Turkish J. Eng. Env. Sci.
30 (2006) , 359 – 365.
© TÜBİTAK

Investigation of Non-Newtonian Micropolar Fluid Flow with Uniform Suction/Blowing and Heat Generation

Hazem Ali ATTIA
Al-Qasseem University, Department of Mathematics, College of Science, Buraidah-SAUDI ARABIA
e-mail: ah1113@yahoo.com

Received 09.03.2006

Abstract

The steady laminar flow with heat generation of an incompressible non-Newtonian micropolar fluid impinging on a porous flat plate is investigated. A uniform suction or blowing is applied normal to the plate, which is maintained at a constant temperature. A new mathematical model is developed taking into account the new elements introduced, such as uniform suction and heat generation. Numerical solutions for the governing nonlinear momentum and energy equations are obtained. The effect of the uniform suction or blowing and the characteristics of the non-Newtonian fluid on both the flow and heat transfer is presented and discussed.

Key words: Stagnation point flow, Stretching sheet, Heat generation, Finite difference method.

Introduction

The two-dimensional flow of a fluid near a stagnation point is a classical problem in fluid mechanics. It was first examined by Hiemenz (1911), who demonstrated that the Navier-Stokes equations governing the flow can be reduced to an ordinary differential equation of the third order using similarity transformation. Owing to the nonlinearities in the reduced differential equation, no analytical solution is available and the nonlinear equation is usually solved numerically, subject to two-point boundary conditions, one of which is prescribed at infinity.

Later, the problem of stagnation point flow was extended in numerous ways to include various physical effects. Axisymmetric three-dimensional stagnation point flow was studied by Homann (1936). The results of these studies are of great technical importance: for example in the prediction of skin-friction, as well as heat/mass transfer near stagnation regions of bodies in high speed flows, and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling, and thermal oil recovery. In either the two- or three-dimensional case, Navier-Stokes equations governing the flow are reduced to an ordinary differential equation of the third order using a similarity transformation. The effect of suction on the Hiemenz flow problem has been considered in the literature. Schlichting and Bussman first gave the numerical results in 1943. More detailed solutions were later presented by Preston (1946). An approximate solution to the problem of uniform suction is given by Ariel (1994). In hydromagnetics, the problem of Hiemenz flow was chosen by Na (1979) to illustrate the solution of a third-order boundary value problem using the technique of finite differences. An approximate solution of the same problem has been provided by Ariel (1994). The effect of an externally applied uniform magnetic field on the two- and three-dimensional stagnation point flow was given, respectively, by Attia (2003, 2003) in the presence of uniform suction or injection.

On the other hand, the study of heat transfer in boundary layer flows is of importance in many engineering applications, such as the design of thrust bearings and radial diffusers, transpiration cooling, drag reduction, and thermal recovery of oil. Massoudi and Ramezan (1990) used a perturbation technique to solve for the stagnation point flow and heat transfer of a second grade non-Newtonian fluid. Their analysis is valid only for small values of the parameter that determines the behavior of the non-Newtonian fluid. Later, Massoudi and Ramezan (1992) extended the problem to nonisothermal surfaces. Garg (1994) improved the solution obtained by Massoudi and Ramezan (1992) by numerically computing the flow characteristics for any value of the non-Newtonian parameter, using a pseudo-similarity solution.

Non-Newtonian fluids were considered by many researchers. Thus, among the non-Newtonian fluids, the solution of the stagnation point flow for viscoelastic fluids has been given by Arial (1992) and others; for power-law fluid by Djukic (1974); and for second grade fluids in the hydromagnetic case by Attia (2000). Stagnation point flow of a non-Newtonian micropolar fluid with zero vertical velocity at the surface or heat generation was studied by Nazar et al. (2004). It should be mentioned that the theory of micropolar fluids has potential importance in industrial applications due to its capability to describe complex fluids, such as particle suspensions, liquid crystals, animal blood, lubrication, and turbulent shear flows (Nazar et al., 2004).

The purpose of the present paper is to study the effect of uniform suction or blowing directed normal to the wall on the steady laminar flow of an incompressible non-Newtonian micropolar fluid at a two-dimensional stagnation point with heat generation. The wall and stream temperatures are assumed to be constants. A numerical solution is obtained for the governing momentum and energy equations using finite difference approximations, which takes into account the asymptotic boundary conditions. The numerical solution computes the flow and temperature fields for the whole range of the non-Newtonian fluid characteristics, and also such parameters as the suction or blowing parameter, and the Prandtl number.

