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Optimization of vertical alignment of highways utilizing discrete dynamic programming and weighted ground line

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Abstract

Earthwork optimization plays an important role for reducing the total cost of highway projects. Weighted Ground Line Method (WGLM) is an earthwork optimization technique based on a hypothetical ground line balancing cut-fill volumes and minimizing the total amount of earthwork. Current form of the method describes only a hypothetical reference line guiding project engineers to locate the optimal roadway for the fixed horizontal alignment. Therefore, there are numerous grade line alternatives available for a given Weighted Ground Line. The aim of this study is to present a method to integrate discrete dynamic programming with WGLM to achieve the best vertical alignment of highways in terms of earthwork optimization.

Key Words: Highway alignment; Earthwork; Optimization; Grade line, Dynamic programming.

Introduction

Basically, highway alignment is a process for the determination of two- or three-dimensional route location on the ground surface using contour maps. In general, two separate two-dimensional alignments, namely horizontal or vertical alignments, are preferred for the simplicity. Likewise, in three-dimensional approach, horizontal and vertical planes are simultaneously considered to find out the optimal route location. Three-dimensional alignment is a complex optimization problem that is subject to large amount of design variables, several nonlinear constraint equations, and numerous solution alternatives (Jong and Schonfeld 2003). As a result, in most highway projects, horizontal alignment is determined and then vertical alignment is adjusted by a set of design controls (Schoon 2001).

The Weighted Ground Line Method (WGLM) is a technique proposed by the authors for cut-fill balancing and earthwork minimization. The main principal is to calculate a hypothetical center elevation for each cross section that optimizes the earthwork construction in terms of cut-fill balancing and total earthwork minimization. Furthermore, swelling and shrinkage factors, can be incorporated in the method to achieve better results (Goktepe and Lav 2003; Goktepe and Lav 2004).

Current form of WGLM describes a methodology to create Weighted Ground Line (WGL) by connecting Weighted Ground Elevations (WGEs). Successively, project engineer may fix the final alignment by establishing the grade line as close as possible to this reference ground line. Consequently, the aim of this study is the determination of optimal grade line by discrete dynamic programming and WGL.

Overview of Previous Highway Alignment Studies

Highway alignment is a constrained optimization problem, in which constraints and objective function are nonlinear. Therefore, it is possible to solve this problem using a constrained optimization technique. In the literature, there are numerous studies focused on the two-dimensional solution of the highway alignment optimization problem (Trietsch 1987; Easa 1988; Goh and Chew 1988; Chew et al. 1989; Fwa 1989; Moreb 1996; ReVelle et al. 1997; Fwa et al. 2002; Kim at al. 2004). The major discrepancies among developed two-dimensional models are the types of optimization techniques and model definitions (constraints and cost functions) that were employed. For that reason, several optimization techniques i.e., linear programming, quadratic programming, gradient search, state parameterization, and dynamic programming, were utilized in these studies.

Furthermore, some researchers focused on three-dimensional optimization of highway alignment. In this manner, dynamic programming, state parameterization, and genetic algorithm approaches applied to find out optimal route candidates in three-dimensional manner (Chew et al. 1989; Jong and Schonfeld 1999; Jong and Schonfeld 2003; Jha 2003). Because of complex and comprehensive structures of the three-dimensional models, they are not practical for most alignment problems (Easa 1997; Jong and Schonfeld 2003; Jha 2003).

Weighted Ground Line Method (WGLM)

Generally, centerline elevation (ground line) of a roadway is the guide for determination of final vertical alignment (fixing the grade line) of a highway project. However, it is not likely that the center ground elevation to characterize the whole cross section (template) in terms of variation of elevations. Fundamentals of WGLM comes from this fact, and WGE, which is a hypothetical center elevation utilized to represent whole template in terms of cut-fill balance. Connected WGEs form a reference ground line, on which earthwork balancing and total earthwork minimization are both satisfied. Calculation of WGE's is carried out as follows:

$$\sum_{i=1}^{n} \mathbf{S_C}(i) = \sum_{i=1}^{m} \mathbf{S_F}(i) \qquad (for \quad y = h_w) \tag{1}$$

where, h_w is weighted ground elevation (WGE), $\mathbf{S}_C$ is cut area vector, $\mathbf{S}_F$ is fill area vector, n is the number of cut areas in cross sections, and m is the number of fill areas in cross sections. For a cross section having p nodes, the calculation procedure of WGE is indicated in Fig.1. Mathematical expression of h_w is as given below:

