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Effects of slip conditions on forced convection and entropy generation in a circular channel occupied by a highly porous medium: Darcy extended Brinkman-Forchheimer model

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Abstract

Fully developed forced convection in a circular channel filled with a highly porous medium saturated with a rarefied gas and uniform heat flux at the wall is investigated in the slip-flow regime, using the Darcy extended Brinkman-Forchheimer momentum equation and the entropy generation due to heat transfer and fluid friction is formulated. The expressions for velocity and temperature distribution have been obtained in terms of an asymptotic expansion for large Darcy numbers, assuming both velocity and temperature slip at the wall. The effects of slip and other parameters are examined on the Nusselt number and entropy generation rate. For small slip at the wall, it is noted that the velocity slip increases the Nusselt number and the temperature slip decreases it. The entropy generation number attains high values in the region close to the channel wall and the velocity and temperature slip parameters reduce it.

Key Words: Forced convection, Highly porous medium, Slip flow regime, Permeability, Entropy generation.

Introduction

Rapid progress in science and technology has led to the development of an increasing number of flow devices that involve the manipulation of gases in various geometries. The continuum assumption in the Navier-Stokes equations is only valid when the mean free path of the molecules is smaller than the characteristic dimension of the flow domain. If this condition is violated, the flow will then be influenced by non-continuum effects and the conventional no-slip boundary condition imposed at a solid-gas interface becomes invalid. The degree of rarefaction of the gas and the validity of the continuum hypothesis is determined by the Knudsen number, which is the ratio of the mean-free-path to a characteristic macroscopic length scale. For $10^{-2} \leq K_n \leq 10^{-1}$, commonly referred to as the slip-flow regime, the flow can be modeled by the Navier-Stokes equations with limited velocity slip. Thus, in such case, tangential slip-velocity boundary conditions must be implemented at the solid wall of flow domains. The effects of slip at the solid-gas interface for Couette flow are considered by Marques et al. (2000). Khaled and Vafai (2004) examined the effect of the slip condition on Stokes and Couette flows due to an oscillating wall. Some mathematical studies on non-Newtonian fluid flows in porous medium with the slip condition are discussed by Hayat et al. (2007) and Khan et al. (2008).

Convection is of fundamental interest in many engineering, industrial, and environmental applications such as cooling of electronic devices, air-conditioning systems, atmospheric flows, and security of energy systems, and in designs related to thermal insulation. Forced convective heat transfer through fluid saturated porous media has been discussed in detail and reviewed by Nield and Bejan (2006). A closed form solution of the Brinkman-Forchheimer equation for the forced convection in a fluid saturated porous medium with isothermal and isoflux boundaries was obtained by Nield et al. (1996), valid for all values of the Darcy number. They found that when the Darcy number is large and simultaneously the Forchheimer number is small, the velocity profile is approximately parabolic, and the effect of an increase in the viscosity ratio(μ_{eff}/μ)is to decrease the Nusselt number. Here the viscosity ratio depends on the structure of the porous matrix. Bear and Bachmat (1990) obtained a relation between the viscosity ratio and the tortuosity of the porous medium by the process of averaging, as $(\mu_{eff}/\mu)\,\varepsilon = 1/T$, where ε is the porosity and T is the tortuosity of the porous medium. Several authors investigated flow and heat transfer in channels filled (or partially filled) with porous medium, i.e. Chauhan and Gupta (1999), Al-Hadhrami et al. (2002), Al-Nimr and Khadrawi (2003), Liu (2006), Barletta et al. (2007), Kuznetsov and Nield (2008), Zahrani and Kiwan (2009), and many others.

Circular tubes or parallel-plates channels are the most useful and therefore widely used geometries in fluid flow and heat transfer devices. Estimation of local heat transfer in parallel plates and circular ducts filled with porous materials was obtained by Haji-Sheikh (2004). An analytical study of thermally developing forced convection in a parallel plate or circular channel filled with a porous medium, with walls at constant heat flux and with isothermal walls, was conducted by Nield et al. (2003, 2004), whereas such problems of forced convection with axial conduction have been studied by Kuznetsov et al. (2003), Hooman et al. (2003), and Minkowyez and Haji-Sheikh (2006). Hooman et al. (2006) and Barletta et al. (2008) investigated viscous dissipation effects on thermally developing forced convection in a circular duct filled with a saturated porous medium. A perturbation based analysis was presented by Hooman and Ranjbar-Kani (2004) on the forced convection in a porous saturated tube. Nield and Kuznetsov (2006) studied forced convection flow of a rarefied gas through a channel filled by a hyper-porous medium in the 'slip-flow regime', using the Brinkman model. The problem is solved using limited velocity slip and temperature slip at the walls. The velocity slip coefficient α and the temperature slip coefficient β are related to the values of the Knudsen number, and the knowledge about their values has been discussed and summarized by Schaaf and Chambre (1961) and Harley et al. (1995). Forced convection has been investigated by Hooman and Gurgenci (2007) in a circular tube filled with saturated porous medium, with no-slip boundary conditions and uniform heat flux at the wall on the basis of a Brinkman-Forchheimer model in the absence of viscous dissipation. The study of convection processes, as discussed here, in porous media, is a well-developed field of investigation due to its importance in a variety of situations. One potential application is found in thermoacoustic engines, which make use of the thermoacoustic phenomena and provide cooling or heating using environmentally benign gases as the working fluid. In these devices a porous medium, such as a fine wire mesh, may be embedded inside the fluid gap to enhance the thermal contact and heat transfer.