Formulation of the Problem

Consider the two-dimensional stagnation point flow of an incompressible non-Newtonian micropolar fluid impinging perpendicular on a permeable wall and flowing away along the x-axis. This is an example of a plane potential flow that arrives from the y-axis and impinges on a flat wall placed at $y = 0$, divides into 2 streams on the wall, and leaves in both directions. The viscous flow must adhere to the wall, whereas the potential flow slides along it. (u,v) are the components for the potential flow of velocity at any point (x,y) for the viscous flow, whereas (U,V) are the velocity components for the potential flow. A uniform suction or blowing is applied at the plate with a transpiration velocity at the boundary of the plate given by $-v_o$, where $v_o > 0$ for suction. The velocity distribution in the frictionless flow in the neighborhood of the stagnation point is given by

$$U(x) = ax, V(y) = -ay$$

where the constant $a(> 0)$ is proportional to the free stream velocity far away from the surface. The simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar, and incompressible micropolar fluid are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = (\mu + h)\left(\frac{\partial^2 u}{\partial y^2}\right) + h\frac{\partial N}{\partial y}, \tag{2}$$

$$\rho\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \frac{\gamma}{j}\frac{\partial^2 N}{\partial y^2} - \frac{k}{j}\left(2N + \frac{\partial u}{\partial y}\right), \tag{3}$$

where N is the microrotation or angular velocity whose direction of rotation is in the $x - y$ plane, μ is the viscosity of the fluid, ρ is the density, and j, γ, and h are the micro-inertia per unit mass, spin gradient viscosity, and vortex viscosity, respectively, which are assumed to be constant (Nazar et al., 2004). The appropriate physical boundary conditions of Eqs. (1)-(3) are (Nazar et al., 2004)

$$u(x,0) = 0, v(x,0) = -v_o, N(x,0) = -n\frac{\partial u}{\partial y}, \tag{4a}$$

$$y \to \infty : u(x,y) \to U(x) = ax, v(x,y) \to 0, N(x,y) \to 0, \tag{4b}$$

where n is a constant and $0 \le n \le 1$. The case $n = 1/2$ indicates the vanishing of the anti-symmetric part of the stress tensor and denotes weak concentration of microelements (Nazar et al., 2004), which will be considered here. The governing Eqs. (1)-(3), subject to the boundary conditions Eqs. (4a) and (4b), can be expressed in a simpler form by introducing the following transformation

$$\eta = \sqrt{\tfrac{a}{\nu}}y, u = axf'(\eta), v = -\sqrt{a\nu}f(\eta), N = ax\sqrt{\tfrac{a}{\nu}}g(\eta), g(\eta) = -\tfrac{1}{2}f''(\eta) \tag{5}$$

so that Eqs. (2) and (3) reduce to the single equation

$$\left(1+\frac{K}{2}\right)f''' + ff'' - f'^2 + 1 = 0 \tag{6}$$

subject to the boundary conditions

$$f(0) = A, f'(0) = 0, f'(\infty) = 1, \tag{7}$$

where $K = h/\mu(>0)$ is the material parameter, $A = v_o/\sqrt{a\nu}$ is the suction parameter, and primes denote differentiation with respect to η. For micropolar boundary layer flow, the wall skin friction τ_w is given by (Nazar et al., 2004)

$$\tau_w = \left[(\mu+h)\frac{\partial u}{\partial y} + hN\right]_{y=0} \tag{8}$$

Using $U(x) = ax$ as a characteristic velocity, the skin friction coefficient, C_f, can be defined as

$$C_f = \frac{\tau_w}{\rho U^2}, \tag{9}$$

Substituting Eqs. (5) and (8) into Eq. (9), we get

$$C_f Re_x^{1/2} = (1+K/2)f''(0) \tag{10}$$

where $Re_x^{1/2} = xU/\nu$ is the local Reynolds number.

Using the boundary layer approximations and neglecting the dissipation, the equation of energy for temperature T with heat generation or absorption is given by Massoudi (1992),

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + Q(T - T_w) \tag{11}$$

where c_p is the specific heat capacity at constant pressure of the fluid, k is the thermal conductivity of the fluid, and Q is the heat generation/absorption coefficient. A similarity solution exists if the wall and stream temperatures, $T_w$and T_∞, are constants—a realistic approximation in typical stagnation point heat transfer problems (Massoudi, 1992).

