

The Effects of Wall Properties on Peristaltic Transport of a Dusty Fluid

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Received 20.10.2008

Abstract

The interaction between peristaltic transport of a dusty fluid (Saffman's model) in a 2-dimensional uniform channel and the elasticity of the channel's flexible walls was studied under long wavelength approximation. Perturbation solutions were obtained for the stream functions of both the fluid particles and solid particles, in terms of the wall slope parameter. The expressions for average velocity of the fluid particles and solid particles, and average fluid flow rate were derived. The effects of various elastic parameters and mass concentration of dust particles on the streamline pattern and average flow rate were studied. The phenomenon of trapping was observed and the area of the trapped bolus increased along with the tension parameter, but decreased with damping and mass concentration of dust particles.

Key Words: Peristaltic transport, Wall properties, Dusty fluid.

Introduction

Peristaltic pumping is the transport of a fluid via traveling waves imposed on the walls of a tube or channel. This is known to be a major mechanism in biological systems, including urine transport from the kidneys to the bladder through the ureter, the movement of chyme in the gastro-intestinal tract, the movement of spermatozoa in the ductus deferens of the male reproductive tract, movement of ova in the fallopian tubes, vasomotion of small blood vessels, and in many other glandular ducts. The industrial use of peristaltic pumping in roller/finger pumps is well known. This principle has been adopted by engineers to pump corrosive materials and fluids that must be kept away from pumping machinery. In particular, the peristaltic transport of toxic liquid is used by the nuclear industry so as not to contaminate the environment. Peristaltic pumping is used in biomedical devices such as the heart-lung machine to pump blood. It is also speculated that peristalsis

may be involved in the translocation of water in tall trees. The translocation of water involves its motion through the porous matrix of the trees.

Several investigators mathematically and experimentally studied the peristaltic transport of Newtonian fluid to understand its fluid mechanical aspects, both in mechanical and physiological situations, under various approximations. Many of these studies explained the basic fluid mechanical aspects of peristalsis and 2 important phenomena—trapping and reflux. Reflux refers to the net retrograde motion of some part of a fluid in a direction opposite that of wave propagation on the wall, and trapping is the development and transport of an internally circulating bolus of fluid (Shapiro et al., 1969). Fung and Yih (1968) observed that pumping against a positive pressure gradient greater than a critical value results in a backward flow in the central region of the stream. An elaborate review of the earlier literature regarding peristalsis is provided by Jaffrin and Shapiro (1971). As the behavior of most phys-

iological fluids is known to be non-Newtonian, attempts have been made to analyze the peristaltic transport of non-Newtonian fluids. Peristaltic transport of blood in small vessels was investigated by several researchers (Picologlou et al., 1973; Srivatsava, 1986; Philip and Peeyush, 1995; Misery et al., 1996; Srinivasacharya et al., 2003) that described the non-Newtonian behavior of fluids as power law, couple stress, simple micro, micropolar, and generalized Newtonian fluid models, respectively.

Most studies have treated blood as a single homogeneous mixture having the properties of a Newtonian fluid or non-Newtonian fluid, or as 2 fluids consisting of a boundary layer of pure plasma near the wall and whole blood elsewhere in the blood vessel. For a realistic description of the flow of blood it is essential to treat blood as a binary system of plasma and blood cells. Saffman's (1962) dusty fluid model serves as a good model for describing blood as a binary system. Kaimal (1978) studied peristaltic transport of a solid-fluid mixture at a low Reynolds number under long wavelength approximation. Radhakrishnamacharya (1978) studied the pulsatile flow of a fluid containing small solid particles through a 2-dimensional constricted channel.

However, the interaction of peristalsis and the elastic properties of the channel wall have not received much attention. Mitra and Prasad (1973) studied peristaltic transport of a Newtonian viscous fluid in a 2-dimensional uniform channel while considering the elasticity of the wall. They reported that flow reversal occurs at the center of the channel if the walls of the channel are elastic and that the position may shift to the boundaries if the viscous damping forces are considered. Srinivasulu and Radhakrishnamacharya (2002) studied peristaltic transport in a non-uniform channel with elastic effects. Study of this interaction under different conditions may lead to a better understanding of the role of peristalsis in the transport of physiological fluids.

The present research aimed to study the effects of channel wall properties on peristaltic transport of a dusty fluid (Saffman's model) through a 2-dimensional channel. Stream functions for both fluid particles and solid particles were obtained under long wavelength approximation. The expressions for average velocity of the fluid particles and solid particles, and average flow rate of the fluid were derived. The effects of various parameters on the streamline pattern, average velocity of fluid particles, and average flow rate were studied.

Formulation of the Problem

Consider the laminar flow of an incompressible fluid that contains small solid particles, whose number density (N) (assumed to be a constant N_0) is large enough to define average properties of the dust particles at a point through a symmetrical 2-dimensional channel. Peristaltic waves of long wavelength are assumed to travel along the walls of the channel. The geometry of the wall surface is described by

$$\eta = d + a \sin \left(\frac{2\pi}{\lambda} (x - ct) \right) \quad (1)$$

where d is the half width of the channel, a is the amplitude of the wave, λ is the wave length, and t is time (Figure 1).

