Turkish J. Eng. Env. Sci.
31 (2007) , 225 – 233.
© TÜBİTAK

Unsteady Natural Convection Heat Transfer of Micropolar Fluid over a Vertical Surface with Constant Heat Flux

Rebhi A. DAMSEH
Al-Balqa Applied University, Al-Huson University College, Mechanical Engineering Department, P.O.B (50), Irbid-JORDAN
e-mail: Rdamseh@yahoo.com
Tariq A. AL-AZAB
Al-Balqa Applied University, Al-Al-Salt Community College, Mechanical Engineering Department, P.O.Box 7003 (19117), Salt-JORDAN
Benbella A. SHANNAK
Al-Balqa Applied University, Al-Huson University College, Mechanical Engineering Department, P.O.B (50), Irbid-JORDAN
Mahmoud AL HUSEIN
Al-Balqa Applied University, Faculty of Engineering, P.O.Box 7003 (19117), Salt-JORDAN

Received 29.11.2006

Abstract

This paper presents a numerical investigation of the natural heat transfer problem of a micropolar boundary layer flow near a vertical surface with constant heat flux. The governing equations are solved numerically using McCormack's technique. Many results are obtained and a representative set is displaced graphically to illustrate the influence of the various physical parameters on the velocity profiles, rotation profiles, and transient wall coefficient of friction. It was found that the temperature increases inside the boundary layer for the micropolar flows, as compared to the Newtonian flows. Moreover, increasing the vortex viscosity parameter increased the rotation inside the boundary layer, which has a propensity to increase the coefficient of friction and to decrease the local Nusselt numbers. Comparisons with previously published work in the limits are performed and the results are found to be in excellent agreement.

Key words: Micropolar fluid, Transient, Natural convection.

Introduction

In recent years, the dynamics of micropolar fluids has been a popular area of research. As fluids consist of randomly oriented molecules and as each volume element of the fluid has translation as well as rotation motions, the analysis of physical problems in these fluids has revealed several interesting phenomena not found in Newtonian fluids. The theory of micropolar fluids and thermo micropolar fluids developed by Eringen (1966, 1972) can be used to explain the characteristics in certain fluids such as exotic lubricants, colloidal suspensions, or polymeric fluids, liquid crystals and animal blood. The micropolar fluids exhibit certain microscopic effects arising from local structure and microrotation of fluid elements. An excellent review about micropolar fluid mechanics was provided by Ariman et al. (1973, 1974).

In the past decades, researchers have focused mainly on the heat transfer of micropolar fluid flow over flat plates (Ahmadi, 1976; Gorla et al., 1983; Agarwal and Dhanapal, 1988) or regular surfaces (Lien et al., 1986, 1990). Yao (1983) studied natural convection heat transfer from wavy surfaces. Cheng and Wang (2000) studied the effect of wavy surfaces on micropolar fluids' forced convection heat transfer, and they found that increasing the micropolar fluid parameter results in decreasing heat transfer rates and increasing local coefficient of friction and hydrodynamic and thermal boundary layer thicknesses.

More recently, Bhargava and Takhar (2000) studied the micropolar boundary layer near a stagnation point on a moving wall; it was found that the temperature increases inside the boundary layer compared to the Newtonian flows. Mansour et al. (2000) studied heat and mass transfer effects on the magnetohydrodynamic flow of micropolar fluid on a circular cylinder with uniform heat and mass flux, and the results indicated that micropolar fluids display a reduction in drag as well as heat transfer when compared with Newtonian fluids. Kelson and Desseaux (2001) set self-similar solutions for the boundary layer flow of micropolar fluids driven by a stretching sheet with uniform suction or blowing through the surface. Ibrahim and Hassanien (2001) obtained local similarity solutions for mixed convection boundary layer flow of a micropolar fluid on horizontal flat plates with variable surface temperature. Kim and Lee (2003) performed analytical studies on MHD oscillatory flow of a micropolar fluid over a vertical porous plate, and the effects of non-zero values of micro-gyration vector on the velocity and temperature fields across the boundary layer were studied using small perturbation approximation. Elbarbary and Elgazery (2004) studied the effect of thermal radiation and variable viscosity and thermal conductivity on micropolar fluids using the Chebshev finite difference method. The results showed that the variable viscosity and thermal conductivity in the presence of thermal radiation had significant influences on the velocity, the angular velocity and temperature profiles, shear stress, couple shear stress, and Nusselt numbers.

The aim of the present work was to analyse the transient and steady natural convection heat transfer problems adjacent to a vertical finite plate for incompressible, micropolar fluid with constant heat flux. The governing equations are written in their dimensionless forms using a set of dimensionless variables and solved numerically using McCormack's technique. Numerical results of velocity profiles, microrotation profiles, temperature profiles, local coefficient of friction and local Nusselt numbers under the effect of vortex viscosity parameter, spin gradient viscosity parameter, material parameter and microgyration parameter are presented in graphs and tables.

