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An Adaptive Parametric Study on Mesh Refinement during Adaptive Finite Element Simulation of Sheet Forming Operations

Mohd AHMED
PBA division, SB Group, Post Box –9887, Jeddah-21423, K. SAUDI ARABIA
e-mail: ahmed1md_69@hotmail.com
Devender SINGH
Prasar Bharati. Delhi-INDIA

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Abstract

The paper presents an adaptive parametric study on mesh refinement during adaptive finite element analysis of sheet forming operations. The selected adaptivity parameters for the study are field variable recovery, refinement techniques, and accuracy limit. Influences of parameters on performance of adaptive procedure have been studied by simulating the sheet stretching operation. The post-processing and refinement techniques used in the adaptive analysis are discussed. The adaptively refined meshes for various adaptive parameters at different stages of deformations and CPU times are studied and discussed.

Key words: Finite Element, Adaptive Parameter, Post-processing Procedure, Refinement Technique, Sheet Forming.

Introduction

Popularity of the finite element method has stimulated the development of error estimation and adaptive procedures for improving the efficiency of finite element analysis and ensuring that the discretization error is within the permissible limit. The adaptive finite element procedure is a cyclic process consisting of the following steps:

(1) Perform a finite element analysis on a given mesh.

(2) Determine error of the solution and find whether it is within the specified limit.

(3) If necessary, refine the mesh.

A recent trend in the development of the finite element technique is the use of adaptive procedures based upon error estimators. Ahmed et al. (2005) has presented the developments in the simulation of sheet metal forming. Zienkiwicz (2000) has listed some important achievements in the finite element method and presented an outline of some problems still requiring treatment. The different aspect of sheet metal forming simulation including adaptive mesh refinements, reliability, and sensitivity assessments are examined by Moshfegh (2007). There are 3 main types of error estimators for adaptive procedures, namely the residual type introduced by Babuska and Rheinholdt (1978), the interpolation type propounded by Erikson and Johnson (1988), and the post processing type proposed by Zienkiewicz-Zhu (1987). The post processing type of error estimators rely upon a "recovery" of higher order finite element solution. The recovery techniques are based on the least square fitting of velocity (or the displacement) field or their derivatives (stress field) by a higher order polynomial over a patch of elements or nodes. The recovery of post processed stress field by the least square method had earlier been proposed by Zienkiewicz-Zhu (1992). Singh et al. (1999) proposed a recovery technique based upon the least square fitting of velocity field over an element patch. Gradients (or curvature) of displacement based mesh refinement criterion have been introduced by Zienkiewicz et al. (1995). Lee and Bathe (1994) used point wise error in strain to guide the mesh refinement. Lo and Lee (1998) developed the concept of selective regional refinement. The error norm of state variable based mesh refinement is used by Buscaglia et al. (2001) for elasticity problems. A local a posteriori bending error indicator is developed by Han and Peter (2000) for non-linear h-adaptive analysis. Micheletti and Perotto (2006) have tested reliability and efficiency of an anisotropic Zienkiewicz-Zhu error estimator. A posteriori error estimation for generalized finite element methods based on the partition of unity method has been studied by Strouboulis et al. (2006). Giraud et al. (2004) and Cherouat et al. (2007) have presented a new scheme for simultaneously refining and coarsening a mesh during sheet metal forming process. The process parameter of hydro forming has been studied by Jansson et al. (2007) using adaptive simulations.

The adaptive finite element analysis is influenced by several adaptivity parameters (i.e. parameters related to adaptive mesh refinement) including field variable recovery, degree of polynomial used in post processing, error norm, refinement strategy, accuracy limit, patch size, etc. There are only a few published works on the role of adaptivity parameters in adaptive finite element analysis. Onate and Castro (1994) discuss different refinement strategies that can be used for adaptive analysis. Li and Wiberg (1994) used different accuracy limit and patch size in their adaptive analysis. The present study deals with the effect of adaptivity parameters in the context of finite element simulation of sheet forming problems. Three adaptivity parameters, namely post processed state variable viz. velocity and stress, refinement techniques viz. error equally distributed and square of error equally distributed, and accuracy limit are selected for the present study.

Governing Equations

The governing equations for the solution of the sheet forming problem of rigid visco-plastic materials are summarized as follows.

Virtual work equations

The weak formulation of a boundary value problem can be obtained using the principle of virtual work along with a penalty method to enforce compressibility. It can be expressed by the following equation [Oh and Kobayashi (1980)].

$$\int_{\Omega} \overline{\sigma}\delta\dot{\overline{\varepsilon}}_i d\Omega + K\int_{\Omega} \varepsilon_v \delta\dot{\varepsilon}_i d\Omega - \int_{S3} \tau\delta v_i d\Gamma f = 0 \quad (1)$$

where K, a penalty constant, is a large positive constant of the order of (10^6- 10^8)μ where $\mu = \frac{\overline{\sigma}}{3\dot{\overline{\varepsilon}}}$

Yield criterion, f(σ_{ij}) = C :

$$\overline{\sigma} = \left(\frac{3}{2}\sigma'_{ij}\sigma'_{ij}\right)^{\frac{1}{2}} = \overline{\sigma}(\overline{\varepsilon}, \dot{\overline{\varepsilon}}) \quad (2)$$