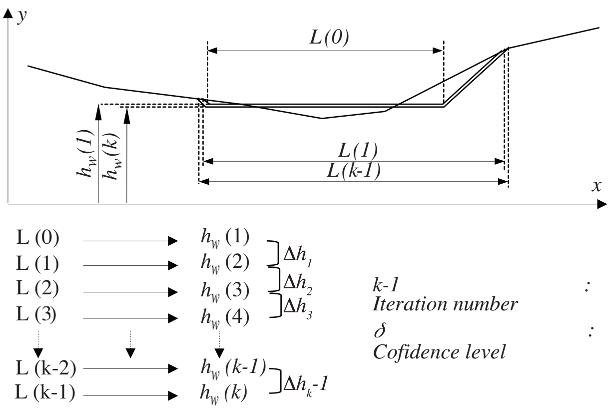

Figure 1. Iterative procedure for the calculation of WGE (Goktepe and Lav 2003).

$$h_w = \frac{\int_0^L f(x)\,dx + \{-S_{CS} + S_{FS}\}}{L} = \frac{\sum_{i=1}^{p-1} [A(i+1,2) - A(i,2)] \times \frac{[A(i,1)-A(i+1,1)]}{2} + \{-S_{CS} + S_{FS}\}}{|A(n,2) - A(1,2)|} \tag{2}$$

where, $f(x)$ is natural surface function, L is earthwork width, A(i, 1) is y coordinate of the i^{th} node, A(i, 2) is x coordinate of the i^{th} node, S_{CS} is triangular shaped excess area due to cut slope, and S_{FS} is triangular shaped absence area due to fill slope. Excess and absence areas are determined with respect to the slope angles of road template. As given in eq. (1), triangular shaped excess-absence areas are subtracted for a cross section in cut, and added for a cross section in fill. In summary, an iterative procedure (Fig.1) is applied to calculate WGEs (h_w) until a pre-determined confidence level (δ) is satisfied.

Although h_w balances cut-fill and minimizes the sum of areas, excavated material may not occupy the same volume when placed in fill. For that reason, there might be considerable error in the calculation of exact cut-fill volumes. In order to calculate earthwork volumes precisely, following material properties were included in WGLM calculations: (1) swelling percent of material after excavation (P_S), (2) appropriateness percentage of material (P_A), and (3) compactibility percent of material (P_C).

In detail, the swelling of excavated material after the excavation is expressed by P_S coefficient, as well as the compaction of excavated and swelled material is determined by P_C parameter. Furthermore, excavated soil cannot be entirely used in fill because of various restrictions (e.g. top soils may not be used). The Appropriateness Percentage (P_A) is developed to take into account these sources of errors. Consequently, C_M coefficient is defined for the combined effect of these material parameters as:

$$C_M = \frac{P_A \times (1 + P_S)}{(1 + P_C)} \tag{3}$$

Therefore, eq. (1) should be modified with the consideration of the Material Coefficient (C_M) as follows:

$$\sum_{i=1}^{m} \mathbf{S_F}(i) = C_M \times \sum_{i=1}^{n} \mathbf{S_C}(i) \tag{4}$$

It should be noted that, if $C_M \geq 1$ then cut areas must be decreased for proper earthwork balancing, and vice versa. In order to modify initial WGE, namely considering C_M parameter, a certain amount of vertical shifting should be made. Calculation of the shifting can be written as follows.

$$h'_w = h_w - \Delta H \tag{5}$$

where, ΔH is the amount of shifting (for $y = h'_w$) and $h'_W$is WGE calculated considering related material properties. Finally, eq. (6) is used for the calculation of ΔH shifting.

$$\Delta H = \frac{C_1 \times g\,(\Delta H)}{L\,(\Delta H)} \tag{6}$$

where,

C_1	$1 - C_M$
$g\,(\Delta H)$	$\sum_{i=1}^{v} \mathbf{S'_C}(i) \quad for \quad y = (h_w - \Delta H)$
$L\,(\Delta H)$	$L' \qquad for \quad y = (h_w - \Delta H)$
$\mathbf{S}'_C$	cut area vector for new positioning (for$y = h'_w$)
v	number of cut areas for new positioning (for y = h_w')
L'	final earthwork width due to shifting

Obviously, the eq. (6) is recursive and varies with ΔH; thus, the root can be calculated by a numerical approach, such as the Method of False Position and Bisection methods (Chapra and Canale 1998). Consequently, after the calculation of ΔH using an appropriate method as mentioned above, the final weighted elevation (h'_w) can be obtained by eq. (5). Details and mathematical background of WGLM can be found in Goktepe and Lav (2003, 2004).