The foregoing discussions that deal with the forced convection problem in porous channels are very much restricted to the aspect of the first law of thermodynamics, and none of them carried out a second law based analysis to discuss the nature of the irreversibility in terms of entropy generation. Bejan (1995) showed that in thermal-flow systems, entropy minimization helps to improve the efficiency of the system. Therefore, many researchers were motivated to perform analysis, based on the second law of thermodynamics, of problems in porous channels useful in engineering devices, such as Demirel and Kahraman (2000), Mahmud and Fraser

(2005), Hooman (2006), and Hooman et al. (2007).

In view of the above, in this paper, forced convection flow in a circular channel occupied by a highly porous medium saturated with a rarefied gas in the Knudsen slip-flow regime is considered for the case of uniform heat flux on the boundary wall. In the case of forced convection in highly porous medium, in which the solid phase is sparse, e.g. when a channel is filled by metallic foam, it is reasonable to assume that on a macroscopic scale there is limited slip at the walls of the channel and this flow situation can be modeled using the Darcy-extended Brinkman-Forchheimer equation. Thus in this research the Darcy-extended Brinkman-Forchheimer model is used, assuming both limited velocity slip and temperature slip on the wall. An asymptotic solution to this problem that includes viscous dissipation effects is presented here considering the case of a large Darcy number. By incorporating the entropy generation analysis, the research work presented here further contributes to the fluid flow and heat transfer solutions discussed by other researchers. The variations of the viscosity and temperature fields, Nusselt number, entropy generation number, and Bejan number are investigated for various values of the slip parameters, Darcy and Forchheimer numbers, viscosity ratio, and other parameters.

Formulation of the Problem

We consider a circular channel of radius R, occupied by a highly porous medium saturated with a rarefied gas in the slip-flow regime, for the case of uniform heat flux at the boundary. For fully developed steady flow the velocity is $u^*(r^*)$in the axial direction z^* of the circular channel. The governing equations are

$$\mu_{eff}\left(\frac{d^2u^*}{dr^{*2}}+\frac{1}{r^*}\frac{du^*}{dr^*}\right)-\frac{\mu}{K_0}u^*-\frac{C_F\rho u^{*2}}{\sqrt{K_0}}+G=0 \tag{1}$$

$$\rho C_p u^*\frac{\partial T^*}{\partial z^*}=\frac{k}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right)+\frac{\mu u^{*2}}{K_0}+\mu_{eff}\left(\frac{du^*}{dr^*}\right)^2 \tag{2}$$

Equation (1) is the Darcy-extended Brinkman-Forchheimer momentum equation, whereas Eq. (2) is the steady-state thermal energy equation in the absence of thermal dispersion and axial thermal conduction. However, the viscous dissipation term for porous medium is retained following Al-Hadhrami et al. (2002). Local thermal equilibrium has been assumed.

Here μ_{eff} is the effective viscosity; μ the fluid viscosity; K_0 the permeability; ρ the fluid density; C_F the Forchheimer coefficient; T^* the temperature; C_p the specific heat at constant pressure; k the thermal conductivity; and (-G) the applied pressure gradient.

The corresponding boundary and symmetry conditions are

$$\text{at } r^*=1; u^*=-\alpha^*\frac{du^*}{dr^*}, T^*=-\beta^*\frac{dT^*}{dr^*},$$

$$\text{at } r^*=0; \frac{du^*}{dr^*}=0, \frac{dT^*}{dr^*}=0, \tag{3}$$

where α^* and β^* are the velocity slip and the temperature slip coefficients, respectively.

We introduce the following dimensionless quantities:

$$z=z^*/PeR,\ \ r=r^*/R,\ \ u=\mu u^*/GR^2,\ \ M=\mu_{eff}/\mu,\ \ Da=K_0/R^2,$$

$$F=\rho C_F GR^3/\mu^2,\ \ Pe=\rho C_p RU^*/k,\ \alpha=\alpha^*/R,\ \beta=\beta^*/R, \tag{4}$$

where M is the viscosity ratio; Da the Darcy number; F the Forchheimer number; Pe the Péclet number; α the dimensionless velocity slip parameter; and β the dimensionless temperature slip parameter.

Using the above, the dimensionless form of Eq. (1) is given by

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{s^2}{M}u-\frac{Fs}{M}u^2+\frac{1}{M}=0, \tag{5}$$

where $s=(1/Da)^{1/2}$ is the porous medium parameter.

The corresponding flow boundary/symmetry conditions become

$$\text{at } r=1; u=-\alpha\frac{du}{dr},$$
$$\text{at } r=0; \frac{du}{dr}=0. \tag{6}$$

Further, by defining the mean velocity U^* and the bulk mean temperature T_m^* as

$$U^*=\frac{2}{R^2}\int_0^R u^*r^*dr^*, \text{ and } T_m^*=\frac{2}{R^2U^*}\int_0^R u^*T^*r^*dr^*, \tag{7}$$

new dimensionless variables are introduced as

$$\hat{u}=\frac{u^*}{U^*}, \text{ and } \theta=\frac{T^*-T_w^*}{T_m^*-T_w^*}. \tag{8}$$

The Nusselt number Nu is defined by

$$Nu=2Rq^{"}/k\,(T_w^*-T_m^*), \tag{9}$$

where T_w^* and $q^{"}$ are the temperature and heat flux on the wall.