Constitutive equations :

$$\dot{\varepsilon}_{ij} = \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}}\dot{\lambda} : \quad \sigma'_{ij} = \frac{2}{3}\frac{\overline{\sigma}}{\dot{\overline{\varepsilon}}}\dot{\varepsilon}_{ij} \quad (3)$$

with

$$\dot{\varepsilon}_{ij} = \left(\frac{2}{3}\dot{\varepsilon_{ij}}\dot{\varepsilon}_{ij}\right)^{\frac{1}{2}}$$

Compatibility conditions :

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad (4)$$

Incompressibility condition :

$$\dot{\varepsilon}_v = 0 \quad (5)$$

Boundary friction treatment

The magnitude of the frictional stress at a point on the blank is dependent on the magnitude of the relative sliding velocity u_s between the blank and the die. The direction of the frictional stress τ and relative sliding velocity u_s is opposite to each other. The relationship between τ and u_s is expressed as follows.

$$\tau = -m_f\ k\frac{u_s}{\|u_0\|} \cong -m_f\ k\left(\frac{2}{\pi}\tan^{-1}\left[\frac{u_s}{u_0}\right]\right) \quad (6)$$

where k is the shear yield stress and u_0 is a small threshold velocity.

Error Estimation

The error in computed stress or displacement (velocity), e^*_σ , e^*_u , is defined as the difference between the exact values of stress or displacement (velocity), σ, u and respective computed values, σ^h , u^h i.e.

$$e^*_\sigma = \sigma - \sigma^h \; ; e^*_u = u - u^h \tag{7}$$

The error can be evaluated in any appropriate norms. Since the finite element solution minimizes the error in the energy norm, the magnitude of the error in energy norm is a good measure of the overall quality of solution. The integral measure of the error in energy norm may be defined as follows.

$$\|e\| = \left(\int_\Omega e^{*T}_\sigma D \, e^*_\sigma \, d\Omega \right)^{1/2} \tag{8}$$

The relative error in energy norm, η, is defined as

$$\eta = \frac{\|e\|}{\|u\|} \times 100 \text{ percent} \tag{9}$$

The finite element formulation for a metal forming problem is of the mixed type in which incompressibility is ensured by means of a penalty constant. It is, therefore, convenient to concentrate on the energy norm of the deviatoric part only. The estimated error norm for a finite element solution, $\|e\|$, may be obtained by replacing the exact stresses in the above equation by some recovered smooth stresses, σ^h, i.e.

$$\|e\|^2 = \int_\Omega (\sigma^{*\prime} - \sigma^{\prime h})^T (2\mu)^{-1} (\sigma^{*\prime} - \sigma^{\prime h}) d\Omega \tag{10}$$

Post Processing Techniques

Different post-processing techniques can be used to improve the quality of the derivatives such as simple averaging, local or global projection and those exploring the super-convergence phenomenon. The post-processing techniques employed for the recovery of stress and velocity in the present study, are presented below.

Stress recovery

According to so-called ZZ super-convergent patch recovery technique (Zienkiewicz-Zhu, 1992), the nodal values of stress belong to a polynomial expansion of the same complete order as that present in the basis function and is valid over a patch of nodes surrounding the particular given node. Following polynomial expansion may be used for each component of stress:

$$\sigma^*(x) = \mathbf{Q}(x).\mathbf{a} \tag{11}$$

in which **Q(x)** is the basis of the assumed polynomial, $\mathbf{X}$ = (x_1, x_2), the spatial variable and **a** the vector of the unknown parameters. A least square fit of σ^h_s values over the nodal patch may be made by minimizing the following functional:

$$\pi_f(a) = \frac{1}{2} \sum_{i=1}^{np} [\sigma^h_s(x_i, y_i) - \mathbf{Q}(x_i, y_i). \; a]^2 \tag{12}$$

On simplification, it leads to the following equation:

$$\mathbf{Aa} = \mathbf{b} \tag{13}$$

where

$$\mathbf{A} = \sum_{i=1}^{np} \mathbf{Q}^T_i(x_i, y_i) \; . \; Q_i(x_i, y_i);$$

$$\mathbf{b} = \sum_{i=1}^{np} \mathbf{Q}^T_i(x_i, y_i).\sigma^h_s(x_i, y_i) \tag{14}$$

Velocity recovery

The recovery of velocity field is effected by least squares fit of the computed nodal velocity using a higher order polynomial over an element patch that consists of all the elements surrounding the element under consideration. To perform least squares fitting, the following functional may be minimized:

$$\pi_f(a) = \tfrac{1}{2} \sum_{i=1}^{np} [d^h_i(x_i, y_i) - d_i(x_i, y_i)]^2 \tag{15}$$

where

$$\mathbf{d}_i(x_i, y_i) = \mathbf{Q}_i(x_i, y_i).\mathbf{a} \tag{16}$$

$$\mathbf{d}_i = [u_i v_i]^T \; ; \mathbf{a} = [a_u a_v]^T \; ; \tag{17}$$

and

$$\mathbf{Q}_i = \begin{bmatrix} \mathbf{q}_i & 0 \\ 0 & \mathbf{q}_i \end{bmatrix} ; \mathbf{q}_i = [1, x_i, y_i, x^2_i, x_i y_i, y^2_i, ...] \tag{18}$$

The total numbers of sampling points is equal to the total number of nodes in the element patch.