Final Alignment by Discrete Dynamic Programming

After the calculation of modified WGEs ($h_{w'}$) for each cross section (template) by means of WGLM, the ultimate goal is to fit the grade line as close as possible to compute WGEs complying with design controls. Goh, et al. (1988) was first to employ dynamic programming approach for the solution of this constrained nonlinear optimization problem in two-dimensional manner. In their study, centerline elevations were used for the calculation of earthwork cost. In addition, it should be emphasized that dynamic programming found inefficient for three-dimensional calculations by several researchers (Goh et al. 1988; Fwa 1989; Kim and Jha 2004). However, in this study, discrete dynamic programming and WGLM approaches were combined in one model to achieve precise optimization. The reason of choice of discrete dynamic programming in this study is its applicability to the existing problem because linear programming is not applicable for nonlinear constraints as well as not appropriate for the objective function treated in this study. Quadratic programming is also not suitable for the model considered in this article. Although genetic algorithm is also applicable for the problem, it was not preferred in order to simplify the solution methodology. In summary, discrete dynamic programming found efficient for this problem domain; nevertheless, genetic algorithm may be applied in further studies involving more sophisticated constraints and objective functions.

Basically, dynamic programming is a constrained optimization method, in which the problem is divided into k phases and k sub-problem with single variable that are iteratively solved in each phase. Throughout the iterative calculations of dynamic programming algorithm, the solution of each sub-problem is the input of successive step, namely next sub-problem. Therefore, overall optimization is achieved when the last sub-problem is solved. Essentially, dynamic programming falls into two categories with respect to the type of the objective function, namely, discrete and continuous. Discrete dynamic programming concerns with the minimization of an objective function defined by discrete data. Therefore, this is preferred when there is no continuous mapping for the objective function as in several other optimization techniques. Obviously, there are two possible calculation directions for the problem to be solved by dynamic programming algorithm, forward and backward. Backward computation is performed from the end point to the start point. Further information on dynamic programming can be found in the literature (Bertsekas 2001; Taha 2002).

In order to adapt the discrete dynamic programming to optimization of road profiles, it is necessary to split the length of highway into sections. Therefore, each section can be considered as the finite stages of the multistage optimization problem, in which the objective is to minimize the cumulative cost (to achieve the most economical combination of earthwork) of all the stages. In Fig.2, L x H dimensional calculation gird and the illustration of multistage vertical alignment is given. As can be drawn from the figure, the size (h x l) of a finite element is calculated by:

$$l = \frac{L}{d+1} \quad \text{and} \quad h = \frac{H}{s+1} \tag{7}$$

in which, d and s are the numbers of horizontal and vertical grid lines, respectively. In Fig.2, weighted ground line (WGL), which expresses the hypothetical natural surface, is indicated by $\boldsymbol{y}_i$ as well as $\boldsymbol{f}_i$ denotes the grade line obtained by discrete dynamic programming. As a result, final vertical alignment (road profile) is the combination of d+1 straight lines connected to each other at grid points. Therefore, the value of l and h, namely the size of a finite element, is significant for the precision and the computational complexity of the model (Goh et al. 1988).

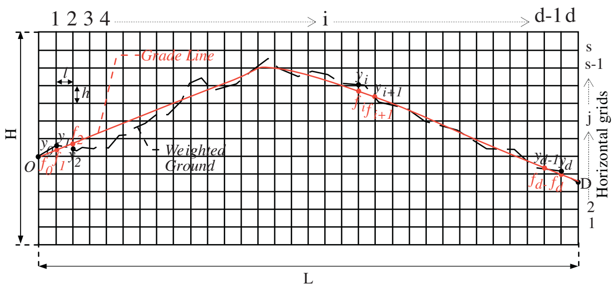

Figure 2. Profile for the illustration of dynamic programming model.