From the first law of thermodynamics, for the case of uniform heat flux on the wall, we have

$$\frac{\partial T^*}{\partial z^*}=\frac{dT_w^*}{dz^*}=\frac{2q^{"}}{\rho C_pRU^*}=constant. \tag{10}$$

Using Eqs. (7)-(10), Eq. (2) in non-dimensional form becomes

$$\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}+\hat{u}Nu+s^2Br\hat{u}^2+BrM\left(\frac{d\hat{u}}{dr}\right)^2=0, \tag{11}$$

where $Br=\mu U^{*2}/k\,(T_m^*-T_w^*)$ is the Brinkman number.

The corresponding boundary/symmetry conditions for temperature become

$$\text{at } r=1; \theta=-\beta\frac{d\theta}{dr},$$
$$\text{at } r=0; \frac{d\theta}{dr}=0. \tag{12}$$

The definition of the dimensionless temperature leads to the integral compatibility condition,

$$\int_0^1 \hat{u}\theta r dr=1/2, \tag{13}$$

resulting from the first law of thermodynamics (Bejan, 1984).

Solution of the Problem

We consider the case of a large Darcy number, and write the following asymptotic expansion for the velocity distribution, assuming that $s << 1$,

$$u = u_0 + s u_1 + \cdots . \tag{14}$$

Substituting (14) in Eqs. (5) and (6) and comparing the coefficients of s on both sides, we obtain the set of ordinary differential equations and boundary conditions for different orders, which are then solved considering the first 2 orders to give

$$u = \frac{1}{4M}\left(a_1 - r^2\right) + \frac{Fs}{1152M^3}\left[2r^6 - 9a_1 r^4 + 18a_1^2 r^2 - a_2\right] + O\left(s^2\right). \tag{15}$$

Using Eqs. (7), (8), and (15), we obtain the mean velocity,

$$U^* = \frac{GR^2}{\mu}\frac{1}{8M}\left[(1+4\alpha) - \frac{Fsa_3}{32M^2}\right], \tag{16}$$

and

$$\hat{u} = \frac{2}{a_4}\left(a_1 - r^2\right) + \frac{Fs}{16M^2}\frac{a_3}{a_4^2}\left(a_1 - r^2\right) + \frac{Fs}{144M^2}\frac{1}{a_4}\left[2r^6 - 9a_1 r^4 + 18a_1^2 r^2 - a_4\right] + O\left(s^2\right). \tag{17}$$

Similarly we proceed to find the temperature distribution by writing the following asymptotic expansion:

$$\theta = \theta_0 + s\theta_1 + \cdots . \tag{18}$$

Using the expansion (18) in (11) and (12), and considering only the first 2 orders, we obtain

$$\begin{aligned}\theta \;=\; & \frac{Nu}{8a_4}\left[r^4 - 4a_1 r^2 + a_5\right] + \frac{BrM}{a_4^2}\left(1 + 4\beta - r^4\right) \\ & - \frac{sFNu}{4608M^2 a_4^2}\left[\begin{array}{c} a_4\left\{r^8 - 8a_1 r^6 + 36a_1^2 r^4 - 8a_2 r^2 - a_6 - 8\beta a_7\right\} \\ -\,18a_3\left\{r^4 - 4a_1 r^2 + a_8 + 4\beta a_4\right\}\end{array}\right] \\ & + \frac{sFBr}{288Ma_4^3}\left[3a_4 r^8 - 16a_1 a_4 r^6 + 36a_1^2 a_4 r^4 - 18a_3 r^4 - a_9 - 24\beta a_{10}\right].\end{aligned} \tag{19}$$

Finally, the Nusselt number can be found by substituting for $\hat{u}$ and θ, using (17) and (19) in the integral compatibility condition (13), as

$$Nu = \frac{48a_4^4 - 8BrMa_4 a_{16} - (sFBra_{22}/180M)}{a_4^2 a_{15} + \left(sFa_{23}/1440M^2\right)}, \tag{20}$$

where

$a_1 = 1 + 2\alpha,\quad a_2 = 11 + 66\alpha + 144\alpha^2 + 144\alpha^3,\quad a_3 = 1 + 8\alpha + 24\alpha^2 + 32\alpha^3,$

$a_4 = 1 + 4\alpha,\quad a_5 = 3 + 8\alpha + 4\beta(1 + 4\alpha),\quad a_6 = 1 - 8a_1 + 36a_1^2 - 8a_2,$

$a_7 = 1 - 6a_1 + 18a_1^2 - 2a_2,\quad a_8 = 4a_1 - 1,\quad a_9 = 3a_4 - 16a_1 a_4 + 36a_1^2 a_4 - 18a_3,\quad a_{10} = a_4 - 4a_1 a_4 + 6a_1^2 a_4 - 3a_3,$

$a_{11} = a_5 + a_8 + 4\beta a_4,\quad a_{12} = a_2 + 9a_1 a_5 + 72a_1^3,$

$a_{13} = a_6 + 8\beta a_7,\quad a_{14} = a_9 + 24\beta a_{10},\quad a_{15} = 12a_1 a_5 - 24a_1^2 - 6a_5 + 20a_1 - 3,$