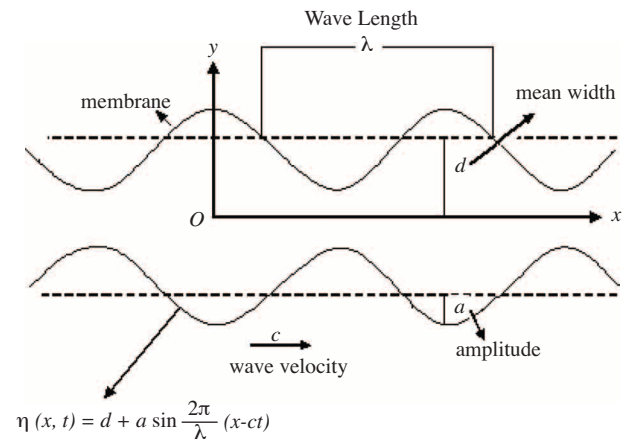


Figure 1. Geometry of 2-dimensional peristaltic motion of channel walls.

The equations of motion of a viscous incompressible fluid with uniform distribution of solid particles are given by Saffman (1962). The flow of fluid is governed by the continuity and momentum equation

$$\nabla \cdot \mathbf{V} = 0 \quad (2)$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V} + \frac{KN_0}{\rho} (\mathbf{V}_s - \mathbf{V}) \quad (3)$$

where $\mathbf{V} = [u, v]$ is the velocity of the fluid particles, p is the fluid pressure, ρ is the density of the fluid, ν is the kinematic coefficient of the viscosity of fluid, and K is the resistance coefficient for the dust particles—a constant. The first 2 terms on the right side of Eq. (3) are, respectively, the pressure gradient and viscosity terms. The last term represents the force due to the relative motion between

the fluid and solid particles. It is assumed that the Reynolds number of the relative velocity is small. In such a case the force between the solid and fluid is proportional to the relative velocity (Saffman, 1962).

The motion of the dust particles is governed by Newton's second law

$$\frac{\partial V_s}{\partial t} + (V_s \cdot \nabla)V_s = \frac{K}{m}(V - V_s) \quad (4)$$

and the continuity equation

$$\nabla \cdot \mathbf{V}_s = 0 \quad (5)$$

where $\mathbf{V}_s = [u_s, v_s]$ is the velocity of the solid particles and m is the mass of the solid particles.

The governing equation of motion of the flexible wall may be expressed as

$$L(\eta) = p - p_0 \quad (6)$$

where L is an operator that is used to represent the motion of the stretched membrane with damping forces such that

$$L = -T \frac{\partial^2}{\partial x^2} + m' \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t} \quad (7)$$

Here, T is the tension in the membrane, m' is mass per unit area, C is the coefficient of viscous damping forces, and p_0 is the pressure on the outside of the wall due to tension in the muscles. This tension may be due to the constitutive relations of the muscles when the displacements are known. Certain terms may be added to Eq. (7) to account for spring foundations, but they do not change the mathematical nature of the problem; therefore, to keep the analysis simple they are not considered herein (Mitra and Prasad, 1973).

It is assumed that $p_0 = 0$ and the walls of the channel are inextensible, so that only their lateral motion takes place and the horizontal displacement of the wall is zero. Thus, the no-slip boundary condition for the velocities is

$$u = 0, u_s = 0 \text{ at } y = \pm\eta \quad (8)$$

Continuity of stresses requires that at the interfaces of the walls and fluid p must be the same as that which acts on the fluid at $y = \pm\eta$. The use of the x - momentum equation yields

$$\frac{\partial}{\partial x} (L(\eta)) = \frac{\partial p}{\partial x} = \rho\nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \rho \left[\frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} + v \frac{\partial u}{\partial y} \right] + \frac{KN_0}{\rho}(u_s - u)$$

$$\text{at } y = \pm\eta \quad (9)$$

Introducing the stream functions, ψ and ϕ , such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, u_s = \frac{\partial \phi}{\partial y}, v_s = -\frac{\partial \phi}{\partial x} \quad (10)$$

and the following non-dimensional variables

$$x' = x/\lambda, y' = y/d, \eta' = \eta/d, t' = (vt)/(\lambda d),$$

$$\psi' = \psi/\nu, \phi' = \phi/(Kd^2/m) \quad (11)$$

into Eqs. (2)-(5), we obtain the following equations (after dropping primes and eliminating the pressure term)

$$\delta \left[\frac{\partial}{\partial t} (\nabla_1^2 \psi) + \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} (\nabla_1^2 \psi) - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} (\nabla_1^2 \psi) \right] = \nabla_1^4 \psi + P \left[\frac{1}{R} \nabla_1^2 \phi - \nabla_1^2 \psi \right] \quad (12)$$

$$\delta \left[R \frac{\partial}{\partial t} (\nabla_1^2 \phi) + \frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} (\nabla_1^2 \phi) - \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y} (\nabla_1^2 \phi) \right] = R \nabla_1^2 \psi - \nabla_1^2 \phi \quad (13)$$

where $\nabla_1^2 = \left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$, $\varepsilon = a/d$, $\delta = d/\lambda$ are geometric parameters, and $R = \frac{\nu m}{Kd^2}$ and $P = \frac{KN_0 d^2}{\rho \nu}$ are non-dimensional parameters.

