A Field Study of Scour at Bridge Piers in Flood Plain Rivers

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Abstract

Scour at bridge piers has long been recognized as an issue of considerable importance as it pertains to pier hydraulic design, according to a large number of scour studies conducted over the past few decades. Despite this long record of study, pier scour remains the most significant cause of bridge failure. The current study, which was based on field evaluations of pier scour failures in rivers with cohesionless beds, focused on a general view of the scouring process. The study was based on the results obtained from the investigation of 6 bridges on 3 rivers in Fars Province, Iran. The hydraulic effects of flow depth and velocity, sediment characteristics such as specific gravity, internal friction angle, particle size, and particle size distribution, and bridge pier geometry were considered. A certainty analysis based on field data was performed to provide a very rough estimate of risk. Statistical and physico-mathematical methods were used in analyzing the data. When possible, a simple comparison was made between the depth of scour and some existing empirical formulae. Among these formulae, the Hanco, CSU, Veiga, and Neill equations exhibited rather good agreement with the field data; however, the Laursen and Inglis equations overestimated scour depth.

Key words: Field study, Iranian rivers, Pier scour, Scouring process

Introduction

When bridge piers are set on erodible beds the locally high velocity of flow caused by the fluid-structure interaction and the related contraction often cause scour to occur in the vicinity of the piers. Flood flow in natural rivers scours the river bed and creates large holes around bridge piers that gradually extend beneath them, eventually destroying them. As reported by many investigators, such as Lefter (1993), Antunes (2005), and FHWA (1998), pier scour has been linked to the most severely damaged and collapsed highway bridges in the United States; therefore, scouring is one of the main factors that cause the destruction of hydraulic structures, especially at bridges on rivers. According to Yanmaz (2001), modeling of the scouring mechanism is so complex that no single method for universal conditions concerning flow, sediment, river, and pier characteristics has been developed to date. Therefore, the absence

of comprehensive mathematical methods for predicting scour depth for pier design is a significant reason that causes certain bridges to collapse, resulting in adverse financial impact, increased travel time due to the disruption of travel routes, and, occasionally, in loss of life (Antunes, 2005).

Many laboratory pier scour experiments have been conducted, but field investigations are needed to increase our knowledge of the subject. The aim of the present study was to present a general view of the scouring process at bridge piers on the ground. This case study was based on the investigation of several bridges on flood plain rivers in Fars Province, Iran. For the purposes of this study, data were collected from the study sites and subsequently analyzed using statistical and physico-mathematical methods. Field observations and photographic methods were also utilized. What makes this study particularly unique is the large flood event of November 1986, which occurred when the study was under way. Three of the bridges in the study failed during the event, resulting in thousands of dollars of damage (Ghorbani, 1988).

Scour Equations

Scour can be defined as the erosion of a channel bed such that the bed level is lowered. The change from the primary to secondary bed level resulting from the erosion is referred to as scour depth. Overall, scour in a river involves 3 components: general scour, contraction scour, and local scour; however, local scour, which occurs in response to fluid-structure interaction in the presence of an erodible boundary material, is the most important. It is of considerable importance in respect to the design of many types of hydraulic structure, particularly bridge piers. This is because when bridge piers, bridge abutments, erosion control devices, or other structures that obstruct the flow are set on erosive beds, the high velocity of flow, shear stress, down-flow, and horse shoe and wake vortices create a scour hole (Chow, 1959; Shen, 1971; Melville and Raudkivi, 1977; Melville, 1984; Raudkivi and Ettema, 1986). The depth of the scour hole is typically much larger than that resulting from general or contraction scour, often by a factor of 10 (Fischenich and Landers, 2000). This phenomenon is one of the main factors that cause hydraulic structures to fail, especially bridges on flood plain rivers.

Researchers have extensively studied local scour depth and recommended many empirical formulae for prediction purposes, but the results for a specific pier and even for similar equations vary widely. Because of the complexity of the scouring process in rivers, development of a single mathematical relation between local scour depth, sediment factors, geometric and hydraulic parameters, and types of scour that occur (i.e. clear water or live bed scour) in river beds is not feasible for universal conditions (Shen, 1971; Raudkivi and Ettema, 1986; Yanmaz and Cicekdag, 2001).