The boundary conditions for the temperature field are

$$y = 0 : T = T_w, \tag{12a}$$

$$y \to \infty : T \to T_\infty, \tag{12b}$$

Introducing the non-dimensional variable

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},$$

and using the similarity transformations given in Eq. (5), we find that Eqs. (11) and (12) reduce to

$$\theta'' + \Pr f\theta' + \Pr B\theta = 0 \tag{13}$$

$$\theta(0) = 1, \theta(\infty) = 0, \tag{14}$$

where $\Pr = \mu c_p/k$ is the Prandtl number and $B = Q/a\rho c_p$ is the heat generation/absorption parameter. The heat transfer from the surface to the fluid is computed by application of Fourier's law

$$q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Introducing the transformed variables, the expression for q becomes

$$q = -k(T_w - T_\infty)\sqrt{a/\nu}\theta'(0) \tag{15}$$

The heat transfer coefficient, in terms of the Nusselt number, Nu, can be expressed as

$$Nu = \frac{q}{k(T_w - T_\infty)\sqrt{a/\nu}} \tag{16}$$

where $\sqrt{a/\nu}$ plays the role of a characteristic length. Using Eq. (15), Eq. (16) becomes

$$Nu = -\theta'(0) \tag{17}$$

The flow Eqs. (6) and (7) are decoupled from the energy Eqs. (13) and (14), and need to be solved before the latter can be solved. The flow Eq. (6) constitutes a non-linear, non-homogeneous boundary value problem (BVP). In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and natural choice. The BVP given by Eqs. (6) and (7) may be viewed as a prototype for numerous other situations, which are similarly characterized by a BVP having a third order differential equation with an asymptotic boundary condition at infinity. Therefore, its numerical solution merits attention from a practical point of view. The flow Eqs.

(6) and (7) are solved numerically using finite difference approximations. A quasi-linearization technique is first applied to replace the non-linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence. The quasi-linearized form of Eq. (6) is

$$(1+K/2)f_{n+1}''' + f_n f_{n+1}'' + f_n'' f_{n+1}$$
$$-f_n'' f_n - 2f_n' f_{n+1}' + f_n'^2 + 1 = 0$$

where the subscript n or $n+1$ represents the nth or $(n+1)$th approximation to the solution. Then, the Crank-Nicolson method is used to replace the different terms by their second order central difference approximations. An iterative scheme is used to solve the quasi-linearized system of difference equations. The solution for the Newtonian case is chosen as an initial guess and the iterations are continued until convergence within prescribed accuracy. Finally, the resulting block tri-diagonal system was solved using the generalized Thomas algorithm.

The energy Eq. (13) is a linear second order ordinary differential equation with a variable coefficient, $f(\eta)$, which is known from the solution of the flow Eqs. (6) and (7), and the Prandtl number, Pr, is assumed to be constant. Equation (13) is solved numerically under the boundary condition Eq. (14) using central differences for the derivatives and the Thomas algorithm for the solution of the set of discretized equations. The resulting system of equations has to be solved in the infinite domain $0 < \eta < \infty$. A finite domain in the η-direction can be used instead with η chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. Grid-independence studies show that the computational domain $0 < \eta < \eta_\infty$ can be divided into intervals, each of uniform step size, which equals 0.02. This reduces the number of points between $0 < \eta < \eta_\infty$ without sacrificing accuracy. The value $\eta_\infty = 10$ was found to be adequate for all the ranges of parameters studied here. Convergence is assumed when the ratio of every one of f, f', f'', or f''' for the last 2 approximations differed from unity by less than 10^{-5} at all values of η in $0 < \eta < \eta_\infty$. It should be mentioned that the results obtained herein reduce to those reported by Nazar et al. (2004) when $A = 0$ and $B = 0$, which gives validity of the present solution.

Results and Discussion

Figures 1 and 2 present the velocity profiles of f and f', respectively, for various values of K and A. The figures show that increasing the parameter K decreases both f and f', but increasing A increases them. The figures also indicate that the effect of K on f and f' is more pronounced for higher values of A (case of suction); however, the effect of A on f and f' becomes more pronounced for smaller values of K. Moreover, increasing K increases the velocity boundary layer thickness, while increasing A decreases it.

