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The Effect of Surface Roughness on the Performance of Hydromagnetic Squeeze Films between Two Conducting Rough Porous Elliptical Plates

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Abstract

The present study analyzed the performance of hydromagnetic squeeze films between 2 conducting rough porous elliptical plates. The bearing surfaces were assumed to be transversely rough. The roughness of the bearing surfaces was characterized by a stochastic random variable with non-zero mean, variance, and skewness. The associated Reynolds equation was stochastically averaged with respect to the random roughness parameter. This equation was then solved with appropriate boundary conditions to obtain the bearing performance characteristics, such as load carrying capacity and response time. The results are presented graphically. It was observed that the transverse surface roughness. The results show that hydromagnetic lubrication significantly increased the load carrying capacity. With this type of bearing system porosity effects were almost negligible, up to a certain porosity. Additionally, the load carrying capacity increased due to the negatively skewed roughness and conductivity of the plates. Furthermore, the negative effect induced by the porosity, standard deviation, and variance (+ve) were compensated for, to a considerable extent, by the combined effect of conductivity and magnetization in the case of negatively skewed roughness. Results of the present study suggest that roughness must be given due consideration while designing such bearing systems.

Key words: Squeeze film, Hydromagnetic lubrication, Reynolds equation, Roughness, Load carrying capacity

Introduction

Various advantages are provided by liquid metals like sodium and mercury over conventional lubricants because of their ability to withstand high temperatures (very high). Accordingly, the lubrication properties of liquid metals have been investigated theoretically as well as experimentally. It is advantageous to employ them where very high temperature and speed occur, such as in space entry vehicles. Additionally, the high thermal conductivity of liquid metals suggests that heat generated by viscous dissipation at high speeds is readily conductible from the source of generation, thus resulting in a tendency towards uniformity of temperature and viscosity of the lubricant film.

Investigations regarding the properties of liquid metals make it clear that if liquid metals such as mercury and sodium could be pumped or held between the moving surfaces of a bearing, bigger loads could be supported by applying a large magnetic field. Due to the high electrical conductivity of liquid metals the possibility of electromagnetic pressurization from the application of an external magnetic field has been investigated. This electromagnetic pressurization comes into effect when a large external electromagnetic field is applied to the electrically conducting lubricant to induce circulating currents, which in turn interact with the magnetic field to create a body force that pumps the fluid between the bearing surfaces.

As liquid metals are good electrical conductors, one can increase load carrying capacity by the utilization of electromagnetic force, thus overcoming the limitations of lubricants at high temperature and minimizing the effect of low viscosity. Considerably high increases in load carrying capacity are possible by making use of super conducting magnets, which require very little power to provide the magnetic field. A cursory glance at the literature reveals that a good amount of theoretical and experimental work has been done concerning the hydromagnetic lubrication of porous and plain metal bearings.

Theoretical study of magnetohydrodynamic pressurization in liquid metal lubrication has been conducted by Elco and Huges (1962). Kuzma (1964) and Kuzma et al. (1964) studied the behavior of magnetohydrodynamic squeeze films. Shukla (1965) investigated the hydromagnetic theory of squeeze films for conducting lubricants between non-conducting nonporous surfaces in the presence of a transverse magnetic field. Shukla and Prasad (1965) dealt with the behavior of hydromagnetic squeeze films between conducting non-porous surfaces and analyzed the effect of the conductivity of each surface on the performance of the bearing system. A number of theoretical and experimental studies of magnetohydrodynamic lubrication have been conducted (Hughes and Elco, 1962; Snyder, 1962; Dodge et al., 1965; Maki et al., 1966; Patel and Hingu, 1978). Sinha and Gupta (1974) investigated the effect of hydromagnetic lubrication on porous squeeze films between annular plates. Patel and Gupta (1979) made use of the Morgan-Cameron approximation to simplify the analysis of hydromagnetic squeeze films between parallel plates of various geometrical shapes.

Reynolds (1886) considered the configuration of an elliptical plate approaching with relative velocity to another flat plate and obtained an analytical solution. Underwood (1945) first used the term squeeze film for this situation. Prakash and Vij (1973) developed the analysis reported by Wu (1970) and obtained the load carrying capacity leading to the time-height relation for a squeeze film between porous plates of various geometries, including circular, annular, elliptical, and rectangular. Prajapati (1995) analyzed the performance of magnetic fluidbased porous squeeze films between various geometrical shapes, including elliptical plates.