There are a number of factors, such as upstream flow velocity, upstream flow depth, sediment particle size and shape, and pier width, that affect local scour depth. Yet the quantification of some of these factors, such as particle shape, cohesiveness, and sediment or flow regimes is difficult, whereas for some of them it is feasible. Therefore, after summarizing and using the Buckingham pi theorem for dimensional analysis, the following relation was achieved between scour depth and the main factors:

$$\frac{d_s}{b} = f\left(\frac{V}{V_c}, \frac{d}{b}, \frac{D_{50}}{b}\right) \tag{1}$$

where $d_s = \text{local scour depth}, V = \text{mean upstream}$ velocity, $V_c = \text{critical mean velocity}, d = \text{upstream}$ depth of flow, b = pier width, and $D_{50} = \text{median}$ size of bed material (Shen, 1971).

Scour occurs when the shear stress at the flowparticle interface exceeds the critical value necessary for incipient motion of sediment particles in the flow direction. The critical shear stress can be computed using Shields' criterion as follows (Neill, 1975):

$$\tau_c = 0.06 (\gamma_s - \gamma) \quad D_{50} D_{50} > 5mm$$
 (2)

where τ_c = critical shear stress (N/m²), γ_s = specific weight of the sediment particles (N/m³), γ = specific weight of the water (N/m³), and D_{50} = average grain size diameter (m). The critical velocity can be calculated using Hanco's equation (Breusers et al., 1977):

$$\frac{V_{cr}^2}{(S_g - 1) \ gD_{50}} = 1.44 \left(\frac{d}{D_{50}}\right)^{0.4} \quad 2 < d/D_{50} < 100$$
(3)

where V_{cr} = mean threshold velocity (m/s), S_g = sediment specific gravity, and g = acceleration due to gravity (m/s²). Because of the horseshoe vortex and acceleration of flow velocity around a pier, the local velocity adjacent to a pier is larger than the upstream approach flow velocity. Therefore, local velocity is assumed to be twice that of the approach velocity, as recommended by Breusers et al. (1977) for finding the type of scour at bridge piers (i.e. clear water or live bed scour).

There exist many empirical equations that describe local scour depth as a function of other factors. Neill (1965) suggested the following equation for computing scour depth, d_s , at rectangular piers in live bed conditions for $\alpha = 0$:

$$\frac{d_s}{b} = 1.5 \left(\frac{d}{b}\right)^{0.3} \tag{4}$$

where α is the angle of attack.

Hanco (Breusers et al., 1977) recommended an equation for a general case in which sediment particle size is explicitly included for d/b > 1, and $D_{50} = 0.5$, 2, and 5 mm:

$$\frac{d_s}{b} = 3.3 \left(\frac{D_{50}}{b}\right)^{0.2} \left(\frac{d}{b}\right)^{0.13} \tag{5}$$

An equation based on the CSU equation is suggested for both live bed and clear water local scour depth (Richardson et al., 1990), which is currently provided in the HEC-18 manual (NHI, 2001):

$$\frac{d_s}{b} = 2.0K_1K_2K_3K_4\left(\frac{d}{b}\right)^{0.35}F_{r1}^{0.43} \tag{6}$$

where K_1 = the correction factor for pier nose shape (1.0 for round noses and cylindrical piers and 1.1 for square noses), K_2 = the correction factor for flow angle of attack (1.0 for a zero degree angle of attack and a range of L/b = 4-12, where L is pier length), K_3 = the correction factor for bed conditions (1.1 for clear water scour and nearly 1.0 for live bed scour), K_4 = the armoring correction factor, and F_r = the Froude number of the approach flow. More details of Eq. (6) can be found in the HEC-18 manual (NHI, 2001).

Veiga (1970) reported the following equation for estimating the scour depth in a live bed scour condition for a circular pier and 0.5 < d/b < 4, in which the influence of grain size was considered to be negligible for D < 0.5 mm on d_s/b :

$$\frac{d_s}{b} = 1.35 (\frac{d}{b})^{0.3} \tag{7}$$

An alternative relation is given by Indian experiments for model bridge piers set in a sand bed for live bed scour conditions (Laursen, 1962):

$$\frac{d_s}{b} = 1.8(\frac{d}{b})^{0.75} \quad 0 < d_s/b < 7.6 \text{ and } 0 < d/b < 7$$
(8)

There is also another Indian experimental equation expressed by Inglis (1949) in which the Froude number is brought into consideration as follows:

$$\frac{d_s}{b} = 4.2 \left(\frac{d}{b}\right)^{0.78} F_r^{0.52} \quad 0 < d/b < 7 \text{ and } F_r < 1$$
(9)

where F_r is the Froude number, which is written as $F_r = V/(gd)^{1/2}$, where g = acceleration of velocity, and V = mean flow velocity. The abovementioned equations represent only a few samples of the many that have been derived to predict sour depth at bridge piers.