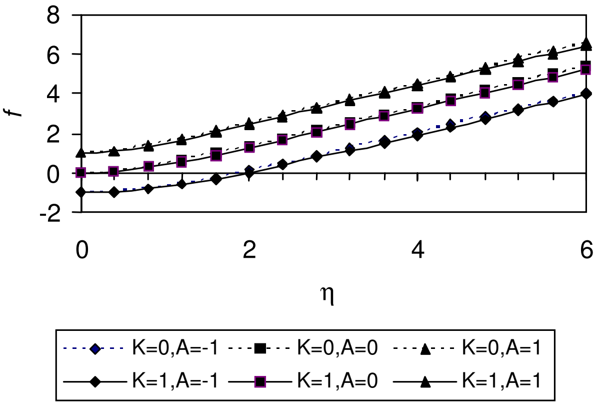

Figure 1. Effect of the parameters K and A on the profile of f.

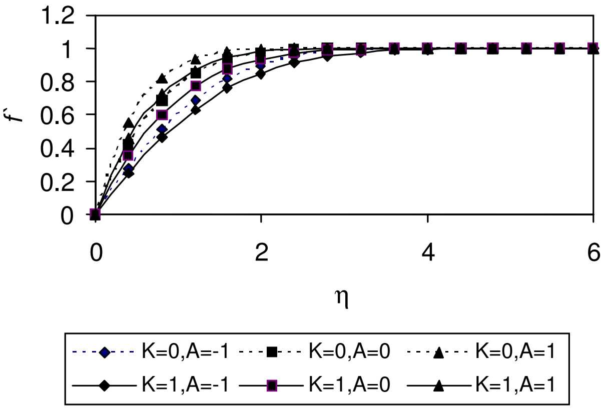

Figure 2. Effect of the parameters K and A on the profile of f'.

Figure 3 presents the profile of temperature θ for various values of K and A and for $Pr = 0.5$ and $B = 0$. It is clear that increasing K increases θ and the thickness of the thermal boundary layer. Increasing

A decreases θ for all K and its influence becomes more apparent for smaller K. This emphasizes the influence of the injected flow in the cooling process. The action of fluid injection ($A < 0$) is to fill the space immediately adjacent to the disk with fluid having nearly the same temperature as that of the disk. As the injection becomes stronger, the blanket extends to greater distances from the surface. As shown in Figure 3, these effects are manifested by the progressive flattening of the temperature profile adjacent to the disk. Thus, the injected flow forms an effective insulating layer, decreasing the heat transfer from the disk. Suction, on the other hand, serves the function of bringing large quantities of ambient fluid into the immediate neighborhood of the disk surface. As a consequence of the increased heat-consuming ability of this augment flow, the temperature drops quickly as we proceed away from the disk. The presence of fluid at near-ambient temperature close to the surface increases the heat transfer.

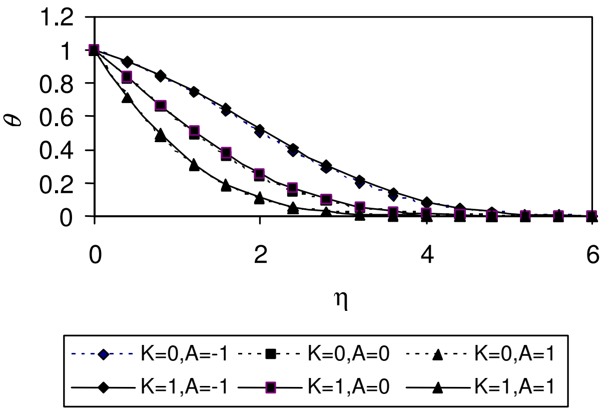

Figure 3. Effect of the parameters K and A on the profile of θ (Pr = 0.5).

Figures 4 and 5 present the temperature profiles for various values of K and Pr, and for A = –0.5 and 0.5, respectively, and for $B = 0$. The figures bring out clearly the effect of the Prandtl number on the thermal boundary layer thickness. As shown in Figures 4 and 5, increasing Pr decreases the thermal boundary layer thickness for all K and A. Figure 4 shows the influence of blowing in flattening of the temperature profiles adjacent to the disk for higher Pr. The effect of K on θ is more pronounced for higher values of Pr for the blowing case (see Figure 4).

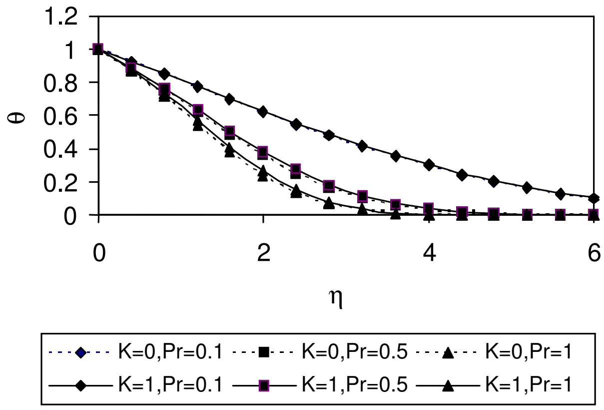

Figure 4. Effect of the parameters K and Pr on the profile of θ (A = –0.5).