Mathematical Formulation

Consider laminar free convection boundary layer flow of micropolar fluid above a heated vertical plate with prescribed wall heat flux in an unsteady manner. The problem is described in a rectangular coordinate system attached to the plate such that the x-axis lies along the plate surface and y-axis is normal to the plate (Figure 1). It is assumed that at time $\bar{t} \leq 0$ the temperatures of the plate and the micropolar fluid are maintained at the constant temperature T_∞, and at time $\bar{t} > 0$ heat is supplied to the plate at a constant rate. The continuity, momentum, microrotation and energy equations under boundary layer approximations can be written as follows:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = (\mu+\kappa)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \kappa\frac{\partial \bar{\omega}}{\partial \bar{y}} + g\beta\,(T - T_\infty) \tag{2}$$

$$\rho j\left(\frac{\partial \bar{\omega}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{\omega}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{\omega}}{\partial \bar{y}}\right) = \kappa\left(-\frac{\partial \bar{u}}{\partial \bar{y}} - 2\bar{\omega}\right) + \gamma\frac{\partial^2 \bar{\omega}}{\partial \bar{y}^2} \tag{3}$$

$$\rho c_p\left(\frac{\partial T}{\partial \bar{t}} + \bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right) = k\frac{\partial^2 T}{\partial \bar{y}^2} \tag{4}$$

with the following initial and boundary conditions:

$$\bar{t} \leq 0,\ \ \bar{u}=0, \bar{v}=0, \bar{\omega}=0, T=T_\infty \ \text{ for all } \bar{x}\geq 0, \bar{y}\geq 0$$

$$\bar{t} > 0, \begin{cases} \bar{u}=0, \bar{v}=0, \bar{\omega}=0, T=T_\infty & \text{for } \bar{x}=0, \bar{y}\geq 0 \\ \bar{u}=0, \bar{v}=0, \bar{\omega}=-n(\partial\bar{u}/\partial\bar{y}), & \\ q=-k(\partial T/\partial \bar{y}) & \text{for } \bar{y}=0, \bar{x}\geq 0 \\ \bar{u}=0, \bar{\omega}=0, T=T_\infty & \text{for } \bar{y}\to\infty \end{cases} \tag{5}$$

In the boundary condition for microrotation variable $\bar{\omega}$, which defines its relation with the surface shear stress, the parameter n is a number between 0 and 1 that describes the microrotation vector to the shear stress at the wall. The value for $n = 0$ corresponds to the case where the particle density is sufficiently large so that microelements close to the wall are unable to rotate. The value of $n = 0.5$ represents a weak representation of the microelements and the value of $n = 1.0$ corresponds to the turbulent flow inside boundary layers of microrotation. Define the non-dimensional variables as

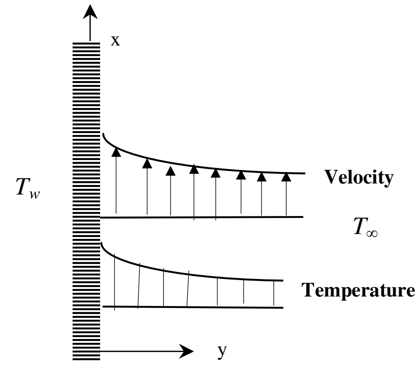

Figure 1. Schematic diagram for flow model and coordinate system.

$$t = Gr^{1/2}(\upsilon/L)\bar{t}, x = \bar{x}/L, y = Gr^{1/4}(\bar{y}/L) \quad (6)$$

$$\begin{aligned} &u = Gr^{-1/2}(\upsilon/L)\bar{u}, v = Gr^{-1/4}(\upsilon/L)\bar{v}, \\ &\theta = k(T - T_\infty)/qL \end{aligned} \quad (7)$$

Substituting Eqs. (6) and (6) into Eqs. (1) to (4) gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (1+R)\frac{\partial^2 u}{\partial y^2} + R\frac{\partial \omega}{\partial y} + \theta \quad (9)$$

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = R.B\left(-\frac{\partial u}{\partial y} - 2\omega\right) + \lambda\frac{\partial^2 \omega}{\partial y^2} \quad (10)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \quad (11)$$

with the corresponding boundary conditions:

$$t \le 0, u = 0, v = 0, \omega = 0, \theta = 1 \text{ for all} x \ge 0, y \ge 0$$

$$t > 0, \begin{cases} u = 0, v = 0, \omega = 0, \theta = 0 & \text{for} x = 0, y \ge 0 \\ u = 0, v = 0, \omega = m(\partial u \partial y), & \\ (\partial\theta/\partial y) = -1 & \text{for} y = 0, x \ge 0 \\ u = 0, \omega = 0, \theta = 0 \text{for} y \to \infty & \end{cases} \quad (12)$$