$a_{16} = 6a_4 (1 + 4\beta) - 4a_1 + 3, \quad a_{17} = 6a_1 a_{11} - 3a_{11} - 24a_1^2 + 20a_1 - 3,$
$a_{18} = 10 - 102a_1 + 15a_5 + 405a_1^2 - 10a_{12} + 60a_1 a_2 + 270a_1^2 a_5 - 30a_2 a_5,$
$a_{19} = 5 - 54a_1 + 330a_1^2 - 80a_2 - 360a_1^3 + 120a_1 a_2 - 15a_{13} + 30a_1 a_{13},$
$a_{20} = (1 + 4\beta)\left(15 - 90a_1 + 270a_1^2 - 30a_2\right) - 10 + 54a_1 - 135a_1^2 + 10a_2,$
$a_{21} = -15a_4 + 114a_1 a_4 - 390a_1^2 a_4 + 135a_3 - 180a_1 a_3 + 360a_1^3 a_4 + 15a_{14} - 30a_1 a_{14},$
$a_{22} = 45a_3 a_{16} + 2a_4 a_{20} + 2a_{21}, \quad a_{23} = 90a_3 a_4 a_{17} + 2a_4^2 a_{18} + a_4^2 a_{19}.$

When there is no-slip ($\alpha = 0,\ \beta = 0$) at the boundary the above results are in agreement with Hooman and Gurgenci (2007) in the case of no viscous dissipation. Further, when $s = 0$ and $Br = 0$, the channel is free of porous material with no viscous dissipation and the Nusselt number is approximately 4.3636, which is in agreement with Ranjbar-Kani (2003).

Entropy Generation

The convection process in a porous medium is essentially irreversible. A continuous entropy generation in the fluid system is caused because of the exchange of energy and momentum within the fluid and at the impermeable boundaries. One part of this entropy generation is due to heat transfer in the direction of finite temperature gradients, and the other part takes place due to fluid friction irreversibility. Following Bejan (1982), the dimensional volumetric rate of entropy generation for the present problem in cylindrical coordinates can be written as

$$S''' = \frac{k}{T_0^{*2}}\left[\left(\frac{\partial T^*}{\partial r^*}\right)^2 + \left(\frac{\partial T^*}{\partial z^*}\right)^2\right] + \frac{1}{T_0^*}\left[\frac{\mu u^{*2}}{K_0} + \mu_{eff}\left(\frac{du^*}{dr^*}\right)^2\right], \tag{21}$$

where $T_0^*$is the reference temperature. In the above equation, the first term represents the entropy generated in the radial direction by heat transfer, the second term accounts for axial conduction, and the last 2 terms are the fluid friction contribution in porous medium.

Using the non-dimensional quantities defined in (4) and (7)-(10), we obtain, in dimensionless form, the entropy generation number

$$Ns = \frac{S'''}{S_0'''} = \left\{\left(\frac{d\theta}{dr}\right)^2 + \left(\frac{Nu}{Pe}\right)^2\right\} + T_0 Br\left\{\frac{\hat{u}^2}{Da} + M\left(\frac{d\hat{u}}{dr}\right)^2\right\}, \tag{22}$$

where $S_0''' = k\left(T_m^* - T_w^*\right)^2 \Big/ T_0^{*2} R^2$ is the reference volumetric entropy generation; and $T_0 = T_0^*/(T_m^* - T_w^*)$ is the dimensionless reference temperature.

Let us denote Ns_1 and Ns_2 as the entropy generation due to heat transfer and viscous dissipation, respectively; then Eq. (22) can be written as

$$Ns = Ns_1 + Ns_2, \tag{23}$$

where

$$Ns_1 = \left(\frac{d\theta}{dr}\right)^2 + \left(\frac{Nu}{Pe}\right)^2, \tag{24}$$

and

$$Ns_2 = T_0 Br\left\{\frac{\hat{u}^2}{Da} + M\left(\frac{d\hat{u}}{dr}\right)^2\right\}. \tag{25}$$

In the above expressions, we have

$$\frac{d\theta}{dr} = \frac{Nu}{2a_4}\left(r^3 - 2a_1 r\right) - \frac{4BrM}{a_4^2}r^3 + \frac{sBrF}{36Ma_4^3}\left[3a_2r^7 - 12a_1a_2r^5 + 18a_1^2a_2r^3 - 9a_3r^3\right]$$
$$-\frac{sFNu}{576M^2a_4^2}\left[a_4\left(r^7 - 6a_1r^5 + 18a_1^2r^3 - 2a_4r\right) - 9a_3\left(r^3 - 2a_1r\right)\right],$$
$$\frac{d\hat{u}}{dr} = -\frac{4r}{a_4} - \frac{sFa_3r}{8M^2a_4^2} + \frac{sF}{12M^2a_4}\left[r^5 - 3a_1r^3 + 3a_1^2r\right],$$

and $\hat{u}$ and Nu are given in (17) and (20) respectively.

We also obtain the Bejan number, the ratio of Ns_1 to the total entropy generation rate, as

$$Be = Ns_1/Ns. \tag{26}$$

By knowing the solutions of the dimensionless velocity and temperature fields, Ns and Be are computed for different values of the various parameters.

Results and Discussion

In the present study an analytic solution is obtained for the velocity profiles, temperature profiles, and Nusselt number for forced convection in a circular duct occupied by a hyper-porous medium saturated with a rarefied gas while the entropy generation due to heat transfer and fluid friction is formulated. It is aimed to investigate the combined effects of Darcy bulk matrix resistance (Darcy effect), porous inertia (Forchheimer effect), boundary friction (Brinkman effect), velocity slip, and temperature slip on the forced convection flow and entropy generation characteristic inside the duct.