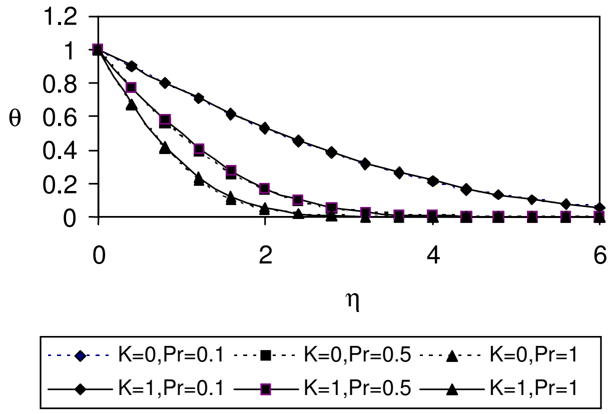

Figure 5. Effect of the parameters K and Pr on the profile of θ (A = 0.5).

Tables 1 and 2 present the variation in the wall shear stress $C_f Re_x^{1/2}$ and the heat transfer rate at the wall $-\theta^{'}(0)$, respectively, for various values of K and A, and for $Pr = 0.5$. Table 1 shows that for $A <$ 0, increasing K steadily increases $C_f Re_x^{1/2}$; however, for $A \geq 0$, increasing K increases $C_f Re_x^{1/2}$, and then additionally increasing K decreases $C_f Re_x^{1/2}$. Increasing A increases $C_f Re_x^{1/2}$ for all K and its effect is more apparent for smaller K. Table 2 shows that increasing K decreases $-\theta^{'}(0)$, while increasing A increases $-\theta^{'}(0)$ for all K.

Table 3 presents the effect of the parameters K and B on $-\theta^{'}(0)$ for $A = 0$ and $Pr = 0.7$. Increasing K decreases $-\theta^{'}(0)$ for all B, but increasing B decreases $-\theta^{'}(0)$ for all K as result of increasing the temperature, which reduces the heat transfer. Table 4 presents the effect of the parameters A and B on

$-\theta^{'}(0)$ for $K = 1$ and $Pr = 0.7$. Increasing A increases $-\theta^{'}(0)$ for all B, but increasing B decreases $-\theta^{'}(0)$ for all A. Table 5 presents the effect of the parameters Pr and B on $-\theta^{'}(0)$ for $K = 1$ and $A = 0$. Increasing Pr increases $-\theta^{'}(0)$ for all B, but increasing B decreases $-\theta^{'}(0)$ for all Pr.

Table 1. Variation in the wall shear stress $C_f Re_x^{1/2}$ with K and $A(Pr = 0.7,\ B = 0.1)$.

A	$K = 0$	$K = 0.5$	$K = 1$	$K = 1.5$	$K = 2$
-2	0.7137	0.7574	0.7986	0.8377	0.8751
-1	1.1349	1.1586	1.1839	1.2103	1.2372
0	1.2326	1.7915	1.7612	1.7469	1.7431
1	1.8892	2.6352	2.5116	2.4312	2.3779
2	2.6699	3.6299	3.3899	3.2271	3.1124

Table 2. Variation in the wall heat transfer $-\theta^{'}(0)$with K and A $(Pr = 0.7,\ B = 0.1)$.

A	$K = 0$	$K = 0.5$	$K = 1$	$K = 1.5$	$K = 2$
-2	–0.0356	–0.0364	–0.0372	–0.0379	–0.0385
-1	0.0809	0.0747	0.0694	0.0647	0.0605
0	0.4387	0.4259	0.4152	0.4060	0.3979
1	0.9722	0.9575	0.9454	0.9352	0.9262
2	1.5881	1.5744	1.5631	1.5536	1.5455

Table 3. Variation in the wall heat transfer rate $-\theta^{'}(0)$ with K and B $(A = 0,\ Pr = 0.7)$.

B	$K = 0$	$K = 0.5$	$K = 1$	$K = 1.5$	$K = 2$
-0.1	0.5495	0.5389	0.5302	0.5228	0.5163
0	0.4959	0.4843	0.4747	0.4665	0.4593
0.1	0.4387	0.4259	0.4152	0.4060	0.3979

Table 4. Variation in the wall heat transfer rate $-\theta^{'}(0)$ with A and B $(K = 1,\ Pr = 0.7)$.