where L is the characteristic length of the plate, $R = \kappa/\mu$ is the vortex viscosity parameter, $B = L^2/jGr^{1/2}$ and $\lambda = \gamma/\mu j$ are dimensionless material parameters, m is the dimensionless micro-gyration vector, $\Pr = \mu c_p/k$ is the Prandtl number, and $Gr = g\beta(T_w - T_\infty)L^3/v^2$ is the Grashof number. The values of local coefficient of friction and local Nusselt numbers are given by

$$C_f Gr^{3/4} = (1+R)\left.\frac{\partial u}{\partial y}\right|_{(x,0,t)} + R\left.\omega\right|_{(x,0,t)} \quad (13)$$

$$NuGr^{-1/4} = \left.\frac{1}{\theta}\right|_{(x,0,t)} \quad (14)$$

The boundary layer equations describe the conservation of mass; momentum, microrotation and energy are formulated and solved in their time-dependent formulation using McCormack's technique, which is an explicit finite difference technique and a second order accuracy in space and time. The details of this solution are clearly explained by Anderson (1995). The numerical solution used is a time marching technique giving the downstream velocity microrotation and temperature profiles using the known upstream profiles. In the present work the above quantities were calculated by obtaining explicitly the flow field variables at grid point (i, j) at time $t + \Delta t$ from the known flow field variables at grid points (i, j),$(i+1, j)$,$(i-1, j)$, $(i, j-1)$ and $(i, j+1)$ at time t. The flow field variables at all other grid points at time $t + \Delta t$ are obtained in a similar fashion. Then the local coefficient of friction and local Nusselt numbers are calculated from Eqs. (12) and (13). In order to verify the accuracy of the present method a comparison of results with similarity solutions presented in Kayes and Crawford (1980) is shown in Table 1 for the steady laminar free convection over a vertical plate of Newtonian fluids. The comparison reveals excellent agreement.

Results and Discussion

The transient laminar free convection heat transfer effects from the vertical surface of micropolar fluids are studied. The governing equations are written in dimensionless form using a set of variables and then solved using a finite difference technique. The micropolar fluid effects on this problem are found to be proportional to material parameters and vortex viscosity parameter. The material parameter $B = L^2/jGr^{1/2}$ is found to be proportional directly to the length of the plate and inversely to microrotation density and Grashof number, while the material parameter $\lambda = \gamma/\mu j$ is directly proportional to the spin-gradient viscosity and inversely to absolute viscosity and microrotation density. Note that the material parameters' effects of micropolar fluid decrease as the Gr or buoyancy effect increases.

Table 1. Correspondence between the values of $Nu_x Gr_x^{-1/4}$ for various Prandtl numbers for the present code and that presented in Kayes and Crawford (1980).

Pr		0.01	0.1	0.72	1.0	10	100	1000
$Nu_x Gr_x^{-1/4}$	Present code	0.0680	0.195	0.408	0.455	0.926	1.73	3.13
	[19]	0.0669	0.189	0.406	0.457	0.931	1.74	3.14

Figure 2a-c shows the dimensionless steady velocity profiles $u(x, y, t)$, temperature profiles $\theta(x, y, t)$ and rotation profiles $\omega(x, y, t)$ for different vortex viscosity parameter $R = 0, 1, 2, 5$ and for certain values of $B = 0.5, \Pr = 1.0, \lambda = 5.0, m = 0.0$ and $x = 0.5$. The increasing of the vortex viscosity parameter increased the velocity and temperature inside the boundary layer and broadened the hydrodynamic and thermal boundary layer thicknesses and increased rotation and rotation boundary layer thickness. This is due to extra mixing of fluid layers due to the new used shear stress. Figure 3a-c shows the

