. For $C < 1$, increasing M increases $-\theta'(0)$; however, for $C > 1$, increasing M decreases $-\theta'(0)$.

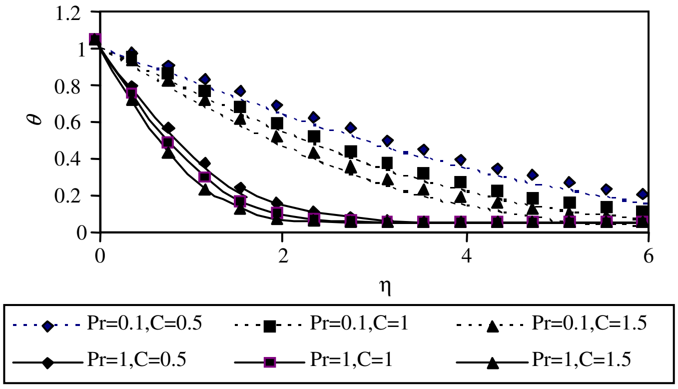

Figure 5. Effect of the parameters C and M on the profile of θ (Pr = 0.7, B= 0.1).

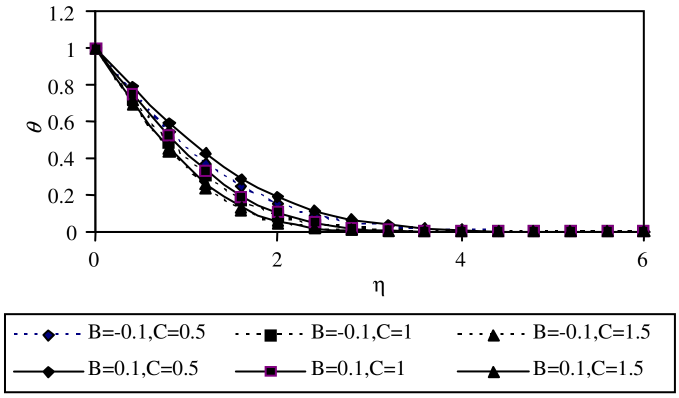

Figure 6. Effect of the parameters C and B on the profile of θ (M= 0.5, Pr = 0.7).

Table 1. Variation of the wall shear stress $f''(0)$with C and M.

M	$C = 0.1$	$C = 0.2$	$C = 0.5$	$C = 1$	$C = 1.1$	$C = 1.2$	$C = 1.5$
0	-1.4541	-1.3772	-1.0009	0	0.2464	0.5066	1.3642
1	-1.3851	-1.3119	-0.9534	0	0.2347	0.4826	1.2996
2	-1.3709	-1.2984	-0.9436	0	0.2323	0.4776	1.2862
3	-1.3795	-1.3065	-0.9495	0	0.2338	0.4806	1.2943

Table 2. Variation of the wall heat transfer rate $-\theta'(0)$ with C and M ($Pr = 0.7$, $B = 0$).

M	$C = 0.1$	$C = 0.2$	$C = 0.5$	$C = 1$	$C = 1.1$	$C = 1.2$	$C = 1.5$
0	0.6454	0.6819	0.7773	0.9109	0.9354	0.9588	1.0263
1	0.5974	0.6493	0.7653	0.9109	0.9365	0.9587	1.0309
2	0.5112	0.5901	0.7421	0.9109	0.9392	0.9661	1.0412
3	0.4402	0.5405	0.7211	0.9109	0.9419	0.9713	1.0522

Table 3 presents the effect of C on $-\theta'(0)$ for various values of Pr and for $M = 1$ and $B = 0$. Increasing C increases $-\theta'(0)$ for all Pr and its effect is more pronounced for higher Pr. Increasing Pr increases $-\theta'(0)$ for all C and its effect is more apparent for higher C. Table 4 presents the effect of the parameters C and B on $-\theta'(0)$ for $M = 0.5$ and $Pr = 0.7$. Increasing C increases $-\theta'(0)$ for all B, but increasing B decreases $-\theta'(0)$ for all C.

Table 3. Variation of the wall heat transfer rate $-\theta'(0)$ with C and Pr ($M = 1$, $B = 0$).

Pr	$C = 0.1$	$C = 0.2$	$C = 0.5$	$C = 1$	$C = 1.1$	$C = 1.2$	$C = 1.5$
0.05	0.1226	0.1292	0.1497	0.1834	0.1899	0.1962	0.2146
0.1	0.1467	0.1599	0.1982	0.2533	0.2631	0.2725	0.2989
0.5	0.3471	0.3840	0.4661	0.5674	0.5849	0.6019	0.6498
1	0.5539	0.5923	0.6839	0.8043	0.8257	0.8465	0.9057

Table 4. Variation of the wall heat transfer rate $-\theta'(0)$ with C and B ($M = 0.5$, $Pr = 0.7$).

B	$C = 0.1$	$C = 0.2$	$C = 0.5$	$C = 1$	$C = 1.1$	$C = 1.2$	$C = 1.5$
-0.1	0.5398	0.5610	0.6217	0.7127	0.7297	0.7462	0.7938
0	0.4667	0.4928	0.5659	0.6676	0.6859	0.7038	0.7548
0.1	0.3736	0.4143	0.5054	0.6201	0.6402	0.6596	0.7143

Conclusion

The 2-dimensional stagnation point flow in a porous medium of a viscous incompressible fluid impinging on a permeable stretching surface is studied with heat generation/absorption. A numerical solution for the governing equations is obtained, allowing the computation of the flow and heat transfer characteristics for various values of the porosity parameter, the stretching velocity, the heat generation/absorption parameter and the Prandtl number. The results indicate that increasing the stretching velocity increases the velocity components but decreases the velocity boundary layer thickness. On the other hand, increasing the stretching velocity decreases the temperature as well as the thermal boundary layer thickness. The effect of the stretching parameter on the velocity and temperature is more apparent for smaller values of the porosity parameter. The variation of velocity components as well as the rate of heat transfer at the wall with the porosity parameter depends on the magnitude of the stretching velocity. The sign of the wall shear stress was shown to depend on the stretching velocity. The effect of the heat generation/absorption parameter B on the rate of heat transfer at the wall becomes more apparent for smaller C.

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