Materials and Methods

For field experiments of the scour process at bridge piers and monitoring riverbed behavior, 9 bridges with a total of 37 piers were chosen from 5 flood plain rivers in Fars Province, Iran (Table 1). The rivers were mostly unstable and had live beds. During the field investigations, 3 bridges, namely Shir Baba on the Ardakan River, Keradeh on the Gharah Aghag River, and Ghotb-Abad on the Jahrom Salt River, collapsed when a large flood event occurred in November 1986. Figure 1 shows the conditions of these rivers before and after the flood event. The study and data collection were carried out on the remaining 6 bridges with a total of 26 round and square nose piers.

	Stream	River		No. of	Pier	Pier	Opening	Situation
River	permanence/	width	Bridge	piers	diameter	shape	width	after 1986
	stability	(m)			(m)		(m)	flood
Ardekan	Permanent,	13.5	Shir Baba*	4	0.85	Round	3.2	Collapsed
	non-stable							
Shesh-Pir	Permanent,	21	Shesh-Pir	2	2.5	Round	6	Scoured, but safe
	non-stable							
		61.5	Bagh-Safa	5	0.42	Round	10	Scoured, but safe
	Ephemeral,							
Shiraz	vertically	60	Horr	5	0.42	Round	10.5	Scoured, but safe
	variable							
		61	Aber-Piadeh	5	0.75	Round	17.4	Scoured, but safe
		32	Brijan	1	1.0	Round	15	Scoured, but safe
Gharah	Ephemeral,							
Aghag	non-stable	533	Choghadeh	8	1.2	Square	9	Scoured, but safe
		-	Keradeh*	3	1.0	Round	4	Collapsed
Jahrom	Ephemeral,		Ghotb-	4		Round	5	Collapsed
Salt	non-stable		Abad*					

Table 1. Characteristics of the rivers and bridges.



Before flood





Before flood



After flood b) Shir Baba, Ardakan River.



Before flood



After flood

c) Ghot Abad, Jahrom Salt River.

Figure 1. The situation at 3 bridges before and after the flood of November 1986: (a) Keradeh, (b) Shir Baba, and (c) Ghot-Abad on the Gharah Aghag, Ardakan, and Jahrom Salt rivers, respectively.

Hydraulic parameters of the rivers were measured/computed where the bridges were located. Upstream mean flow depth was determined using data recorded by a lymnograph and a rating curve available for the river section. The observed mean upstream flow depth with river cross section profiles was used to determine cross-sectional area, wetted perimeter of the flow, as well as top and bottom widths of the flow area located 5 m upstream and downstream of the bridge piers. The water surface slope (\approx energy slope) was determined using water levels measured at 2 points over a distance of 75 and 100 times the mean flow depth from the upstream end face of the bridge piers. Estimation of Manning's roughness coefficient was accomplished using both a

computed value on the basis of the Strickler formula and direct discharge measurements obtained during low flow periods using a current meter and Manning's equation in order to have a primarily rough estimation of n and a rough value obtained from figures of Manning's n for the various sites, as given in Chow (1959). The numerical values of n estimated for reaches involved in this study were in the range of 0.024-0.04, where a constant value was considered for n for a particular reach. Flow rate was determined using Manning's equation based on the measured water surface profile and cross-sectional area, an estimate of Manning's roughness coefficient, as well as by using hydrometric data from local hydrometric stations. Maximum discharge for the study period

varied from 8 m^3/s to 3342 m^3/s in the Shesh-Pir and Jahrom Salt rivers, respectively.

At each bridge site 6 cross sections were surveyed using an engineer's level. Three cross sections were surveyed upstream and 3 cross sections downstream, specifically at 0 m, 1 m, and 3 m end nose of each pier. Changes in bed level in the vicinity of the bridges before and after the flood show the degradation or aggregation that occurred during the flood event. Sediment characteristics at each site (Table 2) include median particle size (D_{50}) and particle size distribution, particle specific gravity (S_g) or density (ρ_s) , and angle of repose (ϕ) . Sieve analysis was used to determine particle size distribution, including the median size and uniformity coefficient. The uniformity coefficient was determined from

$$C_u = \frac{D_{60}}{D_{10}} \tag{10}$$

where D_{60} and D_{10} = particle sizes for which 60% and 10% of the weight of the material is finer, respectively. Specific gravity was measured using a vacuum air removal technique. The angle of repose was determined by measuring the angle formed by a cone of material carefully poured onto a flat surface. Also shown in Table 2 are the critical shear stress (τ_c) and critical velocity (V_{cr}), as determined from Eqs. (2) and (3), respectively.