Minimization condition of π_f (a) implies that **a** satisfies the following relation

$$\sum_{i=1}^{np} Q_i^T(x_i, y_i).\ Q_i(x_i, y_i).a = \sum_{i=1}^{np} Q_i^T(x_i, y_i).\ d_i^T(x_i, y_i) \quad (19)$$

Solving for **a**, the following equation is obtained.

$$\mathbf{a} = \mathbf{A}^{-1}\mathbf{b} \quad (20)$$

where

$$\mathbf{A} = \sum_{i=1}^{np} \mathbf{Q}_i^T(x_i, y_i).\ \mathbf{Q}_i(x_i, y_i);$$

$$\mathbf{b} = \sum_{i=1}^{np} \mathbf{Q}_i^T(x_i, y_i).\ \mathbf{d}_i^T(x_i, y_i) \quad (21)$$

Implementation of Refinement Techniques

The accuracy of the solution is determined from the global error given by Eq. (9). The solution is acceptable if $\eta \leq \eta_{allow}$. If not, h-refinement is carried out. Two usual methods of implementation of h-refinement, namely error equally distributed and square of error equally distributed, are discussed below.

Error equally distributed

In an optimal mesh, it is desirable that the distribution of error should be equal among the elements. The global error is found from the following equation.

$$\|e\|^2 = \left[\sum_{i=1}^{N} \|e_i\|^2\right] \quad (22)$$

The permissible global error is given by

$$\|e\|_{allow} = \frac{\eta_{allow}\ \|e\|}{k} \quad (23)$$

where k is a factor lying between 1.0 to 1.5 to prevent oscillation (Li and Wiberg, 1994). The relation giving the permissible error in the ith element is

$$\|e\|_{allow(i)} = \frac{\|e\|_{allow}}{\sqrt{N}} \quad (24)$$

The so-called element refinement parameter ξ_i given below guides the refinement.

$$\xi_i = \frac{\|e\|_{allow}}{\|e\|_{allow(i)}} \quad (25)$$

If $\xi_i > 1$, refinement is needed. The new element size (h_{new}) is found with the help of the relation.

$$h_{new} = \frac{h_{old}}{\xi_i^{1/p}} \quad (26)$$

Square of the error equally distributed

It is based on the premise that the square of the error should be equally distributed over the whole domain

$$\Omega i.e. \frac{\|e\|_{allow}}{\Omega^{1/2}} = \frac{\|e\|_{allow\ (i)}}{\Omega_i^{1/2}} \quad (27)$$

where Ω is the volume of the domain , Ω_i is the volume of the ith element. Then

$$\|e\|_{allow\ (i)} = \|e\|_{allow} \left(\frac{\Omega_i}{\Omega}\right)^{1/2} \quad (28)$$

The parameter ξ_i serves as a guide for predicting the new element size as explained earlier.

Parametric Study of Adaptive Finite Element Analysis

Numerical simulations of sheet forming operations were carried out to study the performance of the adaptivity parameters (i.e. parameters affecting the adaptive analysis). A 2-dimensional finite element code **AdSheet2** was developed incorporating the above adaptive techniques. Two post-processing techniques namely, post-processing of velocity and stress were adopted. Two refinement strategies, namely error equally distributed and square of error equally distributed throughout the domain, and 2 limits of error, namely 3% and 8%, were used. For post-processed field variable and refinement technique study, an error limit of 8% was used.

The schematic diagram of the sheet forming is shown in Figure 1. Owing to symmetry, the only 1 half of the work piece needs to be modelled. The domain is discretized using 6 nodded triangular elements with 2 degrees of freedom at each node. The initial uniform mesh consisted of 828 elements for different adaptivity parameter study (Figure 2). A new mesh is generated whenever the previous mesh gets overly distorted or error limit exceeded. The von-Mises yield criterion has been employed. The material of the sheet is modelled as rigid-plastic. The friction at die-metal interface is modelled through a constant factor in which frictional stress is related to the shear strength of the material. The punch and die are considered as rigid. The die and blank interface can be modelled as sliding friction or sticking friction. The direct approach has been adopted for contact analysis wherein the contacting nodes are constrained in the normal direction of tool surface and frictional boundary condition are imposed directly. The sticking friction is assigned at die-blank interface for fixed condition i.e. to model sheet stretching. The displacement of punch is modelled in incremental steps. Each displacement increment was such that it caused a maximum strain increment of 1%. The values of sheet forming variable were as follows:

Punch radius (R_p) = 50.8 mm, Blank radius (R_b) = 59.19 mm, Sheet thickness = 2.0 mm

Downward velocity of punch = 1 mm/s , Blank Material = Mild Steel

Stress-strain relation of the blank material:

$$\overline{\sigma} = 589[0.0001 + \overline{\varepsilon}]^{0.216}$$

Although the analysis provides detailed numerical results for each time increments, the deformed mesh at only 2 steps are shown for illustrative purpose. For discussion purposes, the blank is divided into 3 regions. Region I is the portion of the blank between the centre of the blank and a point up to which the punch is in contact with the blank, region II corresponds to the portion of the blank neither in contact with punch nor with the die or blank holder, and region III is at a portion of the blank that is in contact with the die and the blank holder.

The variations in mesh at different stages of deformation, namely initial and final, have been obtained corresponding to adaptive parameters. The computed mesh and deformed shapes at punch displacement of 2.5 mm and 20.0 mm are shown in Figures 3(a) and 3(b) for the adaptive parameters, post-processing of velocity, error equally distributed refinement strategy, and 8% error limit. In the post-processed velocity based adaptive analysis with error equally distributed refinement strategy and 8% accuracy limit, 2 remeshings were required to bring the error below the predefined accuracy limit. In the first remeshing, the numbers of elements were found to increase to 1239 from a mesh of 828 elements. In the next remeshing, the number of elements decreased to 1031. The CPU time required for the analysis was 7 h.