The boundary conditions in non-dimensional form now become

$$\frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \phi}{\partial y} = 0 \text{ at } y = \pm\eta = \pm(1 + \varepsilon \sin 2\pi(x - t)) \quad (14)$$

$$\nabla_1^2 \frac{\partial \psi}{\partial y} - \delta \left[\frac{\partial}{\partial t} \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \frac{\partial \psi}{\partial y} \right] + P \left(\frac{1}{R} \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial y} \right) = \left(E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t} \right) \eta \text{ at } y = \pm\eta = \pm(1 + \varepsilon \sin 2\pi(x - t)) \quad (15)$$

where $E_1 = \frac{Td^4}{\nu \rho}$ is the membrane tension parameter, $E_2 = \frac{m' d^2}{\lambda^3 \rho}$ is the mass characterizing parameter, and $E_3 = \frac{cd^3}{\lambda^2 \nu \rho}$ is the damping parameter.

Method of Solution

Assuming the parameter δ is very small, the stream functions ψ and φ may be expanded in power series of δ as

$$\begin{aligned} \psi &= \psi_0 + \delta\psi_1 + \delta^2\psi_2 + \dots \\ \phi &= \phi_0 + \delta\phi_1 + \delta^2\phi_2 + \dots \end{aligned} \tag{16}$$

Substituting Eq. (16) in Eqs. (12)-(15) and collecting the coefficients of various powers of δ on both sides, we obtain the following sets of coupled linear differential equations for ψ_0, ϕ_0 , and ψ_1, ϕ_1 :

Zerth order in δ

$$\frac{\partial^4 \psi_0}{\partial y^4} + P \left(\frac{1}{R} \frac{\partial^2 \phi_0}{\partial y^2} - \frac{\partial^2 \psi_0}{\partial y^2} \right) = 0 \tag{17}$$

$$R \frac{\partial^2 \psi_0}{\partial y^2} - \frac{\partial^2 \phi_0}{\partial y^2} = 0 \tag{18}$$

and the corresponding boundary conditions at $y = \pm\eta$ are

$$\frac{\partial \psi_0}{\partial y} = \frac{\partial \phi_0}{\partial y} = 0 \tag{19}$$

$$\begin{aligned} \frac{\partial^3 \psi_0}{\partial y^3} + P \left(\frac{1}{R} \frac{\partial \phi_0}{\partial y} - \frac{\partial \psi_0}{\partial y} \right) = \\ \left(E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t} \right) \eta \end{aligned} \tag{20}$$

First order in δ

$$\frac{\partial^4 \psi_1}{\partial y^4} + P \left(\frac{1}{R} \frac{\partial^2 \phi_1}{\partial y^2} - \frac{\partial^2 \psi_1}{\partial y^2} \right) = \tag{21}$$

$$\frac{\partial^3 \psi_0}{\partial t \partial y^2} + \frac{\partial \psi_0}{\partial y} \frac{\partial^3 \psi_0}{\partial x \partial y^2} - \frac{\partial \psi_0}{\partial x} \frac{\partial^3 \psi_0}{\partial y^3}$$

$$R \frac{\partial^2 \psi_1}{\partial y^2} - \frac{\partial^2 \phi_1}{\partial y^2} = R \frac{\partial^3 \phi_0}{\partial t \partial y^2} + \frac{\partial \phi_0}{\partial y} \frac{\partial^3 \phi_0}{\partial x \partial y^2} - \frac{\partial \phi_0}{\partial x} \frac{\partial^3 \phi_0}{\partial y^3} \tag{22}$$

and the corresponding boundary conditions at $y = \pm\eta$ are

$$\frac{\partial \psi_1}{\partial y} = \frac{\partial \phi_1}{\partial y} = 0 \tag{23}$$

$$\frac{\partial^3 \psi_1}{\partial y^3} + P \left(\frac{1}{R} \frac{\partial \phi_1}{\partial y} - \frac{\partial \psi_1}{\partial y} \right) = \tag{24}$$

$$\frac{\partial^2 \psi_0}{\partial t \partial y} + \frac{\partial \psi_0}{\partial y} \frac{\partial \psi_0}{\partial x \partial y} - \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \psi_0}{\partial y^2}$$

The solutions of Eq. (17) and Eq. (18), subject to the boundary condition Eq. (19) and Eq. (20), are

$$\psi_0 = A_1(y^3 - 3\eta^2 y) \tag{25}$$

$$\phi_0 = A_1 R(y^3 - 3\eta^2 y) \tag{26}$$

where

$$\begin{aligned} A_1 &= -\frac{\varepsilon}{6} \left[(2\pi)^3 (E_1 + E_2) \text{Cos} 2\pi(x - t) \right. \\ &\quad \left. - E_3 (2\pi)^2 \text{Sin} 2\pi(x - t) \right] \end{aligned}$$