Statistical and physico-mathematical methods, as well as visual observations, were employed to analyze the collected data. For analysis of the factors that affected scour depth, the following assumptions were made: 1) the bed materials were non-cohesive, degradation was uniform, and the bed material below the surface had the same characteristics as that at the top; 2) beds were flat, there were no bed forms, and the surface roughness depended only on the particle size of the bed material; 3) the flow regime was uniform and steady; 4) the final scour depth was taken into account; 5) single smooth piers were used for experimental purposes.

Analysis of Uncertainty

Although great care was taken when collecting the field data in this study, the data were normally not precise due to progressive changes in sediment flow and hydraulic parameters, as well as measurement difficulties at the bridge sites. Certainty analysis based on field data can provide a very rough estimate of risk, due to the lack of corresponding statistical information. A statistical analysis can be used to examine the statistical randomness and degree of uncertainty of parameters given in Eq. (1). The relevant parameters, i.e. pier width (b), flow depth (d), scour depth (d_s) , and approach flow velocity (V), and statistical parameter data, i.e. mean (μ) , coefficient of variation (Ω), standard deviation (σ), and variance (σ^2) are presented in Table 3. It should be noted that the values of Ω only reflect the range of parameters tested in this study; therefore, they do not express parameter uncertainty.

According to Yanmaz and Cicekdag, (2001), it is essential to determine the level of risk prior to the design of pier footing. Determination of the probability distributions of the governing parameters of local scour provides a rational tool for reliability analysis and will provide the frequency of these parameters over the range of occurrence. For this purpose, frequency histograms of the parameters were plotted at the 95% confidence interval (Figure 2) for scour occurrences around all the bridges examined in this study. It can be seen from this figure that most of the tests were carried out under subcritical flow conditions (i.e. Fr < 1).

Bridge	D_{50}	S_G	ϕ	$ au_c$	V_{cr}	C_u	Local scour depth
	(mm)		(deg)	(N/m^2)	(m/s)		(m)
Choghadeh	36	2.62	38.6	34.3	1.9	23.8	2.38
Braijan	25	2.62	38.5	23.8	2.0	42.5	2.81
Bagh-Safa	47.5	2.67	38.2	46.2	2.1	18.5	1.67
Horr	31.4	2.65	30.5	30.5	1.8	43.4	0.64
Aber-Piadeh	16.5	2.67	38.3	16.2	1.4	35.0	2.25
Shesh-Pir	72	2.50	39.5	63.6	1.9	13.4	0.60

Table 2. Characteristics of river bed materials at bridge sections used in this study.

Bridges	b	d	V	d_s	D_{50}	х	У	Z
	(m)	(m)	(m/s)	(m)	(mm)	(d_o/b)	(d_s/b)	(Fr)
Choghadeh	1.2	1.44	2.49	2.38	36	1.20	1.98	0.66
Braijan	1	3.44	1.61	2.81	25	3.44	2.81	0.28
Bagh-Safa	0.42	1.39	2.32	1.67	47.5	3.31	3.98	0.63
Horr	0.42	1.16	3.1	0.64	31.4	2.76	1.52	0.92
Aber-Piadeh	0.75	0.93	3.47	2.25	16.5	1.24	3.00	1.15
Shesh-Pir	2.5	0.59	1	0.6	72	0.24	0.24	0.42
Mean value (μ)						2.03	2.26	0.68
Standard deviation, (σ)						1.20	1.19	0.29
Variance (σ^2)						1.45	1.42	0.09
Coefficient of						0.59	0.53	0.43
variation (Ω)								

 Table 3. Statistical information for calibration data.



Figure 2. Frequency histograms of the scouring parameters for all bridges.