Referring to Figure 3(a), it is observed that regions I and III have finer elements. In the region I, the element density is maximum at the punch-blank interface i.e. at the top surface of the sheet. The mesh in region II is more or less uniform mesh but its density is smaller. The region III has a band of finer elements throughout its thickness. The deformed mesh at higher punch travel (Figure 3(b)) shows that though regions I and III, continue to have greater number of fine elements but their distribution is quite different. Region I, which is localized at punch travel of 2.5mm, becomes dispersed and the element density tends to decrease towards the region II. In region III also, 2 distinct bands of fine elements develop. The mesh in region II tends to become coarser with the increase of punch travel.

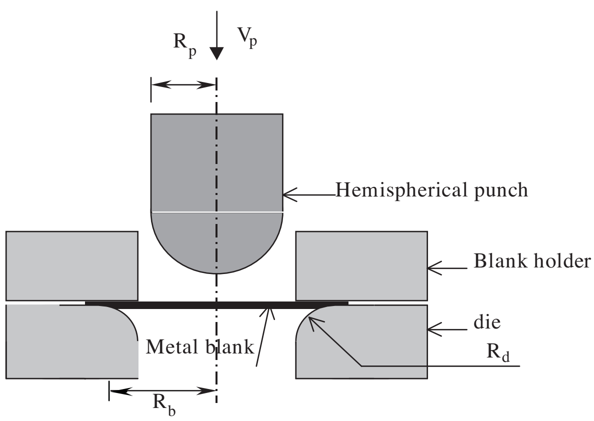

Figure 1. Schematic diagram of sheet forming operations.

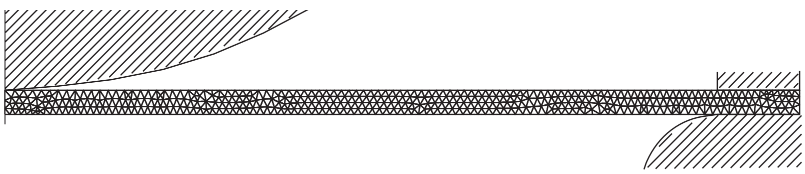

Figure 2. The initial mesh.

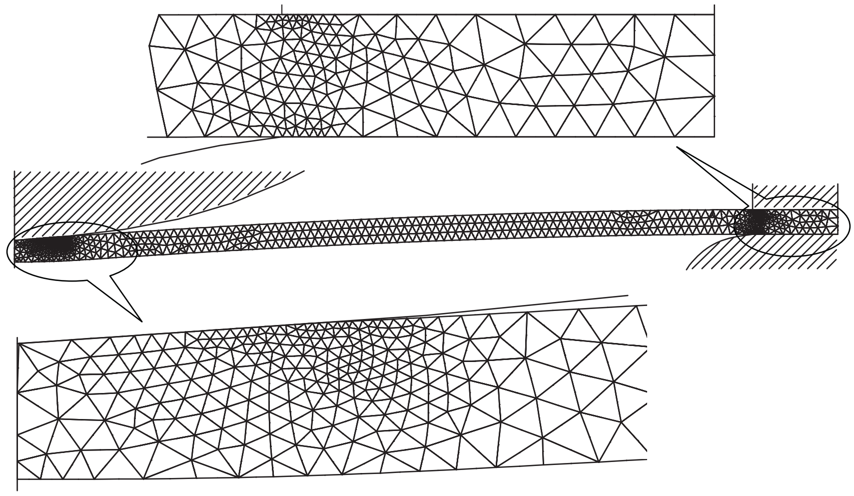

Figure 3(a). Deformed mesh at punch displacement = 2.5 mm [Post-processed field = velocity; Refinement technique = Error-equally-distributed; Accuracy limit = 8%].

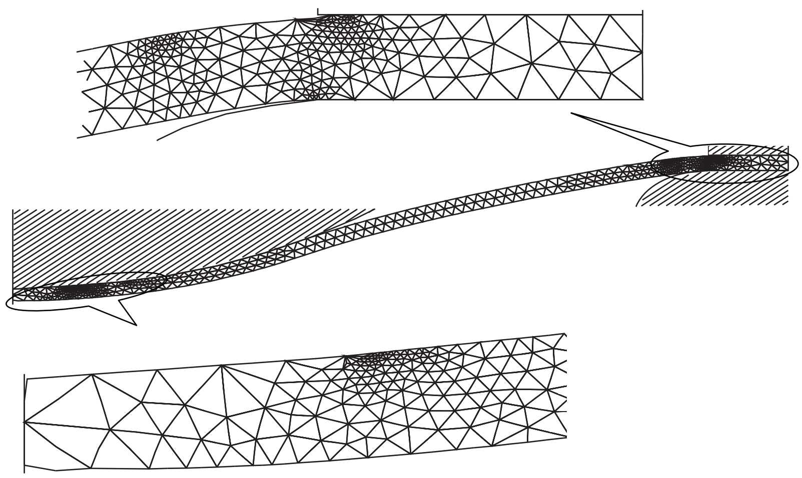

Figure 3(b). Deformed mesh at punch displacement = 20.0 mm [Post-processed field = Velocity; Refinement technique = Error-equally - distributed; Accuracy limit = 8%].