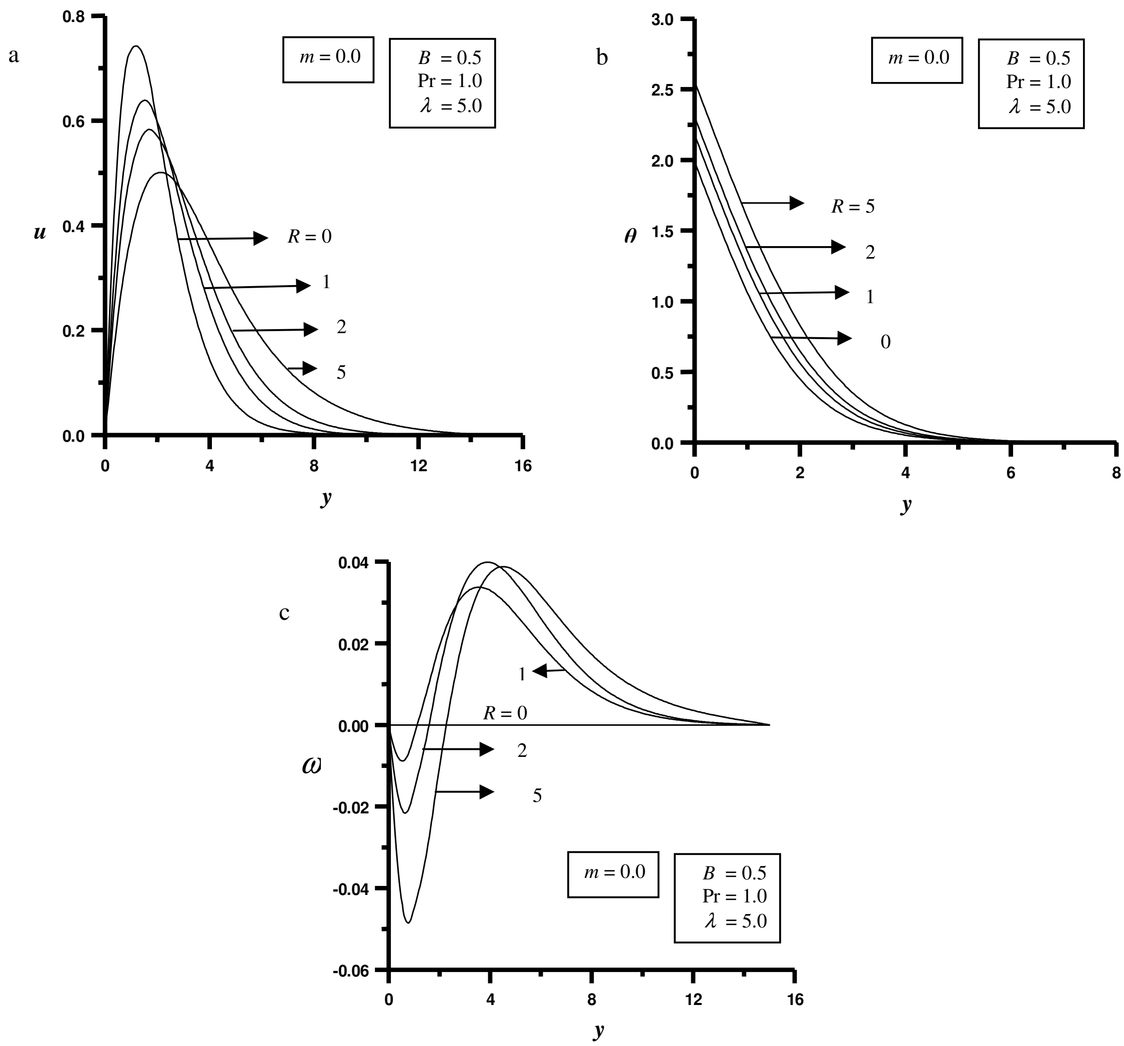

Figure 2. a, b, c, Dimensionless velocity, temperature and angular velocity profiles, respectively, for different values of vortex viscosity parameter R and for certain values of $B = 0.5, \Pr = 1.0, \lambda = 5, m = 0$ and $x = 0.5$.