Figure 1 shows the effects of the slip parameters α and β, Forchheimer number$F$, viscosity ratio parameter M, and the parameter s on the fully developed velocity profiles u plotted against r. As expected, the velocity profile approaches the plane Poiseuille flow when$s \to 0$. The velocity profile is significantly non-linear for small values of s, and changes substantially with the values of α, M, and s. We have also compared our results with those obtained in the case of no-slip boundary conditions. Maximum velocity appears at the centerline of the channel and it is found that velocity slip leads in general to an increase in this velocity. It also increases by decreasing the parameter s, i.e. by increasing the Darcy number Da. However, the viscosity parameter M leads to a decrease in the velocity profile in the channel. It is also found that velocity profile decreases by increasing the Forchheimer number F.

The dimensionless temperature profiles are plotted against r in Figure 2. It is seen that the maximum temperature appears at the centerline of the channel, and it increases by increasing the velocity slip parameter α. We have also compared the results for θ in the case of temperature no-slip ($\beta = 0$) and slip ($\beta = 0.1$) boundary conditions and found that, when there is slip, temperature θ near the wall is greater than that of the no-slip case, whereas it is reduced at the centerline of the channel. Further, it is seen that θ increases near the wall and decreases at the centerline with increasing Brinkman number Br or viscosity ratio parameter M.

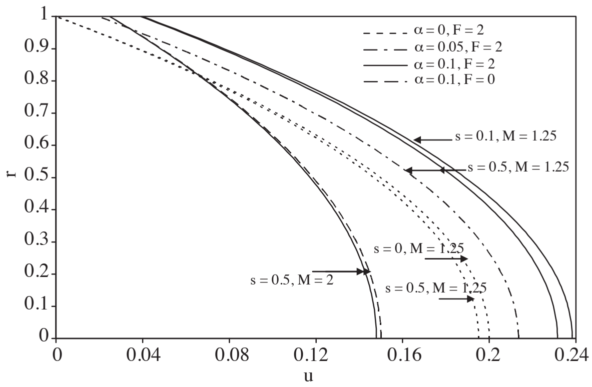

Figure 1. Velocity profiles u vs. r.

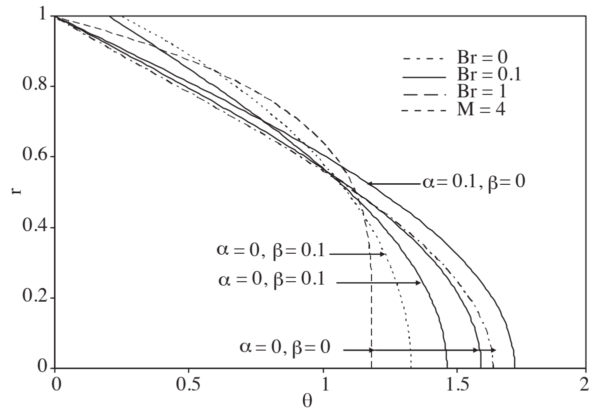

Figure 2. Temperature profiles θ vs. r, for $M = 1.25$.

We have investigated the effects of the parameters α, β, s, M, F, and Br, in detail, on the Nusselt number Nu in Figures 3-5. Nusselt number Nu is plotted against small values of F in Figure 3. It is found that Nu increases with increasing$F$; however, it decreases with increasing Br. It is also found that in the porous channel with no slip($s = 0.3,\ \alpha = \beta = 0$) the Nusselt number Nu is increased by the introduction of the velocity slip $\alpha = 0.1$, as expected. However, when there is slip in both velocity and temperature at the boundary($\alpha = \beta = 0.1$), then the Nusselt number is reduced, which then increases with increasing s. It is also clear from this figure that Nu decreases with increasing temperature slip β.

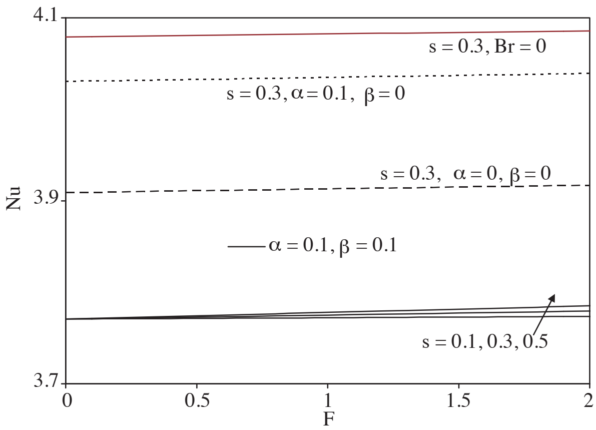

Figure 3. Nu vs. F, for $M = 1.25,\ Br = 0.1$.

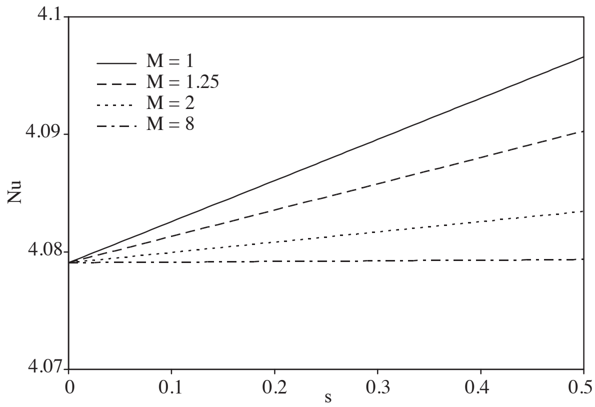

Figure 4. Nu vs. s, for $\alpha = 0.1,\ \beta = 0.1,\ Br = 0,\ F = 2$.