Adaptivity parameter i: post-processed field variables

To study the effect of choice of post-processing of field variable on adaptive analysis, post-processing of velocity and stress are taken. In the post-processed stress based adaptive analysis with error equally distributed refinement strategy, the mesh was automatically regenerated, as the solution error in the energy norm was higher than the predefined error limit of 8%. Only single remeshing was needed to achieve the target error as the global error of the solution remained below the prescribed limit throughout the deformation process. The number of elements consequently increased to 1468. The CPU time for post-processed stress based adaptive analysis was 22 h. The mesh and deformed shapes at punch displacements of 2.5 mm and 20.0 mm are shown in Figures 4(a) and 4(b), respectively. It is observed that after remeshing, the 2 zones, located in regions I and III, have finely distributed elements. In region I, the density of the elements is highest near the punch-blank interface. The elements density decreases from the top surface of the sheet towards the bottom of the sheet. Region III has a band of finer elements across the whole sheet thickness in the vicinity of the inner radius of die-blank and blank-blank holder interface.

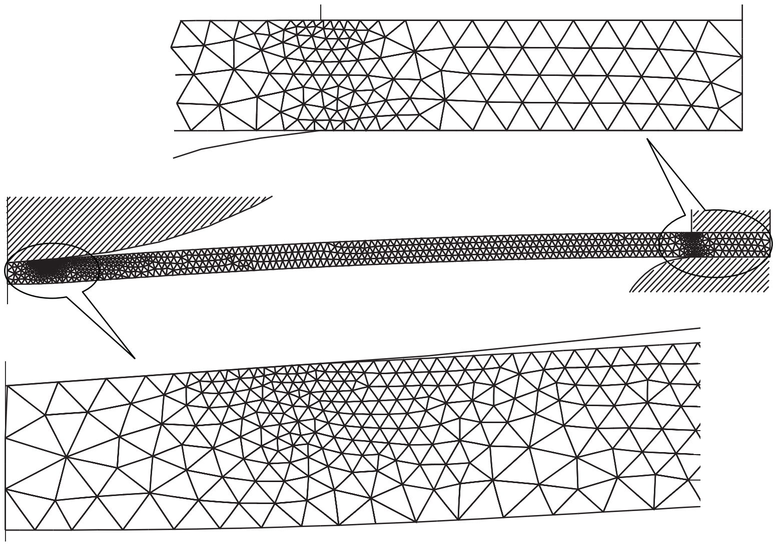

Figure 4(a). Deformed mesh at punch displacement = 2.5 mm [Post-processed field = Stress.

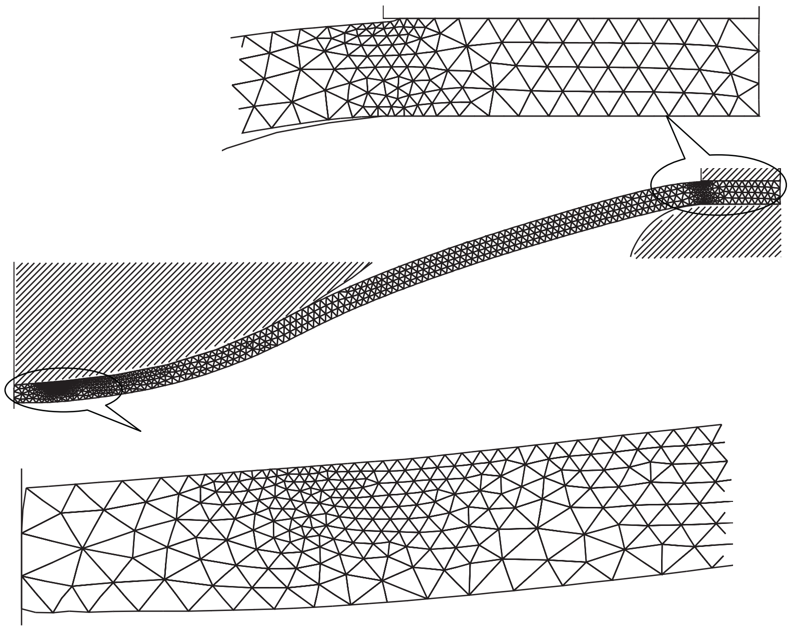

Figure 4(b). Deformed mesh at punch displacement = 20.0 mm [Post-processed field = Stress].

By comparison of results of adaptive parameter recovery of field variable, the CPU time for post-processed velocity based adaptive analysis is smaller compared to post processed stress based adaptive analysis. It can be inferred that the choice of post-processing of basic field variable leads to faster convergence during adaptive analysis, and the patch size of the recovery domain, enhances the recovery process of improved value.

Adaptivity parameter ii: refinement strategy

In order to investigate the influence of implementation of refinement techniques on adaptive analysis, 2 different implementation schemes, namely error equally distributed and square of error equally distributed, schemes are considered. The deformed meshes at 2.5mm and 20.0 mm punch travel for error-equally-distributed refinement strategy are shown in Figures 3(a) and 3(b). As mentioned earlier, 2 remeshings were required to bring the error below the target error when the error-equally-distributed strategy was used. When using square-of-error-equally-distributed strategy, only one remeshing was needed. The numbers of elements were 1459 after remeshing. The CPU time required for the analysis using both the different refinement strategies was equal (nearly 7 h).