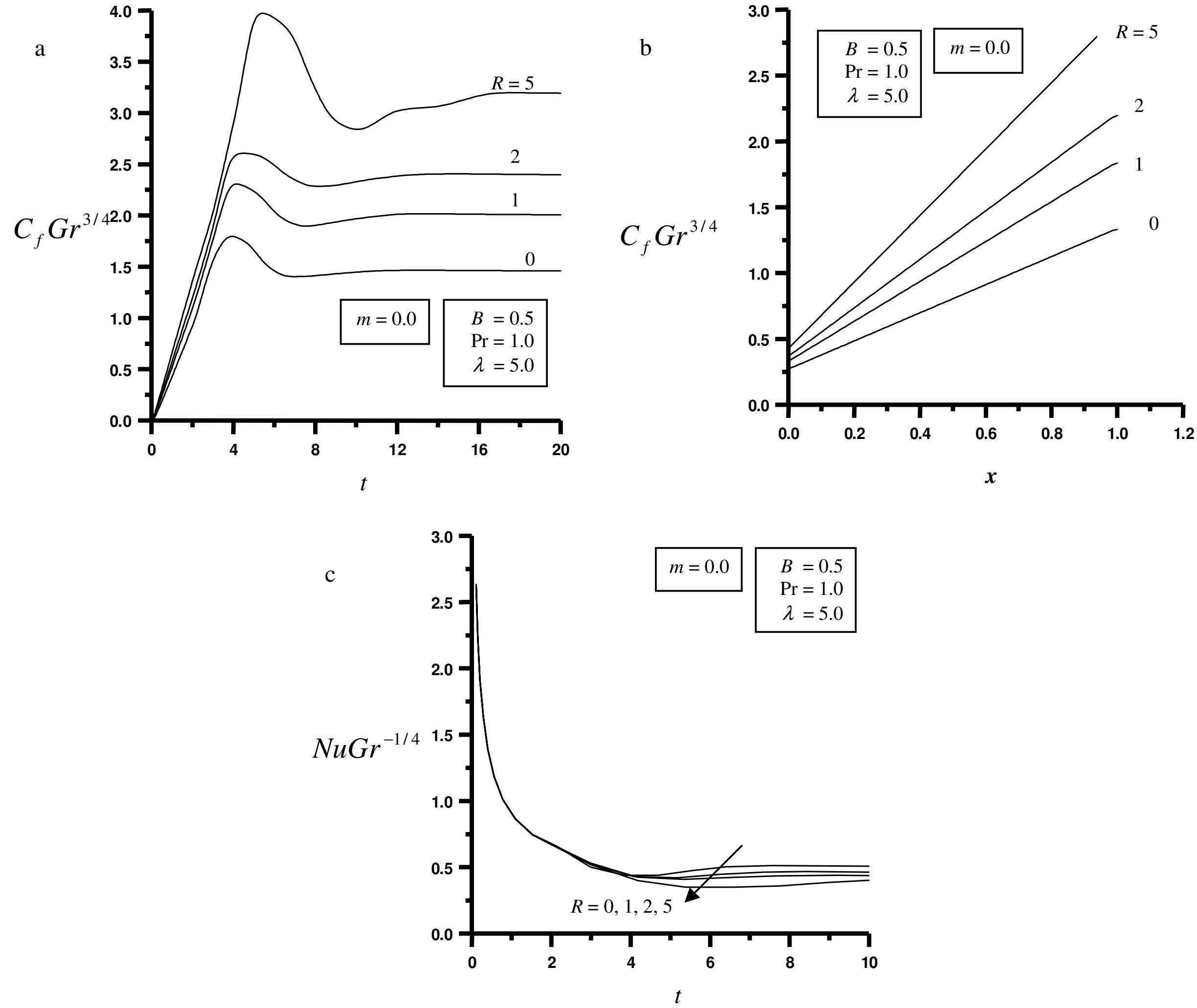

Figure 3. a, b, c, Local coefficient of friction and Nusselt numbers for different values of vortex viscosity parameter R and for certain values of $B = 0.5, \Pr = 1.0, \lambda = 5, m = 0$ and $x = 0.5$.

variation in local coefficient of friction $C_f Gr^{3/4}$and local Nusselt numbers $NuGr^{-1/4}$against transient times ($x = 0.5$) or streamwise direction distance along the plate ($t \to \infty$) for different values of vortex viscosity parameter and for certain values of $B = 0.5, \Pr = 1.0, \lambda = 5.0$ and $m = 0.0$. These figures show that the increasing of the vortex viscosity parameter increased the coefficient of friction due to higher mixing of fluid layers and decreased local Nusselt numbers due to excessive heating of fluid layers. These figures show also how the steady state solutions are approached as the transient time increases at $t \to \infty$. The case of $R = 0$ corresponds to Newtonian fluids.

Figure 4a and b shows dimensionless steady velocity profiles $u(x, y, t)$and rotation profiles $\omega(x, y, t)$ for different dimensionless material parameter $B = 0.2, 0.4, 0.6, 0.8, 1.0$ and for certain values of $R = 2.0, \Pr = 1.0, \lambda = 5.0, m = 0.5$ and $x = 0.5$. The increasing of the dimensionless material parameter decreases velocities inside boundary layers due to the retarding effect of micro inertia density and consequently small buoyancy forces; this is similar to a flow against adverse pressure gradient. The increasing dimensionless material parameter also increased rotation inside boundary layers because of favourable non-Newtonian effects, and has negligible effects on temperature profiles, which are not presented here. Figure 5a and b show the variations in local coefficient of friction against transient times ($x = 0.5$) and streamwise direction distance along the plate ($t \to \infty$) for different values of dimensionless material parameter where the increasing of this parameter increased coefficient of friction, and the figures again show how the steady state solutions are approached.

The effects of the dimensionless material parameter, which represent spin-gradient viscosity, are drawn in Figure 6a and b on both steady state velocity and angular velocity profiles. It is clear from these figures that the increasing of this material parameter increases the velocity and decreases the rotation inside the boundary layer. The reason is that the material parameter λ is a viscosity ratio parameter and decreasing the dynamic fluid viscosity (increasing λ) causes the friction between the fluid layers to decrease and to decrease the rotation inside the boundary layer at the same time. Figure 7a and b show the variations in the local coefficient of friction against transient times t ($x = 0.5$) and streamwise direction distance ($t \rightarrow \infty$) for different values of dimensionless material parameter λ. As can be concluded from Figure 6, the effect of the dimensionless material parameter λ is to decrease the local coefficient of friction.