The variation in the Nusselt number Nu as a function of the parameter s is shown in Figure 4. As mentioned before, it increases with increasing s. As expected, one can observe that increasing M decreases the Nusselt number. This fact is in agreement with the previous results reported in the literature by Nield et al. (1996).

Figure 5 shows that the Nusselt number Nu decreases with the increase in the value of Brinkman number Br, while it increases with the increase in the Forchheimer number F.

The above discussion dealing with the forced convection in a porous circular duct is restricted to first-law (of thermodynamics) analysis. Now we conduct a second-law based analysis to determine the nature of the irreversibility in terms of entropy generation, which affects very much the thermodynamic efficiency of a system.

Figures 6-8 show the variation in entropy generation number (Ns_1) due to heat transfer in the circular duct against radial distance r for different values of the Péclet number (Pe), Brinkman number (Br), viscosity ratio parameter (M), velocity slip parameter (α), and temperature slip parameter (β). Entropy generation number (Ns_1) in the central region of the duct is low due to gradually varying and small temperature gradients in this region, whereas it attains high values close to the duct wall. As the Péclet number or the temperature slip parameter increases, entropy generation number (Ns_1) decreases, while the velocity slip parameter increases it. However, by increasing the Brinkman number or the viscosity ratio parameter, Ns_1 increases in the region close to the duct wall, while it decreases in the central region of the duct.

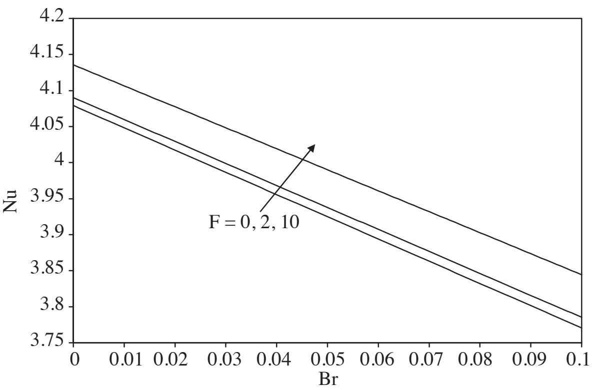

Figure 5. Nu vs. Br, for $\alpha = 0.1,\ \beta = 0.1,\ M = 1.25,\ s = 0.5$.

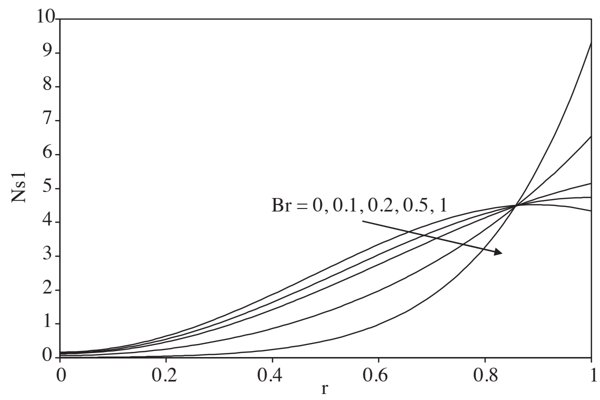

Figure 6. Ns_1 vs. r for $\alpha = 0.1,\ \beta = 0.1,\ M = 1.25,\ s = 0.1, F = 2, Pe = 10$.

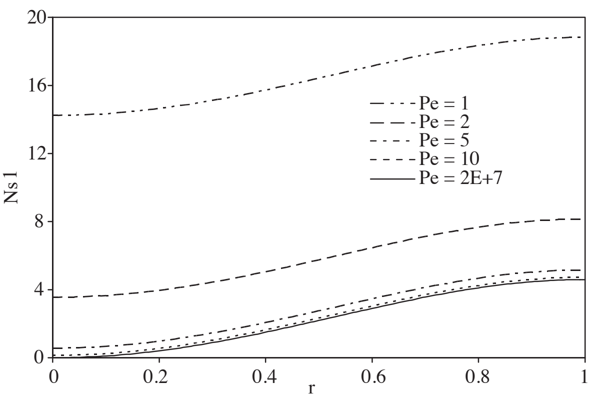

Figure 7. Ns_1 vs. r, for $\alpha = 0.1,\ \beta = 0.1,\ M = 1.25,\ s = 0.1, F = 2, Br = 0.1$.

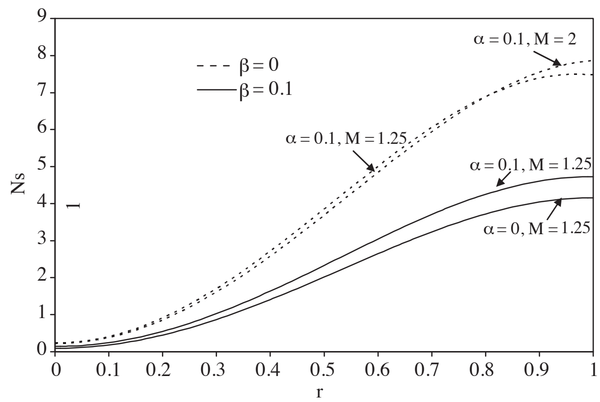

Figure 8. Ns_1 vs. r for $s = 0.1, F = 2, Br = 0.1,\ Pe = 10$.