The deformed meshes at 2.5 mm and 20.0 mm punch travel corresponding to square-of-error-equally-distributed refinement strategy are shown in Figures 5(a) and 5(b). The observation of mesh at punch travel of 2.5 mm indicates that regions I and III also have high density mesh. Region II has uniform mesh throughout. In region III, a band of finer elements throughout the sheet thickness get generated. A uniform coarser mesh is generated at the start of die radius and in the straight portion of die. The mesh distribution at punch travel of 20.0 mm for square-of-error-equally-distributed strategy is similar to the mesh at punch travel of 2.5 mm.

The results obtained with a different choice of refinement strategies suggest that numbers of remeshings are required to achieve an optimal mesh. The well-optimised mesh is obtained in velocity recovery based adaptive analysis using error equally distributed scheme. There is no marked effect of refinement strategies studied on the efficiency of the analysis.

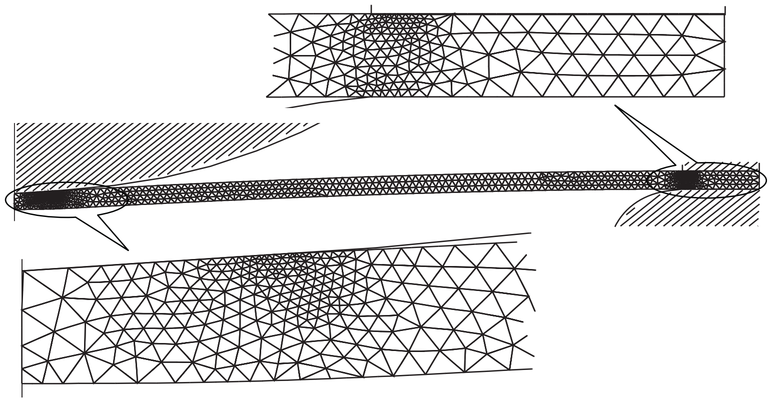

Figure 5(a). Deformed mesh at punch displacement = 2.5 mm [Refinement technique = Square of error-equally-distributed].

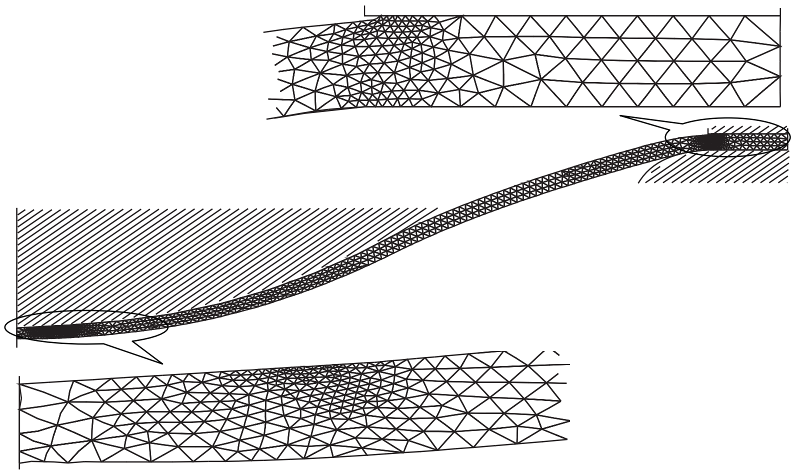

Figure 5(b). Deformed mesh at punch displacement = 20.0 mm [Refinement technique = Square of error-equally-distributed].

Adaptivity parameter iii: limit of accuracy

To study the effect of choice of error limit on adaptive analysis, 2 values of the limit namely, 3% and 8%, were considered. For 3% target error, the number of elements after remeshing increased to 2950. However, for 8% target error, the number of elements was found to increase to 1239 in the first remeshing. A single remeshing was needed for the 20.0 mm deformation process at 3% limit of error. For 8% error limit, however, one more remeshing was required to bring the error below the predefined error limit. In the next remeshing the number of elements was found to decrease to 1031. It can be deduced that with error limit decrease to one-third, the number of elements increases 3 times.

The mesh plots at 2.5 mm and 20.0 mm deformation for 3% error limit are shown in Figures 6(a) and 6(b), respectively. It is observed from Figure 6(a) that regions I and III, have more finer elements compared to region II. But the density of elements in region II is more or less constant throughout. The deformed mesh at punch travel of 20.0 mm for 3% limit of accuracy is similar to the mesh at punch travel of 2.5 mm.

On comparing the deformed mesh at 2.5 mm punch travel (Figures 3(a) and 6(a)), it is inferred that the pattern of distribution of elements in regions I and III corresponding to 3% error limit is similar to that corresponding to 8% error limit. However, the band of finer elements of regions I and III become wider. Moreover, the end portion of region II becomes a mesh of uniformly fine elements and the central portion becomes uniform but of coarser elements. The CPU time required for the analysis having different prescribed error limit was quite different. While the CPU time corresponding to 8% error limit was 7 h, it was as high as 24 h and 40 min when the 3% error limit was imposed. A 5%-10% error limit is an ideal choice for the adaptive analysis of sheet forming processes.