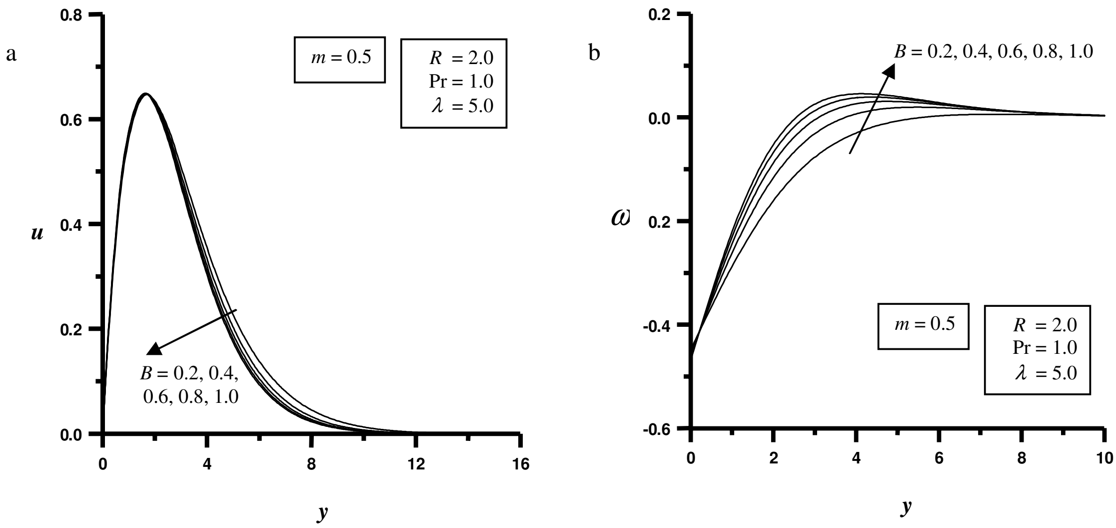

Figure 4. a, b. Dimensionless velocity and angular velocity profiles, respectively for different values of material parameter B and for certain values of $R = 2.0, \mathrm{Pr} = 1.0, \lambda = 5.0, m = 0.5$ and $x = 0.5$.

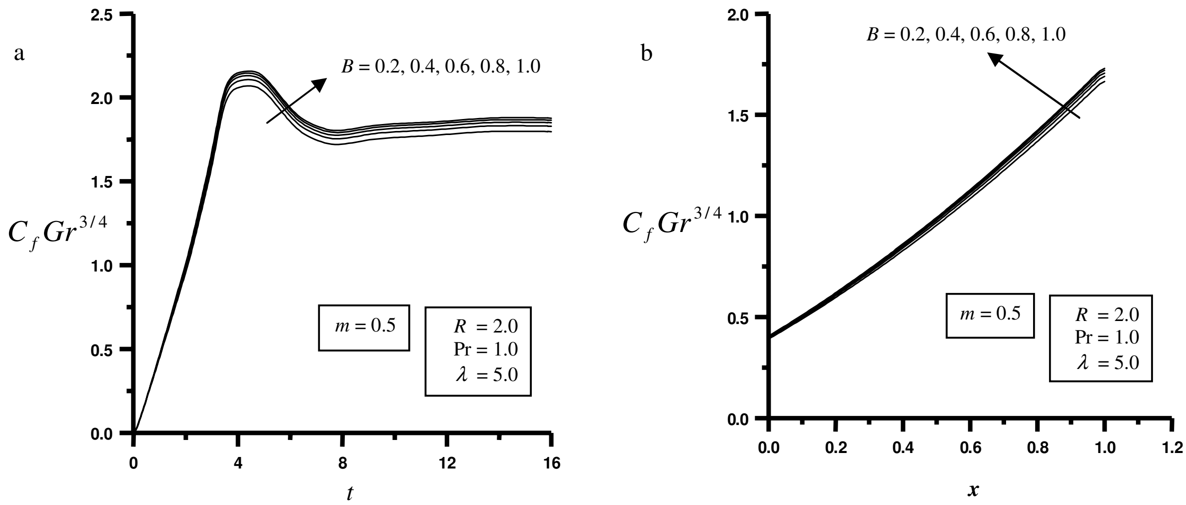

Figure 5. a, b, Local coefficient of friction for different values of material parameter B and for certain values of $R = 2.0$, $\mathrm{Pr} = 1.0, \lambda = 5.0, m = 0.5$ and $x = 0.5$.