Similarly, the solutions of Eq. (21) and Eq. (22), subject to the boundary condition Eq. (23) and Eq. (24), are

$$\psi_1 = \left[\frac{B_1 y^7}{840} + \frac{B_2 y^5}{120} + \frac{B_3 y^3}{6} + B_4 y \right] \tag{27}$$

$$\begin{aligned} \phi_1 &= \left[R \left(\frac{B_1 y^7}{840} + \frac{B_2 y^5}{120} + \frac{B_3 y^3}{6} \right) \right. \\ &\quad \left. - 6R^2 \left(\frac{D_1 y^5}{20} + \frac{D_2 y^3}{6} \right) + D_3 y \right] \end{aligned} \tag{28}$$

where

$$B_1 = 12(1 + S)A_1(\partial A_1/\partial x)$$

$$B_2 = 6(1 + S)[6A_1^2\eta(\partial\eta/\partial x) + (\partial A_1/\partial t)]$$

$$\begin{aligned} B_3 &= 12A_1(\partial A_1/\partial x)\eta^4 + 36A_1^2\eta^3(\partial\eta/\partial x) \\ &\quad - 6A_1\eta(\partial\eta/\partial t) - \frac{B_1}{4}\eta^4 - \frac{B_2}{2}\eta^2 \end{aligned}$$

$$B_4 = -\left(\frac{B_1\eta^6}{120} + \frac{B_2\eta^4}{24} + \frac{B_3\eta^2}{2} \right)$$

$$D_1 = B_1/[6(1 + S)]$$

$$D_2 = B_2/[6(1 + S)]$$

$$D_3 = 6R^2 \left[\frac{D_1\eta^4}{4} + \frac{D_2\eta^2}{2} \right] + RB_4$$

$S = \frac{N_0 m}{\rho}$ is the mass concentration of dust particles.

Average axial velocities of the fluid particle (\bar{u}) and the solid particle (\bar{u}_s) (up to first order) over one period of motion are given by

$$\bar{u} = \int_0^1 u dt \tag{29}$$

$$\bar{u}_s = \int_0^1 u_s dt \tag{30}$$

where

$$u = 3A_1(y^2 - \eta^2) + \delta \left[\frac{B_1 y^6}{120} + \frac{B_2 y^4}{24} + \frac{B_3 y^2}{2} + B_4 \right] \quad (31)$$

$$u_s = 3A_1 R(y^2 - \eta^2) + \delta \left[R \left(\frac{B_1 y^6}{120} + \frac{B_2 y^4}{24} + \frac{B_3 y^2}{2} \right) - 6R^2 \left(\frac{D_1 y^4}{4} + \frac{D_2 y^2}{2} \right) + D_3 \right] \quad (32)$$

The fluid flow rate (Q) defined by

$$Q = \int_0^\eta u dy \quad (33)$$

can be obtained using Eq. (31) as

$$Q = -2A_1 \eta^3 + \delta \left[\frac{B_1 \eta^7}{840} + \frac{B_2 \eta^5}{120} + \frac{B_3 \eta^3}{6} + B_4 \eta \right] \quad (34)$$

The time average flow rate over a period of time defined by

$$\bar{Q} = \int_0^1 Q dt \quad (35)$$

can be obtained using Eq. (34).

Results and Discussion

The average fluid flow rate was obtained from Eq. (35) by carrying out integration with respect to $x-t$.

To explicitly see the effects of various parameters on these flow variables, these quantities were numerically evaluated and the results are graphically presented in Figures 2-8.

The rigid nature of the wall is represented by the parameter E_1 , which depends on the wall tension, and E_2 represents the stiffness property of the wall. The parameter E_3 represents the dissipative feature of the wall. The choice $E_3 = 0$ implies that the wall moves up and down, with no damping force on it and, therefore, indicates the case of elastic walls.

The effect of the rigid nature of the walls on the streamline patterns for the elastic walls ($E_3 = 0$) is shown in Figure 2a and b. It can be seen from this figure that the streamline got closer as the tension parameter (E_1) increased and the phenomenon of trapping was observed. It is significant to note that the trapping phenomenon became predominant and the area of the trapped bolus increased as the tension parameter increased. The effect of dissipative walls on the streamline is given in Figure 2c and d. Figure 2c and d show that, as the dissipative nature of the wall increased, change in the character of the streamlines for fixed values of E_1 was not significant. Though trapping was observed and was predominant, it can be seen that as damping (E_3) increased the area of the trapped bolus in some regions decreased.