It is known that after having some run-in and wear bearing surfaces develop roughness. The effect of surface roughness was analyzed by many investigators (Michell, 1950; Burton, 1963; Davies, 1963; Tzeng and Saibel, 1967; Christensen and Tonder, 1969a, 1969b, 1970; Tonder, 1972; Berthe and Godet, 1973). Christensen and Tonder (1969a, 1969b, 1970) proposed a comprehensive general analysis, both for transverse and longitudinal surface roughness. Christensen and Tonder's approach formed the basis for the study of the effect of surface roughness in a number of investigations (Ting, 1975; Prakash and Tiwari, 1982; Prajapati, 1991, 1992; Guha, 1993; Gupta and Deheri, 1996; Andharia et al., 1997, 1999).

Patel (1975, 1980) studied the behavior of hydromagnetic squeeze film between porous annular and circular disks with velocity slip. More work concerning hydromagnetic lubrication has been contributed by Bhat (1978), Bhat and Hingu (1978), and Hingu (1979). Recently, Vadher et al. (2008) analyzed the performance of hydromagnetic squeeze film between conducting porous transversely rough triangular plates and Vadher et al. (2008) discussed the behavior of hydromagnetic squeeze film between conducting transversely rough porous circular plates.

The present study sought to analyze the performance of hydromagnetic squeeze films between conducting porous transversely rough elliptical plates.

Analysis

Figure 1 describes the configuration of the bearing system.



Figure 1. Configuration of the bearing system.

The lower plate with a porous facing is assumed to be fixed, while the upper plate moves along its normal towards the lower plate. The plates are considered electrically conducting and the clearance space between them is filled by an electrically conducting lubricant. A uniform transverse magnetic field (B₀) is applied between the plates. The flow in the porous medium obeys the modified form of Darcy's law (Ene, 1969), while the equations of hydromagnetic lubrication theory hold for the film region. Film thickness h(x) of the lubricant film is

$$h(x) = h(x) + h_s(x)$$

where $\hat{h}(x)$ is mean film thickness and $h_s(x)$ is the deviation from mean film thickness, which characterizes the random roughness of the bearing surfaces and is considered to be stochastic in nature and governed by the probability density function

$$f(h_s), -C \le h_s \le C$$

where C is the maximum deviation from mean film thickness. The mean (α) , standard deviation (σ) , and parameter (ε) , which is the measure of the symmetry of random variable (h_s) , are defined by the relationships

$$\alpha = E(h_s)$$
$$\sigma^2 = E[(h_s - \alpha)^2]$$

and

$$\varepsilon = E[(h_s - \alpha)^3]$$

where E denotes the expected value defined by

$$E(R) = \int_{-C}^{C} Rf(h_s) dh_s$$

According to the usual assumptions of hydromagnetic lubrication, the modified Reynolds equation for the lubricant film pressure (Prajapati, 1995; Vadher et al., 2008) is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{h}{\left[\frac{2A}{\mu M^3} \left(tanh\frac{M}{2} - \frac{M}{2}\right) - \frac{\psi A}{\mu c^2}\right]}$$
(1)
$$\cdot \frac{1}{\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{tanh(M/2)}{(M/2)}}\right]}$$

where

$$A = h^3 + 3h^2\alpha + 3h(\alpha^2 + \sigma^2) + \varepsilon + 3\sigma^2\alpha + \alpha^3$$

The concerned boundary conditions are

$$p(x_1, z_1) = 0 (2)$$

where

$$\frac{x_1^2}{a^2} + \frac{z_1^2}{b^2} = 1$$

Solving Eq. (1) with boundary conditions (2) one obtains the non-dimensional pressure

$$P = -\frac{ph^{3}}{\mu h \cdot \pi ab} = \frac{\left[\frac{a/b}{(a/b)^{2}+1}\right] \cdot \left(1 - \frac{x^{2}}{a^{2}} - \frac{z^{2}}{b^{2}}\right)}{2\pi \left[\frac{2B}{M^{3}} \left(tanh\frac{M}{2} - \frac{M}{2}\right) - \frac{\psi B}{c^{2}}\right]} \cdot \frac{1}{\left[\frac{\phi_{0} + \phi_{1} + 1}{\phi_{0} + \phi_{1} + \frac{tanh(M/2)}{(M/2)}}\right]}$$
(3)

wherein

$$B = 1 + 3\alpha^* + 3(\alpha^{*^2} + \sigma^{*^2}) + \varepsilon^* + 3\sigma^{*^2}\alpha^* + \alpha^{*^3}$$