Figure 9 shows the entropy generation number (Ns_2) due to fluid friction in the duct for different parameters. It is seen that Ns_2 reduces to a minimum at the centerline of the duct where the magnitude of the velocity is maximum; on the other hand, it attains high values in the region close to the duct wall because of the high rate of fluid strain in this region. Increasing the velocity slip parameter lowers the entropy generation number (Ns_2), while the Brinkman number or the viscosity ratio parameter enhances it.

The variations in total entropy generation number (Ns) are plotted against the radial distance r in Figures 10 and 11. The total entropy generation number is more pronounced in the region close to duct wall due to both enhanced heat transfer rates and fluid friction in this region. It increases with increasing Brinkman number or viscosity ratio parameter. However, it is seen that entropy generation number (Ns) decreases with increasing Péclet number (Pe) or slip parameters(α and β). This result is important in the entropy minimization process.

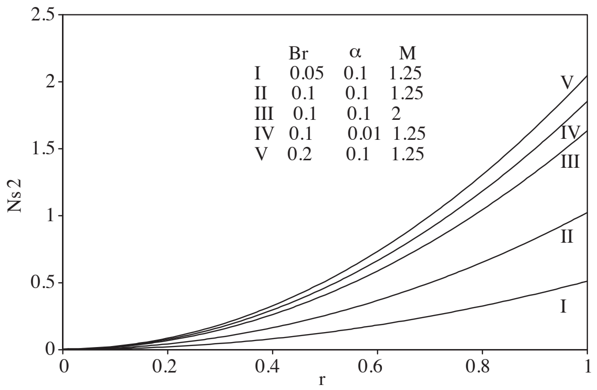

Figure 9. Ns_2 vs. r for $s = 0.1, F = 2,\ T_0 = 1$.

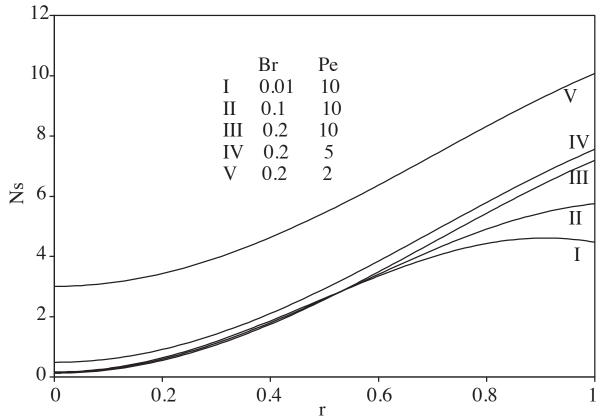

Figure 10. Ns vs. r for $s = 0.1, F = 2,\ T_0 = 1,\ \alpha = 0.1,\ \beta = 0.1,\ M = 1.25$.

Figure 12 shows the variations in Ns against Br for different values of s and F. It is found that total entropy generation increases with increasing s while it decreases with F.

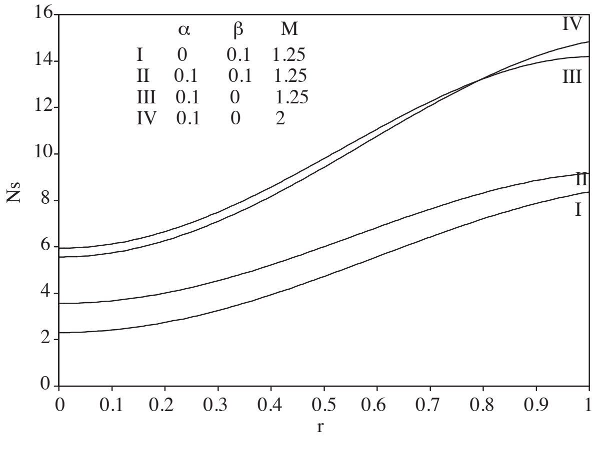

Figure 11. Ns vs. r for $s = 0.1, F = 2,\ T_0 = 1,\ Br = 0.1,\ Pe = 2$.

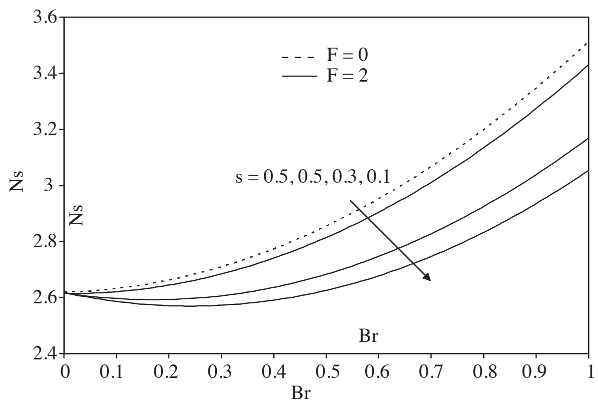

Figure 12. Ns vs. Br for $\alpha = 0.1,\ \beta = 0.1,\ M = 1.25,\ T_0 = 1,\ Br = 0.1,\ Pe = 10$.