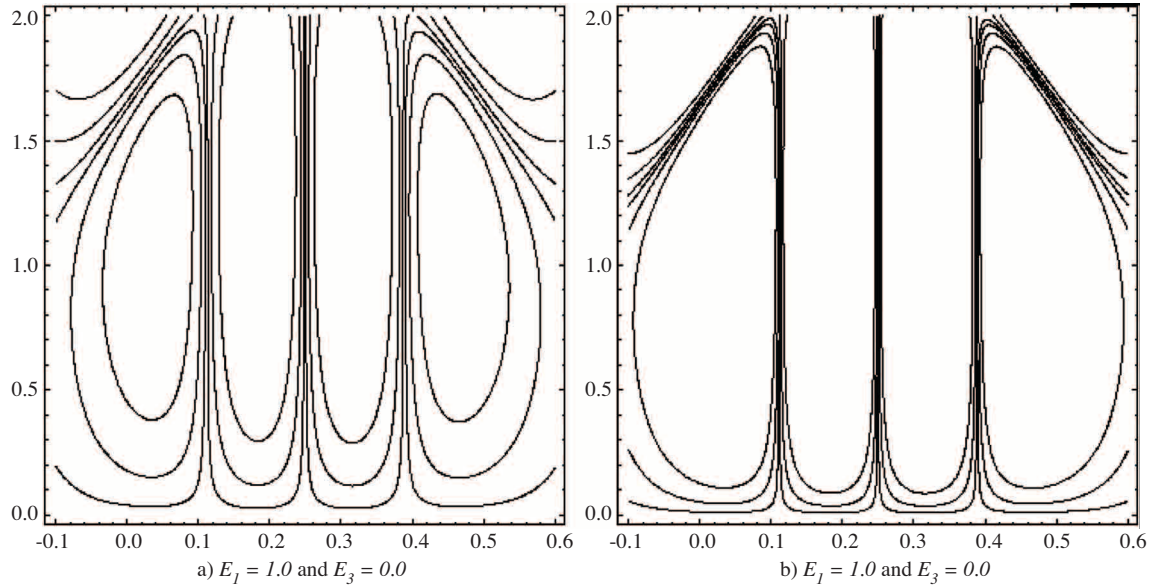


Figure 2. Effect of E_1 and E_3 on the streamline pattern of fluid particles for $E_2 = 0.2$, $\delta = 0.2$, $\varepsilon = 0.4$, $R = 1$, and $S = 0.5$.

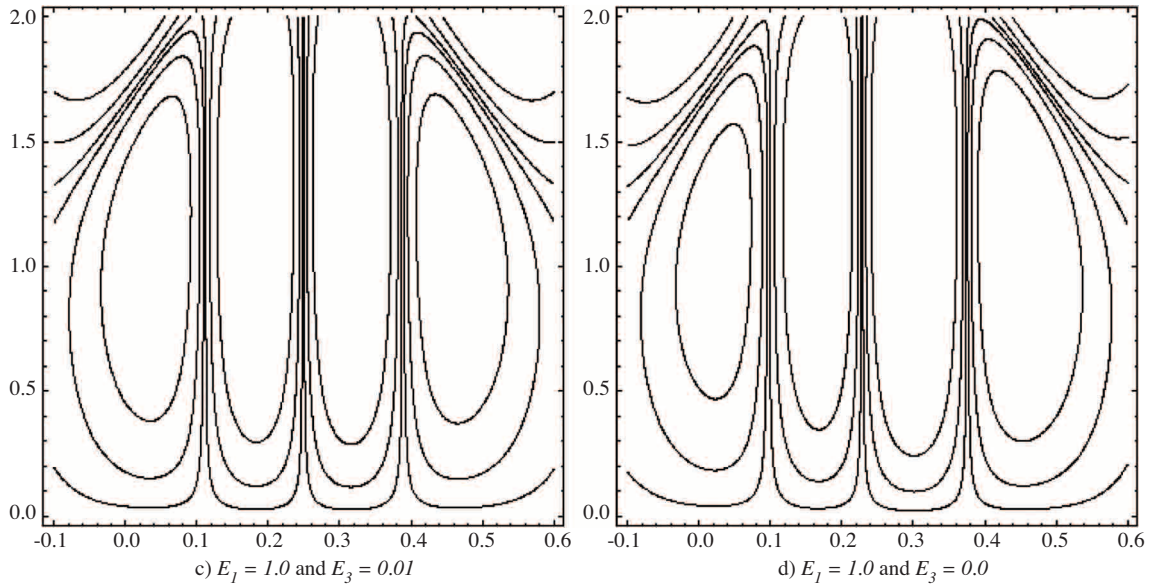


Figure 2. Continued.

Figure 3 shows the effect of the mass characterizing parameter (E_2) on the streamline pattern for elastic and dissipative walls. As the mass concentration parameter (E_2) increases, the streamlines get closure and the area of the trapped bolus increases, both for elastic (Figure 3a and b) and dissipative walls (Figure 3c and d). The effect of the mass concentration parameter (S) on the streamline pattern

is shown in Figure 4; change in the character of the streamlines with variation in the mass concentration of dust particles (S) was not significant. Though trapping was observed and was predominant for all values of S , it can be seen that as the mass concentration of dust particles increased, the area of the trapped bolus in some regions decreased.