Then, the load carrying capacity given by

$$w = \int_{x=-a}^{x=a} \int_{z=-\frac{b}{a}\sqrt{a^2 - x^2}}^{z=\frac{b}{a}\sqrt{a^2 - x^2}} p dx dz$$

is obtained in dimensionless form as

$$W = -\frac{wh^{3}}{\mu h^{\cdot} \pi^{2} a^{2} b^{2}} = \frac{\left\lfloor \frac{a/b}{(a/b)^{2}+1} \right\rfloor}{4\pi \left[\frac{2B}{M^{3}} \left(tanh \frac{M}{2} - \frac{M}{2} \right) - \frac{\psi B}{c^{2}} \right]} \cdot \frac{1}{\left[\frac{\phi_{0} + \phi_{1} + 1}{\left[\frac{\phi_{0} + \phi_{1} + 1}{(M/2)} \right]} \right]}$$
(4)

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Finally, the time-height relation in non-dimensional form becomes

$$\Delta T = \int_{0}^{t_1/t_0} \frac{W h_0^2}{\mu \pi^2 a^2 b^2} dt$$

which suggests that

$$\Delta T = \frac{1}{4\pi} \left[\frac{a/b}{(a/b)^2 + 1} \right] I \tag{5}$$

where

$$\begin{split} I = \ -h_0^2 \int\limits_{1}^{h_1/h_0} \frac{1}{\left[\frac{2A}{M^3} \left(tanh\frac{M}{2} - \frac{M}{2}\right) - \frac{KH}{c^2}\right]} \\ \cdot \frac{1}{\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{tanh(M/2)}{(M/2)}}\right]} \end{split}$$

Results and Discussion

The pressure distribution was determined by Eq. (3), while Eq. (4) accounted for the variation of load carrying capacity. In addition, the response time is given by Eq. (5). It can be clearly seen from Eqs (3), (4), and (5) that the pressure, load carrying capacity, and response time were dependent on various parameters, such as M, ψ , $\phi_0+\phi_1$, k, σ^* , ε^* , and α^* . Taking roughness parameters to be zero, one obtains the investigation carried out by Prajapati (1995) concerning the performance of a hydromagnetic squeeze film between smooth porous conducting elliptical plates.

Further, considering the roughness parameters as well as the conductivities ϕ_0 and ϕ_1 to be zero, the current study reduces to the analysis of Prakash and Vij (1973) when the magnetization parameter $M \rightarrow 0$. In addition, the discussion of Reynolds (1886) is obtained from this investigation by neglecting the porosity effects.

The present analysis describes the behavior of a squeeze film between rough porous elliptical plates when the magnetization parameter and conductivity are considered to be zero. Finally, our investigation leads to the performance of a hydromagnetic squeeze film between rough porous non-conducting elliptical plates when ϕ_0 and ϕ_1 are taken to be zero.

A close look at Eq. (4) reveals that the roughness adversely affected the system. It was observed that the load carrying capacity increased as $\phi_0 + \phi_1$ increased for fixed values of ψ , M, k, σ^* , α^* , and ε^* . Furthermore, the effect of conductivity on the load carrying capacity (W) and response time (ΔT) came through the factor

$$\left(\frac{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}}{\phi_0 + \phi_1 + 1}\right)$$

Nonetheless, for large values of M this tends to $\frac{\phi_0+\phi_1}{\phi_0+\phi_1+1}$, as tanhM ~ 1 and 2/M ~ 0. One can clearly see that both of these functions are increasing functions of $\phi_0 + \phi_1$. Thus, it may be concluded from the mathematical analysis that the pressure, load carrying capacity, and response time increased as the values of conductivity parameter $\phi_0 + \phi_1$ increased. A close glance at the equation reveals that the bearing with the magnetic field can support a load even when there is no flow.