Consequently, the objective function is given as a function of increment in the elevation of grade line (u_i) by:

$$F = \min \left(\sum_{i=1}^{d} C\left(u_i\right) \right) \tag{8}$$

where, $u_i = y_{i+1} - y_i$, and $C(u_i)$ is the earthwork cost at the i^{th} stage. Details of an element and related definitions are depicted in Fig.3. In addition, $C(u_i)$ is calculated for four possible situations using weighted ground elevations (WGEs) as follows:

$$C(u_i) = \begin{cases} \alpha \times h \times \frac{(f_i + f_{i-1} - y_i - y_{i-1})}{2} & , if(f_i - y_i) < 0 \; and \; (f_{i-1} - y_{i-1}) < 0 \\ \beta \times h \times \frac{(f_i + f_{i-1} - y_i - y_{i-1})}{2} & , if(f_i - y_i) > 0 \; and \; (f_{i-1} - y_{i-1}) > 0 \\ \alpha \times h \times (f_{i-1} - y_{i-1}) + \beta \times h \times (f_i - y_i) & , if(f_i - y_i) < 0 \; and \; (f_{i-1} - y_{i-1}) > 0 \\ \beta \times h \times (f_{i-1} - y_{i-1}) + \alpha \times h \times (f_i - y_i) & , if(f_i - y_i) > 0 \; and \; (f_{i-1} - y_{i-1}) < 0 \end{cases} \tag{9}$$

where, α is excavation cost per cubic meter, and β is fill cost per cubic meter.

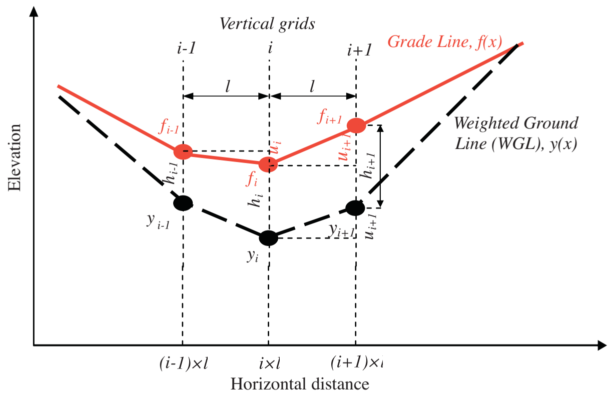

Figure 3. Focusing on successive calculation points in the dynamic programming model.

Obviously, calculated piecewise linear grade line should be subject to several geometric constraints determined by the design rules. On this point of view, in accordance with AASHTO (2001) design specifications, following geometric constraints are considered in this study:

$$u_i = |f_i - f_{i-1}| \le g_{\max} \times \; l \tag{10}$$

where, g_{max} is maximum allowable gradient, and l denotes horizontal distance between successive grid lines. On the other hand, following constraints are given for crest vertical curves $(u_i - u_{i-1} \;\; < 0)$:

$$\begin{array}{ll} g_i \le \frac{100\left(\sqrt{2h_d}+\sqrt{2h_0}\right)^2}{Lv-2S_i} & , if \; Lv \le S_i \\ g_i \le \frac{100L\left(\sqrt{2h_d}+\sqrt{2h_0}\right)^2}{S_i^2} & , if \; Lv > S_i \end{array} \tag{11}$$

and, for sag vertical curve $(u_i - u_{i-1} \;\; > 0)$:

$$\begin{array}{ll} g_i \le \frac{400+3.5S_i}{L-2S} & , if \; Lv \le S_i \\ g_i \le \frac{L\,(400+3.5S_i)}{S_i^2} & , if \; Lv > S_i \end{array} \tag{12}$$

in which, Lv is vertical curve length, h_d is height of driver's head above the roadway surface, h_0 is height of object above the roadway surface, S is the sight distance, and g is the algebraic difference between successive gradients which is given by:

$$g_i = \frac{u_i - u_{i-1}}{l} \tag{13}$$

In addition, the sight distance (S) is calculated as follows:

$$S_i = 3.67\,V + \frac{V^2}{30\,\left(fr + \frac{u_i}{l}\right)} \tag{14}$$

where, V is driving speed, and fr is the road friction factor. It should be noted that, boundary conditions of considered dynamic programming model are given by:

$$f_0 = y_0 \text{ and } f_{d+1} = y_{d+1} \tag{15}$$

Depending on the backward directional optimization methodology of dynamic programming, the accumulated cost from the end point $(x_{d+1},\ f_{d+1})$ to the processing point should be minimized using the optimal control (χ). The goal of optimal control is to iteratively calculate the optimum u_i minimizing the cumulative cost (T_i); thus, the grade elevation on vertical plane is determined. As a consequence, optimal control is an iterative process to adjust the grade elevation by changing u_i. For starting point, optimal grade elevation (f_0) and optimal elevation increment (u_0) are given by:

$$u_0 = \chi\left(0, \frac{f_0}{h}\right), \text{ and } f_0 = y_0 \tag{16}$$

Same parameters can be written for the i^{th} grid as follows:

$$u_i = \chi\left(i, \frac{f_i}{h}\right), \text{ and } f_i = f_{i-1} + u_{i-1} \tag{17}$$

As mentioned before, optimal control procedure seeks to find out the horizontal grid, whose elevation (y coordinate) minimizes the cumulative cost, and grade elevation (f_i) is adjusted to this elevation. In summary, cumulative cost for i^{th} vertical grid (at $i \times l$ distance from the starting point, O) is calculated by:

$$T_i = \sum_{k=1}^{i} C_k \tag{18}$$

Finally, objective function is given by:

$$T_i\,(f_i) = \min_{u_i}\left[C_i(u_i, f_i) + T_{i+1}(f_{i+1})\right] \qquad i = d-1, d-2, \ldots\ldots\ldots, 1 \tag{19}$$

Schematic representation of backward directional discrete dynamic programming model developed for WGLM-based highway alignment optimization is illustrated in Fig. 4.

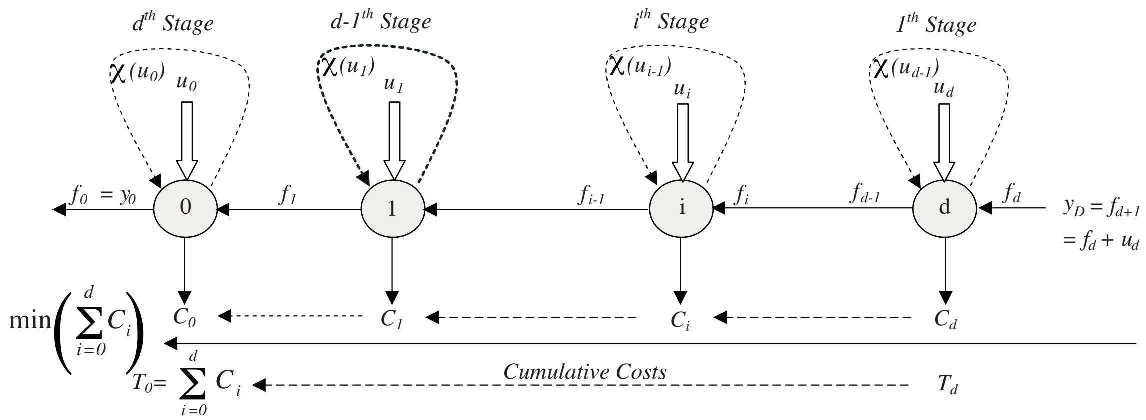

Figure 4. Schematic representation of backward directional dynamic programming model.

Numerical Study

In order to evaluate the performance of WGLM-based dynamic programming model, a numerical example is presented. Details regarding the example are given in Table 1. As mentioned before, the performance of developed model is sensitive to the size of both horizontal and vertical intervals (l and h). In order to evaluate the influence of these parameters, a parametric study is performed using different l and h values. Summary of this parametric study is given in Table 2. Under the light of this parametric study, l and h values were selected as 50m and 0.5m, respectively. It should be noted that, as a consequence of today's computational speeds (in this study hardware having P-IV 2.8 GHz CPU and 2GB DDR-2 Ram is used) computation time (C_T) is quite short, and it was found as 5.2sec for selected parameters (Table 2). In order to illustrate the reason of the selection of l and h parameters, bubble charts for C_M and C_T parameters, respectively. It should be noted fort Figures 6 and 7 that the diameters of the bubbles indicate h parameters, which are 0.25m, 0.50m, 1.00m. As can be seen from Figures 6 and 7, because of the geometric constraints of the problem, the closest C_M value to target values (0.80) can be obtained as 0.84. Considering the computational expense, l: 50m and h: 0.50m, which correspond %1.5 for l parameter and %8 for h parameter, are found to be the optimal values. It should be added for the example that, horizontal alignment is assumed as predetermined, and the aim of this study is to make the final vertical alignment in terms of series of line segments using implemented dynamic programming algorithm.