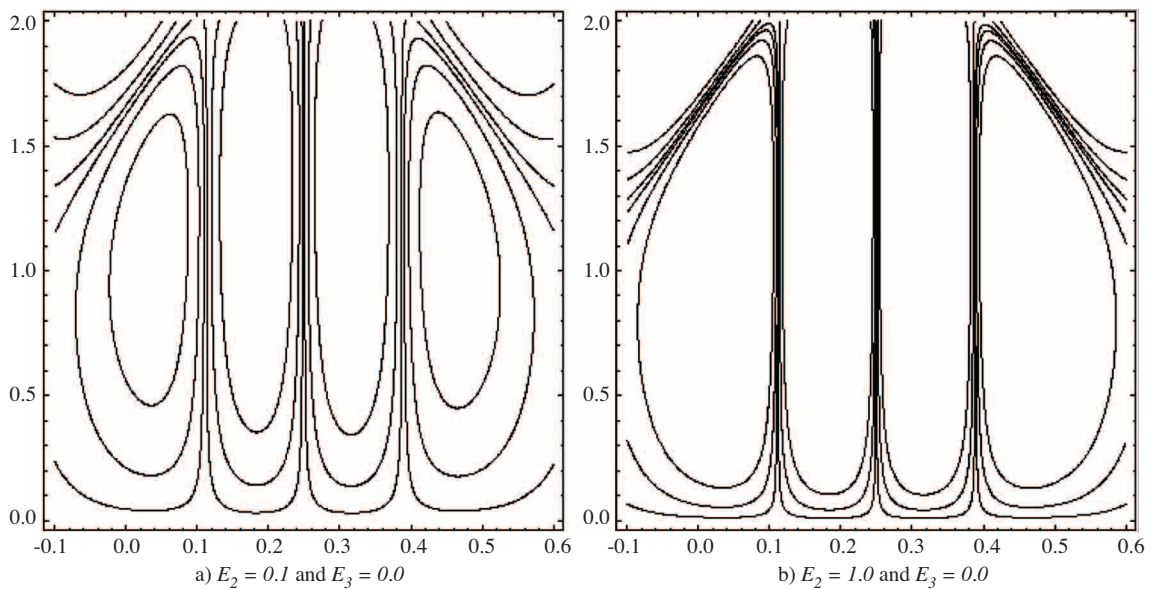


Figure 3. Effect of E_2 and E_3 on the streamline pattern of fluid particles for $E_1 = 1.0$, $\delta = 0.2$, $\varepsilon = 0.4$, $R = 1$, and $s = 0.5$.

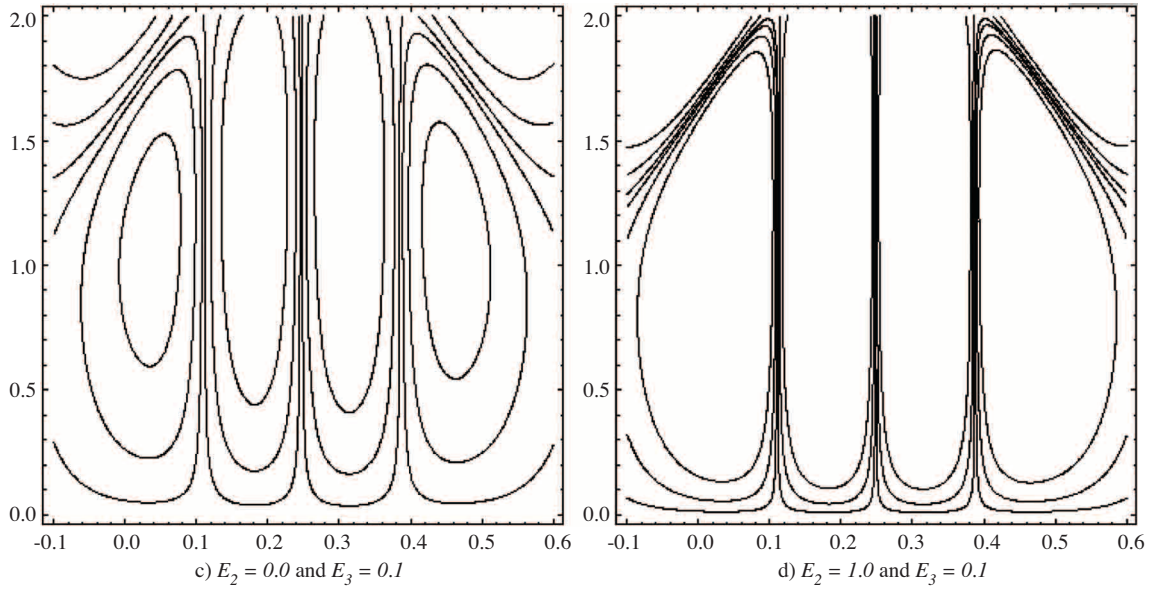


Figure 3. Continued.

The effect of E_1, E_2, E_3 , and S on the average fluid flow rate is shown in Figures 5-8. The phenomenon of reflux, i.e. flow reversal, is observed for all values of the tension parameter; however, for small values of tension ($E_1 = 0.2$), this phenomenon was observed for all values of the amplitude ratio (ε) up to 0.68 (approx.), and after that the flow rate became positive. Yet, for higher values of ten-

sion the amplitude ratio at which the fluid flow rate changed from negative to positive decreased. In particular, for the tension parameter value of 2.0, reflux occurred up to $\varepsilon = 0.56$ (approx.) and after that the flow rate became positive. A similar trend was observed for the mass characterizing parameter (E_2).