Variation in load carrying capacity with respect to the magnetization parameter (M) for various values of porosity (ψ), conductivity parameter ($\phi_0 + \phi_1$), standard deviation (σ^*), variance (α^*), measure of symmetry (ε^*), and aspect ratio (k), respectively, are presented in Figures 2-7. It is clearly seen that the load carrying capacity increased as the magnetization parameter (M) increased and that the bearing suffered, in general, due to transverse surface roughness. Moreover, the combined effect of variance and symmetry (-ve) associated with the roughness was strong.

Equally strong was the effect of the aspect ratio. Negatively skewed roughness increased the load carrying capacity significantly. The nature of the effect of negative variance was similar. Conductivity played an important role in enhancing the performance of the bearing system, as the load carrying capacity increased considerably with respect to conductivity (Figures 8 and 9).

The standard deviation associated with roughness adversely affected the bearing system. Further, it was observed that the effects of porosity were negligible up to the value $\psi \approx 0.001$ and that the rate of decrease in load carrying capacity due to porosity was relatively slower beyond $\psi = 0.1$.

Essentially, the response time adopted the trends of load carrying capacity, which is clear from Eq. (5). It is interesting to note that the time required to squeeze out all the fluid was not infinite, as is the case with the non-porous bearings, because the fluid bled into the porous matrix to allow continuity of the flow.







Figure 3. Variation of load carrying capacity with respect to M and $\phi_0 + \phi_1$.







Figure 5. Variation of load carrying capacity with respect to M and α^* .











Figure 8. Variation of load carrying capacity with respect to $\phi_0 + \phi_1$ and k.



Figure 9. Variation of load carrying capacity with respect to α^* and ε^* .

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It is suggested that for both small and large values of M the bearing suffered when the plates were considered electrically conducting, in comparison with the hydromagnetic case when the plates were non-conducting. Probably, this was due to the occurrence of fringing phenomena, which occurred when the plates were electrically conducting. In addition, it is clearly seen that as plate conductivity and plate thickness increased, lubricant pressure, load carrying capacity, and response time increased.

A comparison of the present study with circular geometry indicates that the compensation of the negative effect induced by porosity and roughness was comparatively more in this case. Furthermore, the negative effect of porosity and standard deviation was relatively less in this case.

Conclusion

A close look at this investigation suggests that the negative effect induced by porosity and standard deviation associated with roughness can be compensated for, to a considerable extent, by the positive effect of the magnetization parameter and conductivity parameter in the case of negatively skewed roughness by choosing a suitable aspect ratio. Furthermore, this study shows that ample scope exists for extending the life period of the bearing system.

Thus, this study shows that it is mandatory that roughness be given due consideration while designing bearing systems, even if suitable choices of aspect ratio, magnetization parameter, and conductivity have been taken into account.

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Nomenclature

- a semi-major axis
- b semi-minor axis
- k aspect ratio (a/b)
- h lubricant film thickness
- H magnetic field component
- K permeability
- m porosity of the porous matrix $(1/2)^{1/2}$

M =
$$B_0 h\left(\frac{s}{\mu}\right)^{1/2}$$
 = Hartmann number

- p pressure distribution
- P non-dimensional pressure
- s electrical conductivity of the lubricant
- w load carrying capacity
- W dimensionless load carrying capacity
- B_0 uniform transverse magnetic field applied between the plates
- $c^2 = 1 + \frac{KM^2}{h^2m}$
- $h_0^{'}$ surface width of the lower plate
- h_1^{\prime} surface width of the upper plate
- s_0 electrical conductivity of lower surface
- s₁ electrical conductivity of upper surface
- Δt response time
- ΔT non-dimensional response time

$$\phi_0 = \frac{s_0 h_0}{s h_{\prime}}$$

$$\phi_1 = \frac{s_1 h_1}{sh}$$

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$$\psi = \frac{KH}{h^3} = \text{Porosity}$$

- μ viscosity
- $\overline{\mu}$ magnetic susceptibility
- μ_0 permeability of the free space
- σ^* non-dimensional standard deviation (σ /h)
- α^* non-dimensional variance (α /h)
- ε^* non-dimensional skewness (ε/h^3)

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