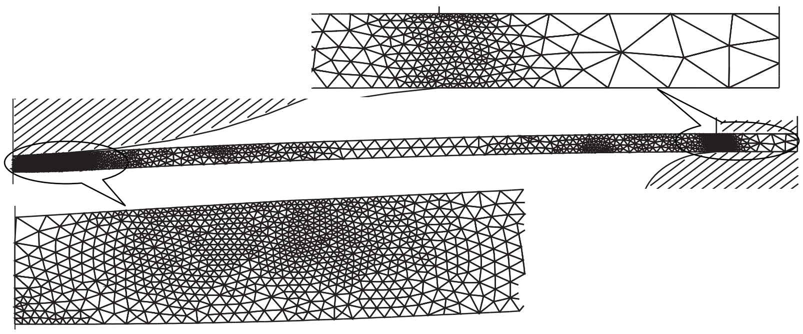

Figure 6(a). Deformed mesh at punch displacement = 2.5 mm [Accuracy limit = 3%].

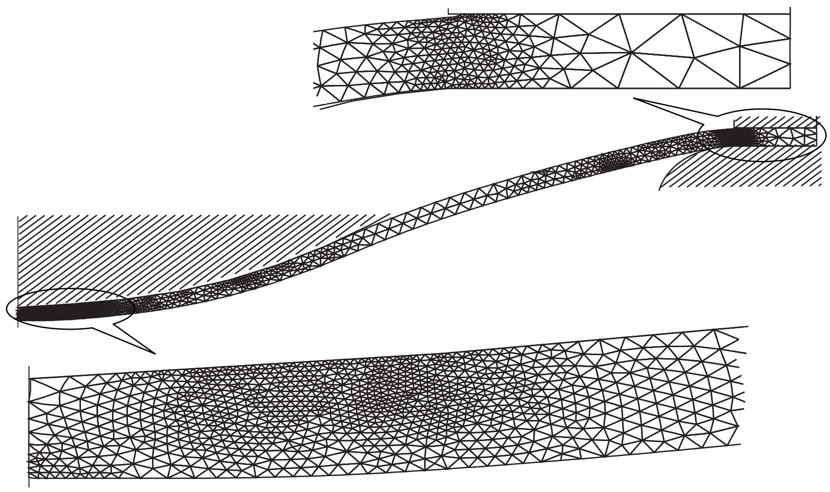

Figure 6(b). Deformed mesh at punch displacement = 20.0 mm [Accuracy limit = 3%].

Validation

The validity of the present model was verified by comparing the predictions of the forming load with those due to Garino and Oliver (1992). Figure 7 shows the plots of forming load at different stages of deformation obtained in the present study and those obtained by Garino and Oliver. The results of the proposed adaptive analyses show good agreement.

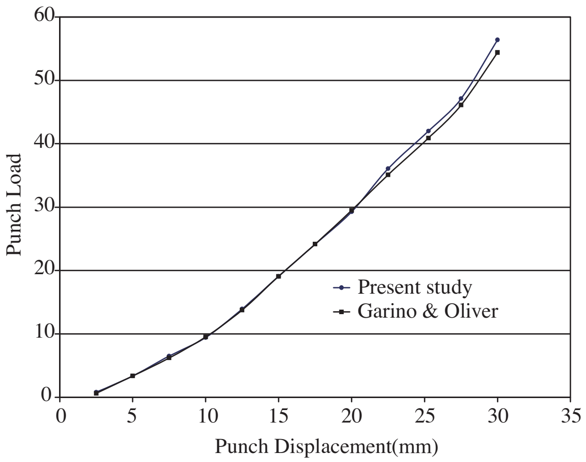

Figure 7. Comparison of load-displacement curves.

In order to show the validity of the present sheet metal stretching study for general sheet forming operations, a deep drawing example, having the same geometrical and mechanical properties, has also been analysed with the proposed adaptive procedures. The velocity recovery technique with error-equally distributed scheme was used. The target error of 8% was adapted. The friction factor was taken as 0.3. A user-defined uniform mesh consisting of 1885 elements was used at the start of the analysis. Four remeshings were required to bring the error below target level.

Figures 8(a) and 8(b) are the mesh plots at 2 deformed locations. The mesh at punch displacement of 2.5 mm shows that the density of elements in regions I and III is high. The end of the region II adjacent to region I has high element density. The density decreases towards the central portion of region II that has more or less a uniformly coarse mesh. The mesh at a punch displacement of 25.0 mm indicates that high-density zone of region I is located in the vicinity of the outer edge of punch. The elements of region II may be divided into 2 zones. The zone near region I has a coarse mesh while the zone near region III consists of a finer mesh. Mesh refinement develops a fine mesh throughout the thickness in region III except at the outer end edge of straight portion of die where coarser elements are formed. The CPU time for adaptive analysis of deep drawing was found to be 4 h.

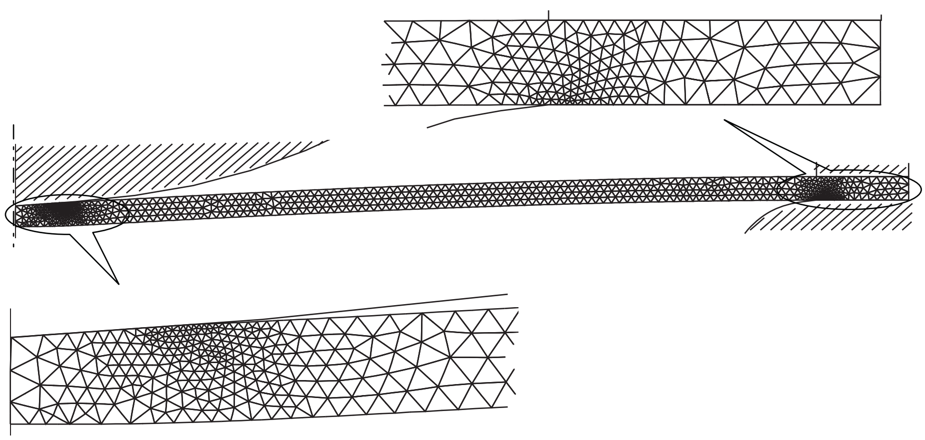

Figure 8(a). Punch displacement = 2.5 mm (Deep drawing example) [Post-processed field = Velocity;Refinement technique = Error-equally-distributed; Accuracy limit = 8%].