B	$A = -2$	$A = -1$	$A = 0$	$A = 1$	$A = 2$
-0.1	0.0637	0.1962	0.5302	1.0346	1.6313
0	0.0152	0.1358	0.4747	0.9910	1.5977
0.1	0.00372	0.0694	0.4152	0.9454	1.5631

Table 5. Variation in the wall heat transfer rate $-\theta^{'}(0)$ with Pr and B $(K = 1,\ A = 0)$.

B	$Pr = 0.05$	$Pr = 0.1$	$Pr = 0.5$	$Pr = 1$	$Pr = 1.5$
-0.1	0.1779	0.2378	0.4772	0.6367	0.7517
0	0.1662	0.2197	0.4329	0.5706	0.6676
0.1	0.1539	0.2007	0.3859	0.4997	0.5770

Conclusion

The two-dimensional stagnation point flow of an incompressible non-Newtonian micropolar fluid with heat generation is studied in the presence of uniform suction or blowing. A numerical solution for the governing equations is obtained, which allows the computation of the flow and heat transfer characteristics for various values of the non-Newtonian parameter K, the suction parameter A, the heat generation/absorption parameter B, and the Prandtl number Pr. The results indicate that increasing the parameter K increases both the velocity and thermal boundary layer thickness, while increasing A decreases the thickness of both layers. The effect of the parameter K on the velocity is more apparent for suction than it is for blowing. The influence of the parameter K on the temperature is more apparent for higher values of the Prandtl number. The effect of the suction velocity on the shear stress at the wall depends on the value of the non-Newtonian parameter K. On the other hand, the influence of the blowing velocity on the heat transfer rate at the wall depends on the value of the non-Newtonian parameter K.

References

Ariel, P.D., "Stagnation Point Flow with Auction: An Approximate Solution", J. Appl. Mech., 61, 976-978, 1992.

Ariel, P.D. , "Hiemenz Flow in Hydromagnetics", Acta Mech., 103, 31-43, 1994.

Attia, H.A., "Hiemenz Magnetic Flow of a Non-Newtonian Fluid of Second Grade with Heat Transfer", Can. J. Phys., 78, 875-882, 2000.

Attia, H.A., "Hydromagnetic Stagnation Point Flow with Heat Transfer over a Permeable Surface", Arab. J. Sci. Eng., 28(1B), 107-112, 2003.

Attia, H.A., "Homann Magnetic Flow and Heat Transfer with Uniform Suction or Injection", Can. J. Phys., 81, 1223-1230, 2003.

Djukic, D.S., "Hiemenz Magnetic Flow of Power-Law Fluids", ASME J. Appl. Mech., 40, 822-823, 1974.

Garg, V.K., "Heat Transfer due to Stagnation Point Flow of a Non-Newtonian Fluid", Acta Mech., 104, 159-171, 1994.

Hiemenz, K., "Die Grenzschicht an Einem in den Gleichformingen Flussigkeitsstrom Eingetauchten Geraden Kreiszylinder", Dingler Polytech. J., 326, 321-410, 1911.

Homann, F., "Der Einfluss Grosser Zahighkeit bei der Stromung um den Zylinder und um die Kugel", Z. Angew. Math. Mech., 16, 153-164, 1936.

Massoudi, M. and Ramezan, M., "Boundary Layers Heat Transfer Analysis of a Viscoelastic Fluid at a Stagnation Point", ASME HTD, 130, 81-86, 1990.

Massoudi, M. and Ramezan, M., "Heat Transfer Analysis of a Viscoelastic Fluid at a Stagnation Point", Mech. Res. Commun., 19, 129-134, 1992.

Na, T.Y., Computational Methods in Engineering Boundary Value Problem. Academic Press, New York, 107-121, 1979.

Nazar, R., Amin, N., Filip, D. and Pop, I., "Stagnation Point Flow of a Micropolar Fluid towards a Stretching Sheet", Int. J. Non-Linear Mech., 39, 1227-1235, 2004.

Preston, J.H., "The Boundary Layer Flow over a Permeable Surface through which Suction is Applied", Reports and Memoirs. British Aerospace Research Council, London, No. 2244, 1946.

Schlichting, H. and Bussmann, K., "Exakte Losungen fur die Laminare Grenzchicht mit Absaugung und Ausblasen", Schri. Dtsch. Akad. Luftfahrtforschung, Ser. B, 7, 25-69, 1943.