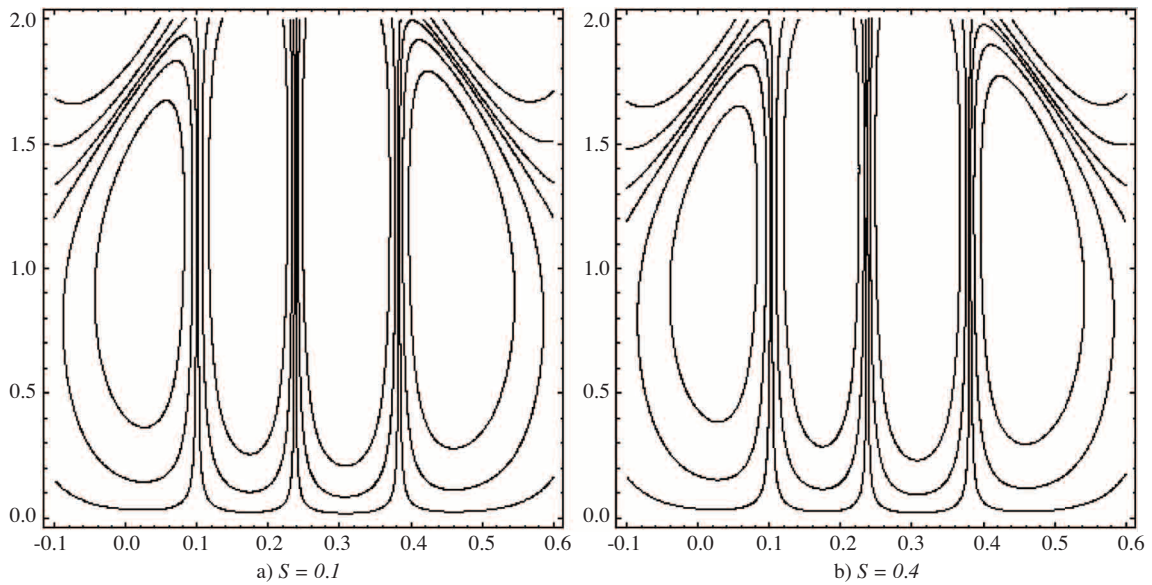


Figure 4. Effect of S on the streamline pattern of fluid particles for $E_1 = 0.5, E_2 = 0.75, E_3 = 0.6, \delta = 0.2, \varepsilon = 0.4$, and $r = 1$.

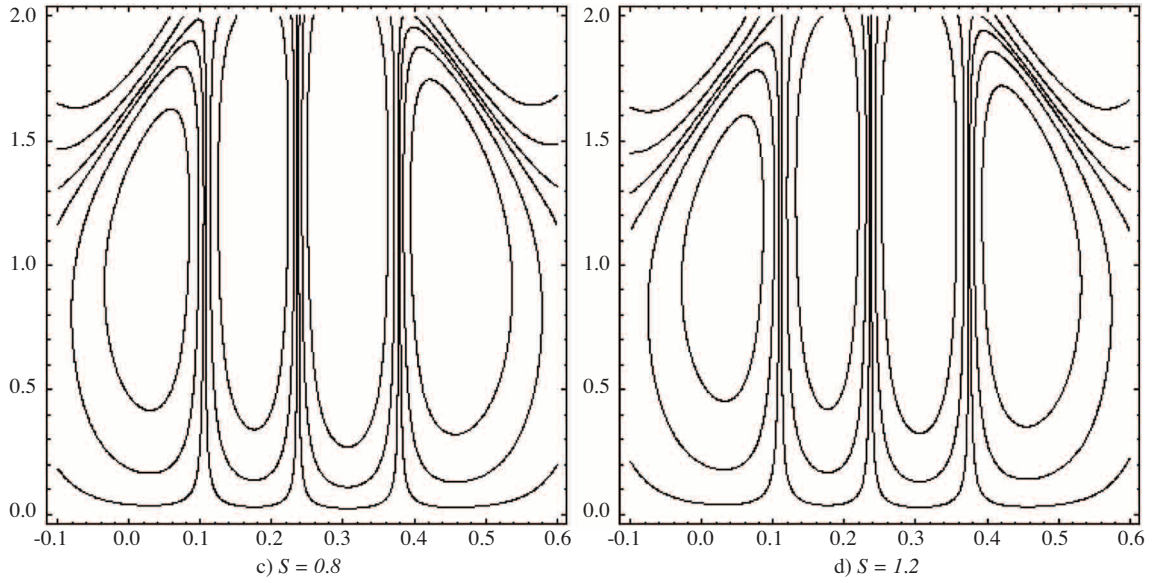


Figure 4. Continued.

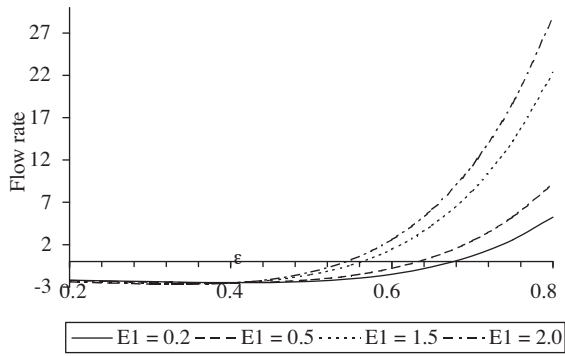


Figure 5. Effect of E_1 on the flow rate of fluid particles for $E_2 = 0.5$, $E_3 = 0.75$, $\delta = 0.1$, $R = 1$, and $S = 0.5$.