Results and Discussion

After the flood event of November 1986, any changes in the river beds or banks, such as degradation or aggradation of sediment, were evidence that scour or erosion had taken place. Figure 3 is an illustra-

tive example of scour at a bridge pier observed after the flood event of November 1986. An example of scour might occur when shear stress exceeds that required for incipient motion of the sediment particles in the flow direction. This phenomenon was accelerated near the pier for several reasons, including the existence of high flow velocity next to the pier, which was nearly 2-fold greater than the approach flow velocity, the horseshoe vortex, and the down-flow in front of the pier face. Table 4 shows 2 types of scour—clear water and live bed scourand the flow properties in the rivers under investigation. As shown by the results presented in Table 4, live bed scour occurred when the approach flow velocity exceeded the critical velocity, which was the case for Choghadeh, Bagh-Safa, Aber-Piadeh, and Horr bridges; however, when the approach flow velocity was less and local velocity around the pier was higher than the critical velocity, clear water scour occurred at Brijan and Shesh-Pier bridges. The critical shear stress was computed using Shields' criterion (Eq. (2)) and the critical velocity was calculated using Hanco's equation (Eq. (3)). The former equation was derived for $D_{50} > 5$ mm and latter was derived for the range of $2 \text{ mm} < d/D_{50} < 100 \text{ mm}$. It is interesting that the field data in this study were in the range and conditions mentioned in these 2 equations (Table 2).

It should be noted that, except for Bagh-Safa and Horr bridges, pier spaces that were protected with stone aprons and Aber-Piadeh bridge piers, which were protected with thin square concrete collars located beneath the riverbed level, the others were exposed and not properly protected. Scouring occurred



(a) Brijan Bridge, Choghadeh River.

(b) Horr Bridge, Shiraz River.

Figure 3. Two illustrative examples of scour occurred at bridge piers (a) Brijan and (b) Horr under investigation after flood event of November 1986.

and bridges failed where the piers were not well protected with countermeasure methods. Due to improper design in some cases, scouring also occurred around the piers protected by countermeasures, such as a concrete apron. Figure 4 shows the profiles of 4



Figure 4. Profile of 4 cross sectional areas: (a) -3, (b) piers nose, (c) between piers, and (d) + 3 of the Shiraz River at Aber-Piadeh Bridge before and after the flood event of November 1986 (assumed datum = 100 m, X axis is located at 0 m upstream end face of pier perpendicular to flow direction and Y axis is parallel to it, and pier length is 1.8 m).



Figure 5. Longitudinal profiles of the river bed next to:
(a) pier 1, (b) pier 2, (c) pier 3, (d) pier 4, and
(e) pier 5 at Aber-Piadeh Bridge before and after the flood event of November 1986 (assumed datum = 100 m, P = pier number beginning from the left bank, d = water depth).

	Flow	Flow	Critical	Approach	Local	Local	Type of
Bridge	rate*	depth	velocity	velocity	velocity	scour depth	scour
	(m^3/s)	(d, m)	$(V_{cr} m/s)$	(V,m/s)	$(V_p m/s)$	(d_s, m)	
Choghadeh	1848	1.44	1.90	2.49	4.98	2.38	Live bed
Braijan	620	3.44	2.03	1.61	3.22	2.81	Clear water
Bagh-Safa	190	1.39	2.08	2.32	4.64	1.67	Live bed
Horr**	192	1.16	1.80	2.75	5.52	0.64	Live bed
Aber-Piadeh	189	0.93	1.40	3.47	6.94	2.25	Live bed
Shesh-Pir	8	0.59	1.88	1.00	2.00	0.60	Clear water

Table 4. Flow characteristics and local scour for the flood event of November 1986 in Fars Province, Iran.

*Flow rates of the Jahrom Salt River and Ardakan River at bridges that failed were 3342 and 27 m³/s, respectively. **Due to low certainty, Horr Bridge data were excluded from further consideration.



Figure 6. General and contraction scour between (a) piers 1 and 2, (b) piers 2 and 3, (c) 3 and 4, and (d) piers 4 and 5 at Aber-Piadeh Bridge before and after the flood event of November 1986 (Shiraz River).

cross sections of the Shiraz River at Aber-Piadeh Bridge before and after the flood event of November 1986. Two of these cross sections are located at the upstream face of the piers at distances of 0 m and 3 m, 1 profile belongs to the points located next to the piers, and the other one was located 3 m away from the downstream end face of the piers. The continuous and intermittent lines in Figure 4 are representative of the riverbed elevations before and after the flood event, respectively. As shown in Figure 4, scour occurred mostly at nearly zero distance from the front of the piers and extended to the piers' sides. Some points between the piers show aggradation of bed load materials where flow velocity was low.