The radial variation in heat transfer irreversibility in terms of the Bejan number (Be) is shown in Figure 13 for various values of the Brinkman number (Br) and the Péclet number (Pe). When $Br = 0$, there is no contribution of fluid friction irreversibility to overall entropy generation, and so the distribution of the Bejan number with respect to r is invariant, as expected. The Bejan number attains its maximum value (i.e. 1) for all r when $Br = 0$. With the increase in Br or Pe, it decreases. Further, it is observed in Figure 14 that Be decreases with increasing s or M. It is also found that the effect of the parameter s on the distribution of Be with respect to r is only seen near the centerline ($r = 0$) of the duct. However, the velocity slip parameter α increases Be while the temperature slip parameter β decreases it.

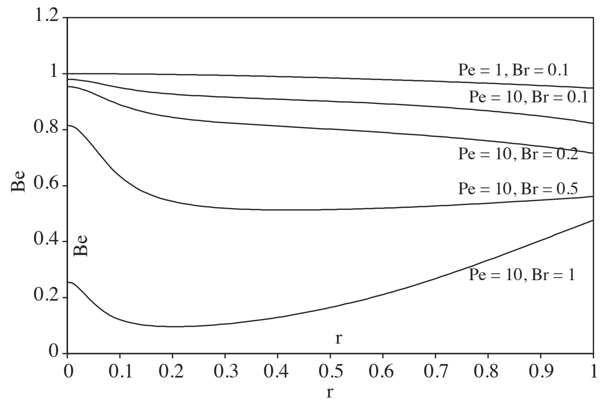

Figure 13. Be vs. r for $s = 0.1, F = 2,\ T_0 = 1,\ \alpha = 0.1,\ \beta = 0.1,\ M = 1.25$.

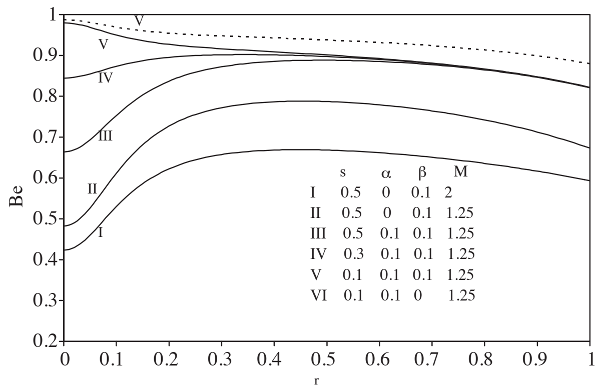

Figure 14. Be vs. r for $F = 2,\ T_0 = 1,\ Br = 0.1,\ Pe = 10$.

Conclusion

The Nusselt number and the entropy generation rate are significantly altered as a result of variation in various parameters involved in the problem, primarily due to the change in velocity and temperature profiles. It is found that the total entropy generation number attains high values in the region close to the circular duct wall.

1. For small slip at the wall, the velocity increases the Nusselt number whereas it decreases with increasing temperature slip.

2. The Nusselt number Nu increases with the increase in s or F while it decreases with increases in Br or M.

3. Increasing Brinkman number or viscosity ratio parameter enhances total entropy generation number (Ns) significantly, particularly in the region close to the duct wall.

4. The Péclet number (Pe) affects Ns through the axial temperature gradient. It is observed that increasing Pe decreases Ns.

5. Velocity and temperature slip parameters (α *and* β) both reduce Ns.

6. Parameter s increases Ns while parameter F reduces it.

7. Bejan number (Be) decreases with the increase in Pe or Br or M or s or β while it increases with α.

These results are useful in many situations for some practical applications in engineering and other fields.

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Nomenclature

Be	Bejan number defined in Eq. (26)
Br	Darcy-Brinkman number $\mu U^{*2}/k\,(T_m^* - T_w^*)$
C_F	Forchheimer coefficient
C_p	specific heat at constant pressure, $[J/kg.K]$
Da	Darcy number defined in Eq. (4)
F	Forchheimer number defined in Eq. (4)
G	negative of applied pressure gradient, $[Pa/m]$
k	porous medium thermal conductivity, $[W/m.K]$
K_0	permeability of the porous media, $[m^2]$
M	viscosity ratio μ_{eff}/μ
Ns	dimensionless entropy generation defined in Eq. (23)
Nu	Nusselt number defined in Eq. (9)
Pe	Péclet number defined in Eq. (4)
q''	heat flux, $[W/m^2]$
r^*, z^*	space coordinates, $[m]$
r, z	dimensionless space coordinates defined in Eq. (4)
R	radius of the circular channel $[m]$
s	porous media parameter $(Da)^{-1/2}$
S'''	entropy generation per unit volume, $[W/m^3.K]$
S_0'''	reference entropy generation per unit volume, $[W/m^3.K]$
T^*	temperature, $[K]$
T_m^*	bulk mean temperature, $[K]$
T_w^*	wall temperature, $[K]$
T_0^*	reference temperature, $[K]$
T_0	dimensionless reference temperature $T_0^*/(T_m^* - T_w^*)$
u^*	axial velocity, $[m/s]$
u	dimensionless axial velocity defined in Eq. (4)
$\hat{u}$	normalized velocity defined in Eq. (8)
U^*	average velocity, $[m/s]$

Greek symbols

α^*	velocity slip parameter, $[m]$
α	dimensionless velocity slip parameter defined in Eq. (4)
β^*	temperature slip parameter, $[m]$
β	dimensionless temperature slip parameter defined in Eq. (4)
μ	fluid viscosity, $[N.s/m^2]$
μ_{eff}	effective viscosity, $[N.s/m^2]$
ρ	fluid density, $[kg/m^3]$
θ	dimensionless temperature defined in Eq. (8)

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