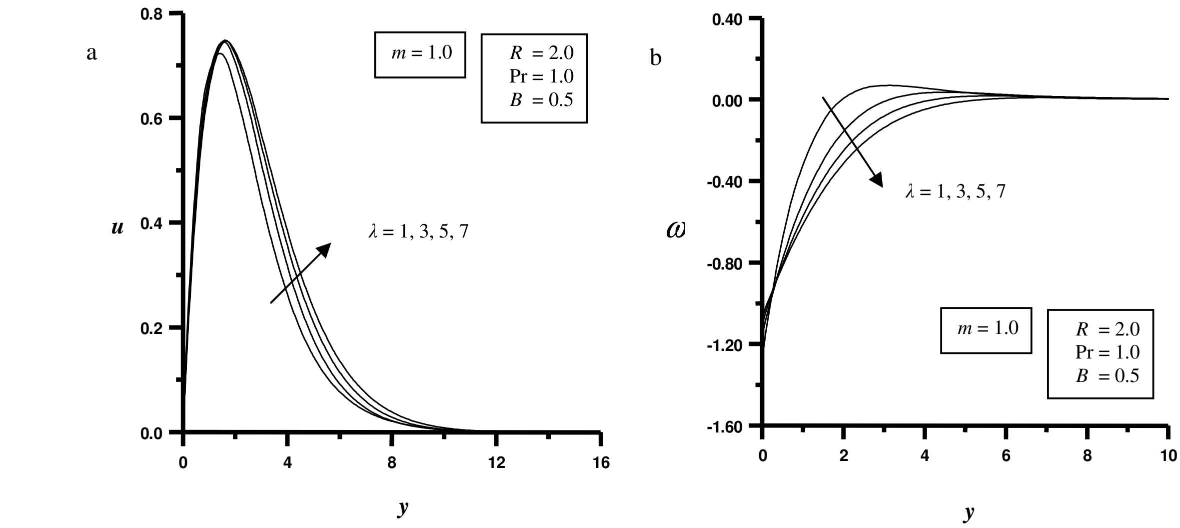

Figure 6. a, b, Dimensionless velocity and angular velocity profiles, respectively, for different values of dimensionless material parameter λ and for certain values of $R = 2.0, \mathrm{Pr} = 1.0, B = 0.5, m = 1.0$ and $x = 0.5$.

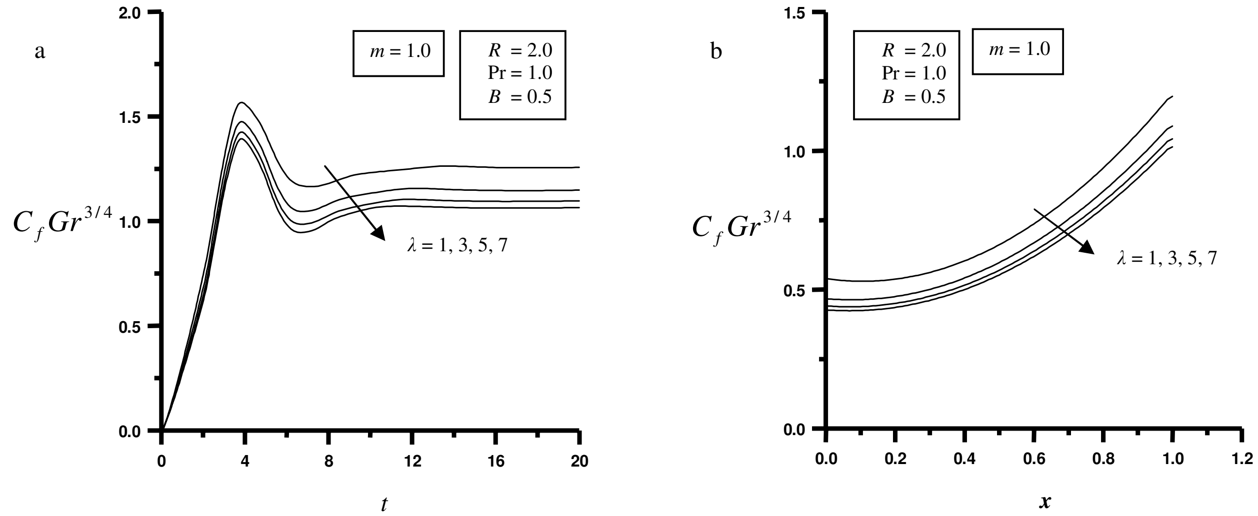

Figure 7. a, b, Local coefficient of friction for different values of dimensionless material parameter λ and for certain values of $R = 2.0, \mathrm{Pr} = 1.0, B = 0.5, m = 1.0$ and $x = 0.5$.

Finally, by comparing Figures 4 and 5 ($m = 0.5$) with Figures 6 and 7 ($m = 1.0$) we can study the effect of micro-gyration vector m on angular velocity profiles and local coefficient of friction. As the micro-gyration dimensionless parameter increases the angular rotation inside the boundary layer and local coefficients of friction decrease. This is due to excessive angular momentum within fluid layers and consequently this enhances local values of Nusselt numbers.

Conclusions

The transient and steady natural convection heat transfer problem of a micropolar fluid past a vertical finite plate with constant heat flux is studied. The governing equations are simplified by using a set of dimensionless variables and then solved numerically using McCormack's technique. It was found that increasing the vortex viscosity parameter R increased the rotation inside the boundary layer, which has a propensity to increase the coefficient of friction due to higher mixing of fluid layers and to decrease the local Nusselt numbers due to excessive heating of fluid layers. The effect of the dimensionless material parameter B is to increase the local coefficient of friction while increasing the material parameter, which represents spin-gradient viscosity λ, and the microgyration vector m causes a decrease in the coefficient of friction.