Figure 5 shows the longitudinal profiles along the rivers in the vicinity of each pier. Each profile started 3 m away from the upstream face of some of the piers and extended to a distance of about 3 m downstream. In this figure, the profile for pier number 4 shows that sediment



Figure 7. Scour depth as a function of bed material particle size.



Figure 8. Comparison of scour equations (CSU, Inglis, Hanco, Neil, Laursen, and Veiga empirical equations) with field measurement data at (a) Choghadeh, (b) Aber-Piadeh, and (c) Bagh-Safa bridges.

Table 2 shows the local scour depths at bridge piers, in terms of bed material characteristics of the rivers in this study. According to the information given in this table and Figure 7, there was linearly an inverse relationship between the relative local scour depth and mean particle size of bed materials ($R^2 =$ 82.3). This means that as sediment particle size increased the local scour depth decreased (Horr Bridge data were excluded from consideration due to their low-level reliability). This was also confirmed by Kells et al. (2001). From this finding it may be concluded that using riprap with a larger size is one of the most effective measures to reduce or control the local scour depth at bridge piers. A comparison of scour equations (Eqs. (4-9)) with field measurements at Choghadeh, Aber-Piadeh, and Bagh-Safa bridges is shown in Figure 8 for F = 0.83, 1.15, and 0.63. The normalized local scour depth (d_s/b) was plotted as a function of dimensionless upstream flow depth (d/b). The widths of the square and round nose piers were 1.2 m, 0.75 m, and 0.42 m. As shown in this figure, Hanco (Breusers et al., 1977), CSU (Richardson et al., 1990), Veiga (Breusers et al., 1977), and Neill (1965) equations were in good agreement with the field data, although the conditions for which Hanco (i.e. for b = 3, 4.7, 6, 13, and 20 cm, and D₅₀ = 0.5, 2, and 5 mm), Veiga (i.e. for rectangular pier) and Neil (i.e. for rectangular pier and $\alpha = 0$) equations were derived are different from those of this study. The reasonable estimate of CSU formulae is also approved by Thamer et al. (2005); however, Indian (i.e. for conditions of $0 < d_s/b < 7.6$ and $0 < y_o/b < 7$) (Laursen, 1962) and Inglis equations (Henderson, 1966) overestimate the scour depth.

Conclusions

Local scour around bridge piers was evaluated using field observation data. Hydraulic parameters, such as water depth and approach flow velocity, and sediment parameters, such as particle size, uniformity coefficient of sediment particle size, angle of repose, and specific gravity were measured on site and in the laboratory. Due to progressive changes in sediment flow and hydraulic parameters, as well as measurement difficulties at the bridge sites, the data were normally not precise. Certainty analysis, therefore, based on field data was performed to provide a very rough estimate of risk. A statistical analysis was carried out to examine the statistical randomness and degree of uncertainty of the parameters given in Eq. (1). From this study, the following observations and conclusions were also drawn. As sediment particle size increased, the depth of local scour decreased. A comparison of scour equations with field measurements revealed that Hanco, CSU, Veiga, and Neill equations exhibited good agreement with the field data; however, Indian and Inglis equations overestimated scour depth.

Nomenclature

b pier width [m]

Cu uniformity coefficient [-]

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- d_o depth of flow [m]
- d_s local scour depth [m]
- D_{50} median size of bed material [mm]
- D_{10} particle sizes for which 10% of the weight of the material is finer [mm]
- D_{60} particle sizes for which 60% of the weight of the material is finer [mm]
- \mathbf{F}_r Froude number [-]
- g acceleration due to gravity (m/s²)
- K_1 correction factor for pier nose shape [-]
- K_2 correction factor for flow angle of attack [-]
- K_3 correction factor for bed conditions [-]
- K_4 armoring correction factor [-]
- R^2 regression coefficient [-]
- L pier length
- S_g specific gravity [-]
- V mean approach flow velocity [m/s]
- V_{cr} critical mean flow velocity [m/s]
- V_p local velocity [m/s]
- γ_s specific weight of the sediment particles $[N/m^3]$
- α angle of attack
- γ specific weight of water [N/m³]
- τ_c critical shear stress [N/m²]
- ϕ angle of repose [degrees]
- ρ_s density [kg/m³]
- Ω coefficient of variation
- σ standard deviation
- σ^2 variance
- μ mean value
- x relative water depth
- y relative local scour depth
- z Froude number

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