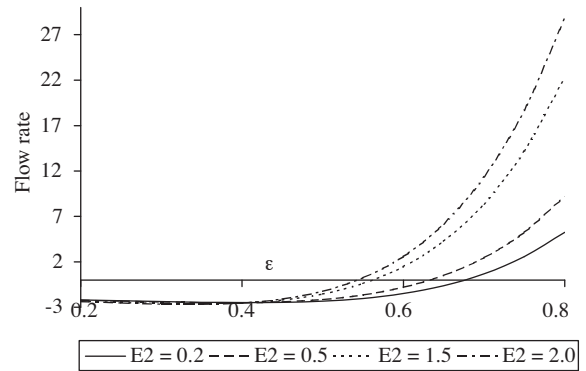


Figure 6. Effect of E_2 on the flow rate of fluid particles for $E_1 = 0.5$, $E_3 = 0.75$, $\delta = 0.1$, $R = 1$, and $S = 0.5$.

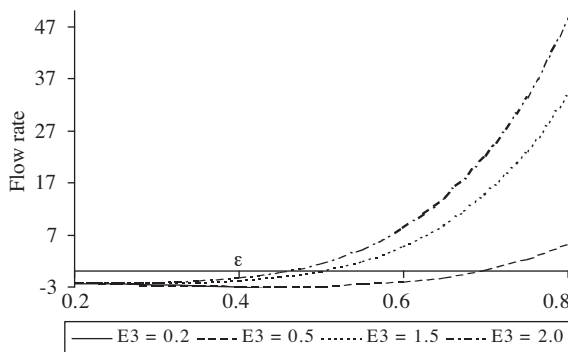


Figure 7. Effect of E_3 on the flow rate of fluid particles for $E_1 = 0.5$, $E_2 = 0.75$, $\delta = 0.1$, $R = 1$, and $S = 0.5$.

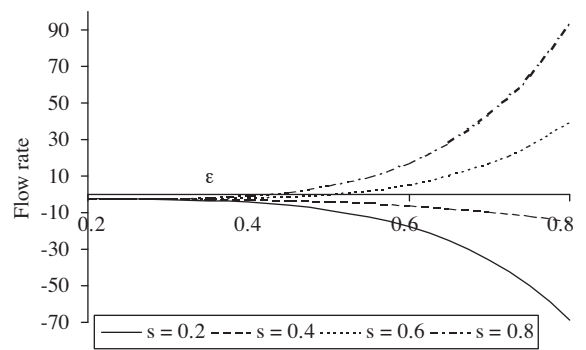


Figure 8. Effect of S on the flow rate of fluid particles for $E_1 = 0.5$, $E_2 = 0.75$, $E_3 = 0.75$, $\delta = 0.1$, and $R = 1$.

It can be seen from Figure 7 that for lower damping values ($E_3 = 0.2$), reflux occurred for amplitude ratios up to 0.68 (approx.), after which the flow rate became positive. Nonetheless, as damping increased ($E_3 = 2.0$) reflux occurred only up to an amplitude ratio of 0.44 (approx.). Figure 8 shows that for values of the mass concentration of dust particles (S) up to 0.6 reflux occurred for all values of amplitude ratios; however, for higher mass concentration of dust particle values ($S = 0.8$), reflux occurred only up to an amplitude ratio of 0.4 (approx.).

Nomenclature

a	amplitude of the wave	N	number density of the small solid particles (assumed to be constant N_0)
C	coefficient of viscous damping forces	p	fluid pressure,
d	half width of the channel	P	non-dimensional parameter ($KN_0d^2/(\rho v)$)
E_1	membrane tension parameter ($Td^4/(v\rho)$)	p_0	pressure on the outside of the wall due to tension in the muscles
E_2	mass characterizing parameter ($m'd^2/(\lambda^3\rho)$)	R	non-dimensional parameter ($vm/(Kd^2)$)
E_3	damping parameter ($E_3 = cd^3/(\lambda^2v\rho)$)	T	tension in the membrane
K	resistance coefficient for dust particles	t	time
L	an operator used to represent the motion of the stretched membrane with damping forces	u, v	velocity components of the fluid particles
m	mass of the solid particles	u_s, v_s	velocity components of the solid particles
m'	mass per unit area	\bar{u}	time-average axial velocity of the fluid particle
		\bar{u}_s	time-average axial velocity of the solid particle
		\mathbf{V}	velocity of the fluid particles
		\mathbf{V}_s	velocity of the solid particles
		Q	fluid flow rate
		\bar{Q}	time-average flow rate over a period of time
		S	mass concentration parameter
		λ	wavelength
		ψ, ϕ	stream functions
		ε	geometric parameter (a/d)
		δ	geometric parameter (d/λ)
		ρ	density of the fluid,
		ν	kinematic coefficient of the viscosity of fluid

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