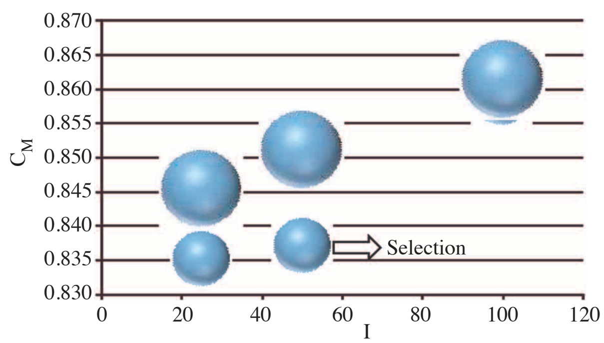

Figure 5. Bubble chart for the illustration of the importance of l and h parameters.

Table 1. Design variables for considered problem.

Design variable	Symbol	Value
Maximum allowable gradient	g_{max}	6.50%
Excavation cost	α	15\$/m^3
Filling cost	β	15\$/m^3
Swelling percent	P_S	0.15
Compactibility percent	P_C	0.30
Appropriateness percent	P_A	0.90
Vertical interval	h	0.5m
Horizontal interval	l	50.0m
Driver's height	h_0	1.2m
Object's height	h_d	0.6m
Length of the roadway	L	3200m
Tangents of cut/fill slopes (X:Y)	$m_C, /m_F$	1:1 / 1:1

Table 2. Effects of l and h values on the analyses.

l (m)	h (m)	C_T (sec)	C_M(%)	Cut Vol. (m^3)	Fill Vol. (m^3)	Earthwork Vol. (m^3)
25	0,50	8,5	0,835	17 775	21 281	39 056
25	1,00	4,9	0,846	17 822	21 077	38 899
50	0,25	11,4	0,836	17 781	21 260	39 041
50	0,50	5,2	0,837	17 785	21 243	39 028
50	1,00	3,4	0,851	17 930	21 065	38 995
100	0,50	3,2	0,859	18 043	21 011	39 054
100	1,00	2,1	0,861	18 089	20 999	39 088

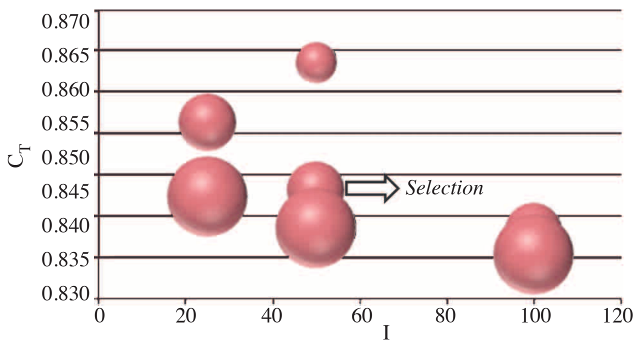

Figure 6. Illustration of the computation time for performed analyses.

The optimized road alignment is shown in Fig.7. Resulting fill and cut volumes are 17785m^3 and 21243m^3, respectively. With respect to the previous experiences of the authors, both cut-fill balancing and total earthwork (17785m^3 + 21243m^3 = 39028m^3) are both outstanding when considering the roadway in 3200m length. Accounting above mentioned soil parameters (Table 1), C_M parameter is calculated by 0.80. On the other hand, the ratio of total cut and fill volumes is 0.837 (17785m^3/ 21423m^3); therefore, this extent is so close to C_M value of 0.80. Assessing the profile given in Fig.5 and total cut-fill volumes obtained, it can be concluded that WGLM-based dynamic programming approach is considered successful.