Nomenclature

B	dimensionless material parameter
C_f	local coefficient of friction
C_p	specific heat of the fluid at constant pressure
g	magnitude of acceleration due to gravity
Gr	Grashof number, $g\beta(T_w - T_\infty)L^3/v^2$
h	heat transfer coefficient
j	microinertia density
k	thermal conductivity
L	characteristic length of plate
N	microrotation
Nu_x	local Nusselt number
Pr	Prandtl number, v/α
q	surface heat flux
R	vortex viscosity parameter
t	dimensionless time
T	temperature
T_∞	ambient fluid temperature
u, v	dimensionless velocity components along x- and y-axes
x, y	dimensionless coordinates

Greek symbols

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
γ	spin gradient viscosity
κ	vortex viscosity
λ	dimensionless material parameter
θ	non-dimensional temperature
μ	dynamic viscosity
v	kinematic viscosity
ρ	fluid density
ω	microrotation component

Subscripts

∞ free stream condition

Superscripts

\- dimensional variables

References

Agarwal, R. S. and Dhanapal, C., "Flow and Heat Transfer in a Micropolar Fluid Past a Flat Plate with Suction and Heat Sources", Int. J. Eng. Sci., 26, 1257-1266, 1988.

Ahmadi, G., "Self-Similar Solution of Incompressible Micro-Polar Boundary Layer Flow Over a Semi-Infinite Plate", Int. J. Eng. Sci., 14, 639-646, 1976.

Ariman, T., Turk, M.A. and Sylvester, N.D., "Microcontinuum Fluid Mechanics—A Review", Int. J. Eng. Sci., 11, 905-930, 1973.

Ariman, T., Turk, M.A. and Sylvester, N.D., "Applications of Microcontinuum Fluid Mechanics—A Review", Int. J. Eng. Sci., 12, 273-293, 1974.

Anderson, J.D., Computational Fluid Dynamics, Chap. 6, McGraw-Hill, New York, 1995.

Cheng, C.Y. and Wang, C.C., "Forced Convection in Micropolar Fluid Flow over a Wavy Surface, Numerical Heat Transfer,Part-A, 37, 271-287, 2000.

Elbarbary, E.M.E. and Elgazery, N.S., "Chebyshev Finite Difference Method for the Effects of Variable Viscosity and Variable Thermal Conductivity on Heat Transfer from Moving Surfaces with Radiation", Int. J. Thermal Sciences, 43, 889-899, 2004.

Eringen, A.C., "Theory of Micropolar Fluids", J. Math. Mech., 16, 1-18, 1966.

Eringen, A.C., "Theory of Thermomicropolar Fluids", J. Math. Anal. Appl., 38, 480-496, 1972.

Gorla, R.S.R., Pender, R. and Eppich, J., "Heat Transfer in Micro-Polar Boundary Layer Flow over a Flat Plate", Int. J. Eng. Sci., 21, 791-798, 1983.

Ibrahim, F.S. and Hassanien, I.A., "Local Nonsimilarity Solutions for Mixed Convection Boundary Layer Flow of a Micropolar Fluid on Horizontal Flat Plates with Variable Surface Temperatures", Applied Mathematics Computation, 122, 133-153, 2001.

Kayes, W.M. and Crawford, M.E., Convective Heat and Mass Transfer, Chap. 16, McGraw-Hill, New York, 1980.

Kelson, N.A. and Desseaux, A., "Effect on Surface Conditions on Flow of a Micropolar Fluid Driving by a Porous Stretching Sheet", In. J. Eng. Sci., 39, 1881-1897, 2001.

Kim, Y.J. and Lee, J.C., "Analytical Studies on MHD Oscillatory Flow of a Micropolar Fluid Over a Vertical Porous Plate", Surface and Coating Technology, 171, 187-193, 2003.

Lien, F.S., Chen, C.K. and Cleaver, J.W., "Analysis of Natural Convection Flow of Micropolar Fluid about a Sphere with Blowing and Suction", ASME J. Heat Transfer, 108, 967-970, 1986.

Lien, F.S., Chen, T.M. and Chen, C.K., "Analysis of Free-Convection Micropolar Boundary Layer about a Horizontal Permeable Cylinder at a Non-Uniform Thermal Condition", ASME J. Heat Transfer, 112, 504-506, 1990

Mansour, M.A., Alhakiem, M.A. and El Kabeir, S.M., "Heat and Mass Transfer in Magnetohydrodynamic Flow of Micropolar Fluid on a Circular Cylinder with Uniform Heat and Mass Flux", J. Magnetism and Magnetic Materials, 220, 259-270, 2000.

Rama Bhargava and Takhar, H.S., "Numerical Study of Heat Transfer Characteristics of the Micropolar Boundary Layer Near a Stagnation Point on a Moving Wall", In. J. Eng. Sci., 38, 383-394, 2000.

Yao, L.S., "Natural Convection along a Vertical Wavy Surface", ASME J. Heat Transfer, 105, 464-468, 1983.