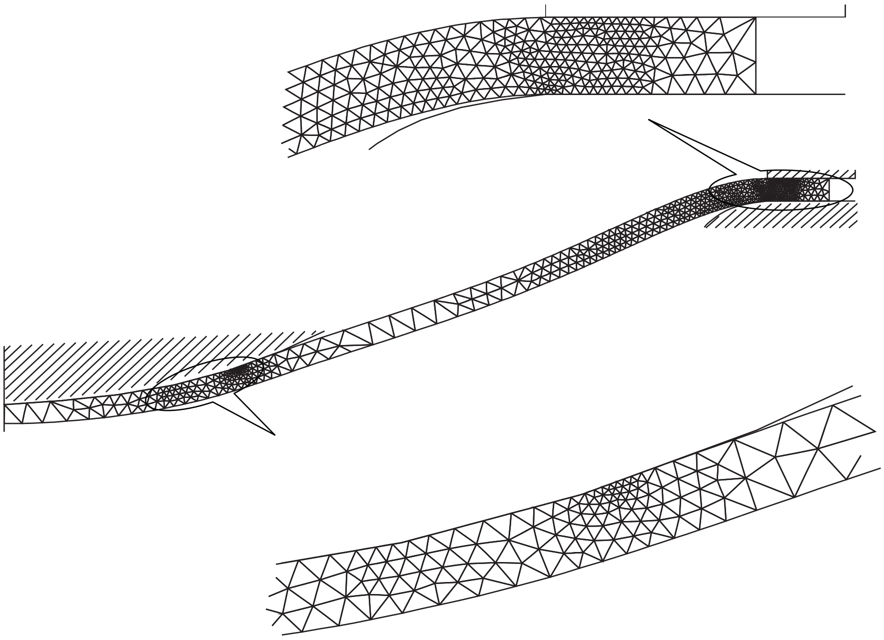

Figure 8(b). Punch displacement = 25.0 mm (Deep drawing example) [Post-processed field = Velocity; Refinement technique = Error-equally-distributed; Accuracy limit = 8%].

Conclusions

In the present study, adaptive finite element analysis of the sheet stretching operation by varying the adaptivity parameters has been carried out. In the analysis, 2 field variables for higher order post-processed values, namely displacement and stress, 2 techniques to refinement the mesh, namely error-equally-distributed and square-of-error-equally-distributed, and 2 error limits, namely 3% and 8%, were used. The adaptive analysis of deep drawing operation was also carried out to test the validity of the present study for general sheet forming operations. The main conclusions of the study are summarized below.

1. In adaptive analyses with different adaptive parameters, the initial uniform mesh got refined with required number of remeshing to bring the error percentage below the target accuracy. It gets finer in places of expected high gradients of strain rate and coarser in places of expected low gradients.

2. The refined mesh in both post-processed techniques has finer elements in the same regions of the blank. However, the distribution of elements in generated adaptive mesh is different. The zones of finer elements are more localized in the velocity post-processed based scheme as compared to those in the stress post-processed scheme. In addition, in the post-processed velocity based approach, the band of refined mesh is of larger width with higher element density.

3. The location of high-density finer elements zones predicted by 2 post-processed scheme also suggest that the performance of velocity post-processed based approach is better compared to that of post-processed stress based approach.

4. The CPU time for velocity recovery based adaptive analysis is smaller compared to stress recovery based adaptive analysis. In other words, the former approach is more efficient.

5. Performance of the error-equally-distributed strategy seems to be better compared to square-of-error-equally-distributed refinement strategy as former strategy results in a well optimised mesh.

6. The efficiency of both refinement techniques studied appears to be same because CPU time required for the analysis using both refinement strategies is same.

7. When the error limit is changed from 8% to 3%, denser mesh is obtained. The numbers of elements are increased 3 times with about one-third decrease in error limit

8. The CPU time is drastically increased with higher accuracy level. A 5%-10% error limit is a good choice for the adaptive analysis of sheet forming operations.

Nomenclature

μ	Viscosity
$\varepsilon_{ij}, \dot{\varepsilon}_{ij}$	Strain and strain rate tensors
$\bar{\varepsilon}$	Effective strain
ε_{ij}	Volumetric strain
σ_{ij}	Stress tensor
σ'_{ij}	Deviatoric stress tensor
$\overline{\sigma}$	Effective stress
. σ_y	Current yield Stress
D	Modulus matrix
h_{old}	Old size of the ith element
K	Penalty constant
N	Number of element in the domain
np	Total number of sampling points
p	Order of the approximating polynomial
x_i , y_i	Coordinates of the sampling points
$\bar{t}$	Surface load vectors
u_i	Displacement Field
e_u, e_σ	Errors in displacement and stress field
$\|e\|$	Energy norm
η_{allow}	Prescribed error percentage

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