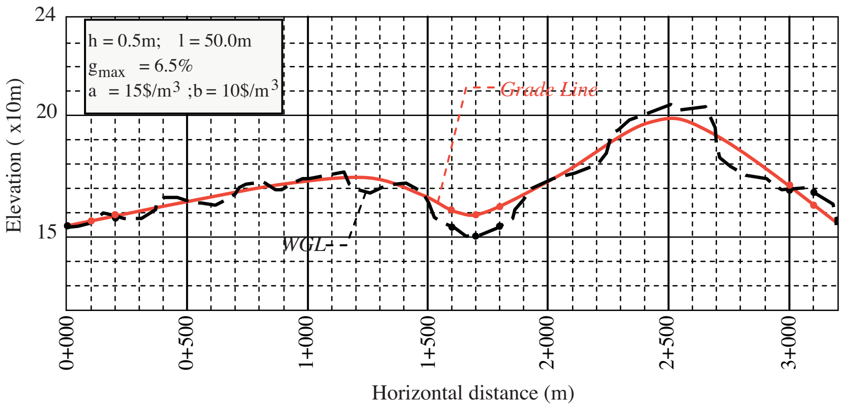

Figure 7. Ground profile and optimized vertical alignment in numerical example.

Conclusions

- The purpose of this study is to present the dynamic programming approach for the determination of vertical alignment of highways incorporated WGLM based earthwork optimization process.
- Although developed methodology is for highway alignment optimization problem, it can also be used for a stepwise regression problem comprising two-dimensional input-output data. Therefore, it can be considered as a new tool for stepwise regression analysis. Nevertheless, further investigations must be made to evaluate this issue.
- This alignment optimization methodology is not only applicable to a highway application, but also applicable to other civil engineering disciplines involving earthwork construction.
- The cost function described here comprises earthwork costs; however other source of costs, such as pavement construction, land acquisition, and vehicle operating costs, can be involved in the objective function.
- The selection of l and h parameters are very important in terms of computation speed and the precision. In this study, optimal parameters are found for horizontal direction (l) around %1 of the roadway length and for vertical direction (h) around %10 of the difference between the highest and the lowest ground elevations. On the other hand, due to the selection of different parameters, computation time may increase up to 5 times. Therefore, it should be considered for complex real world problems including longer roadway sections, additional constraints, and complicated objective functions.
- It should be noted that the proposed approach is dealing only with optimization of vertical alignment, namely the profile. That is, the method is applicable to highway projects where horizontal alignment (the plan) is established previously. Optimization of horizontal alignment is beyond the scope of this study.
- The fundamental inadequacy of this method is that it doesn't involve horizontal alignment. Even though it is capable of treating cross sectional information, it requires predetermined horizontal alignment. In further studies, three-dimensional alignment can be comprised in WGLM based earthwork optimization with proper modifications.
- The results indicated that dynamic programming approach along with WGLM is suitable for optimization of vertical alignment of highways.

Nomenclature

A Coordinate matrix
α Excavation cost
β Fill cost
$C(u_i)$ Earthwork cost at the i^{th} stage
C_M Material coefficient
C_T Computation time
χ Optimal control routine
d Number of horizontal grid lines
δ Confidence level
ΔH Amount of shifting
$f(x)$ Natural surface function
f_i Grade line obtained by dynamic programming
f_0 Optimal grade elevation
fr Road friction factor.
g Algebraic difference between successive gradients
g_{max} Maximum allowable gradient
h Height of the finite element
h_w Weighted ground elevation (WGE)
h'_W Modified WGE
h_d Height of driver's head above roadway surface
h_0 Height of object above roadway surface
l Length of the finite element
L Earthwork width
L' Final earthwork width due to shifting
Lv Vertical curve length
m Number of fill areas in cross section
n Number of cut areas in cross section
P_S Swelling percent of material after excavation
P_A Appropriateness percentage of material
P_C Compactibility percent of material
s Number vertical grid lines
S Sight distance
S_C Cut area vector
S'_C Cut area vector for new positioning
S_F Fill area vector
S_{CS} Triangular shaped excess area due to cut slope
S_{FS} Triangular shaped absence area due to fill slope
T_i Cumulative cost
u_0 Optimal elevation increment
u_i Elevation of grade line
v Number of cut areas for new positioning
V Driving speed
y_i Hypothetical natural surface

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