Turkish J. Eng. Env. Sci.
33 (2009) , 127 – 134.

doi:10.3906/muh-0905-17

Hydromagnetic buoyancy-induced flow of a particulate suspension through a vertical pipe with heat generation or absorption effects

Ali J. CHAMKHA[1] **and Mansour A. AL-SUBAIE**[2]
[1]*Manufacturing Engineering Department, The Public Authority for Applied Education & Training, Shuweikh, 70654, KUWAIT*
e-mail: achamkha@yahoo.com
[2]*Project, Design and Construction Department, Al-Khafji Jiont Operations, P.O. Box: 5143, Khafji 31971, KINGDOM OF SAUDI ARABIA*

Received 15.05.2009

Abstract

This work is focused on the analytical modeling of the problem concerning steady, laminar, natural convection fully developed flow of a particulate suspension through an infinitely long vertical circular pipe in the presence of a transverse magnetic field and fluid heat generation or absorption effects. The wall of the pipe is maintained at a uniform constant temperature. The analytical solutions for the velocity and temperature profiles of both phases are given in terms of Bessel functions. A parametric study of the physical parameters involved in the problem is performed and the obtained results are presented graphically to show special trends of the solutions.

Key Words: MHD; 2-phase flow; heat generation or absorption; pipe flow; free convection.

Introduction

Two-phase (fluid-particle) natural convection flow represents one of the most interesting and challenging areas of research in heat transfer. Such flows are found in a wide range of applications including processes in the chemical and food industries, solar collectors where a particulate suspension is used to enhance absorption of radiation, and cooling of nuclear reactors. In general, all applications of single-phase flow are valid for 2-phase particulate suspension flow because the nature of real life dictates the presence of contaminating particles in fluids. Actually, all research of natural convection flows within vertical pipes is done only for a single phase.

Forced convection 2-phase flows have been considered by many previous investigators. For example, Ritter (1976) reported transient 2-phase fluid-particle flows in channels and circular pipes. He found that the presence of particles caused significant reductions in the flow rates of both phases. Chamkha (1995a, 1995b) reported analytical solutions for transient hydromagnetic 2-phase flow in a channel with constant and oscillating pressure gradients. He confirmed the results of Ritter (1976) and concluded that the presence of a transverse magnetic field normal to the flow direction caused a retardation effect motion of the suspension.

On the other hand, very little work has been reported on natural convection flow of a particle-fluid suspension over and through different geometries. Chamkha and Ramadan (1998) and Ramadan and Chamkha (1999) have reported some analytical and numerical results for natural convection flow of a 2-phase particulate suspension over an infinite vertical plate. They found that increases in either the particle loading or the wall particulate slip coefficient caused reductions in the velocities of both phases. Moreover, Okada and Suzuki (1997) considered buoyancy-induced flow of a 2-phase suspension in an enclosure. However, the present authors were unable to locate any theoretical or experimental work in the literature dealing with natural convection laminar flow of a particulate suspension in vertical channels and pipes. Thus, there is a definite need for investigation of such mentioned problems since it is almost impossible to find non-contaminated fluids in real applications. Hence, the objective of this research is to perform an analytical investigation on steady natural convection laminar flow of particulate suspensions in infinite vertical pipes.

Governing Equations

Consider the physical problem of steady, laminar, natural convective fully developed flow of a 2-phase (fluid-particle) suspension through an infinitely long isothermal vertical circular pipe in the presence of a transverse magnetic field and heat generation or absorption effects. The fluid phase is assumed to be Newtonian, viscous, heat generating or absorbing, and electrically conducting. The particle phase is assumed to be made up of electrically non-conducting spherical solid particles having the same size. The concentration of the particles is assumed to be somewhat low and the suspension is considered dilute in the sense that no particle-particle interactions exist. This means that the particle phase is considered inviscid. Moreover, the particle phase is assumed to be pressureless and that the particles are driven along by the fluid-phase pressure. In addition, the magnetic Reynolds number is considered small, so that the induced magnetic number is neglected. The small magnetic Reynolds number assumption is widely used as it provides a great simplification in the uncoupling of Maxwell's equations from the Navier-Stokes equations of the fluid phase (see Cramer and Pai, 1970). In this study, both phases are treated as interacting continua through interphase momentum and heat transfer. The vector form of the general basic equations (see Marble, 1970; Drew, 1983) that account for the magnetic field effect and the possible presence of heat generation or absorption effects can be written as:

$$\boldsymbol{\nabla}(\rho \mathbf{V}) = 0 \tag{1}$$

$$\rho \mathbf{V}.\boldsymbol{\nabla}\mathbf{V} = -\boldsymbol{\nabla}P + \boldsymbol{\nabla}.(\mu \boldsymbol{\nabla}\mathbf{V}) - \rho_p N(\mathbf{V} - \mathbf{V}_p) + \rho \mathbf{g} + \sigma(\mathbf{V} \times \mathbf{B}) \times \mathbf{B} \tag{2}$$

$$\rho c \mathbf{V}.\boldsymbol{\nabla}T = \boldsymbol{\nabla}(K \boldsymbol{\nabla}T) + \rho_p c_p N_t (T_p - T) \mp Q(T - T_o) \tag{3}$$

$$\boldsymbol{\nabla}.(\rho_p \mathbf{V}_p) = 0 \tag{4}$$

$$\varrho_p \mathbf{V}_p \boldsymbol{\nabla}\mathbf{V}_p = \rho_p N(\mathbf{V} - \mathbf{V}_p) + \rho_p \mathbf{g} \tag{5}$$

$$\rho_p c_p \mathbf{V}.\boldsymbol{\nabla}T_p = -\rho_p c_p N_T (T_p - T) \tag{6}$$

where all variables are defined in the Nomenclature section. The plus "+" or minus "–" sign appearing in Eq. (3) corresponds to heat generation (heat source) or heat absorption (heat sink), respectively.

Since the circular pipe is assumed to be infinitely long, the dependence of flow and thermal variables on the x-direction will be negligible compared with that of the r-direction (see Figure 1). Therefore, all dependent variables in Eqs. (1) through (6) will be functions of r only. Taking into consideration this and all previous assumptions, Eqs. (1) through (6) reduce to

$$-\partial_x P + \mu[\partial_{rr}U + (\frac{1}{r})\partial_r U] - \rho_p N(U - U_p) + \rho g - \sigma B^2 U = 0 \tag{7}$$

$$k[\partial_{rr}T + (\frac{1}{r})\partial_r T] + \rho_p c_p N_T (T_p - T) \mp Q(T - T_o) = 0 \tag{8}$$

$$\rho_p N(U - U_p) - \rho_p g = 0 \tag{9}$$

$$\rho_p c_p N_T (T_p - T) = 0 \tag{10}$$

The continuity equations of both phases (1) and (4) are identically satisfied.

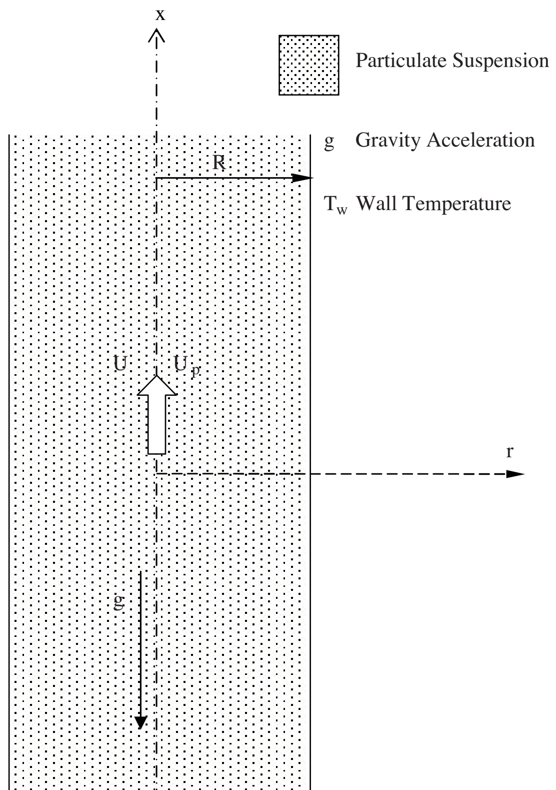

Figure 1. Problem definition.

The pressure gradient term can be eliminated from the linear momentum equation of the fluid phase [Eq. (7)] by evaluating the governing equations at a reference point at the entrance of the pipe. Let "o" be that reference point such that U = 0, T = T_o, $\rho = \rho_o$, $\mu = \mu_o$, $\sigma = \sigma_o$ $U_p = U_{po}$, $T_p = T_{po}$, and $\rho_p = \rho_{po}$. Evaluating Eqs. (7) through (10) at this reference point and employing the Boussinesq approximation gives

$$\rho_{po}/\rho_o g + \mu_o/\rho_o[\partial_{rr}U + (\frac{1}{r})\partial_r U) - \rho_{po}/\rho_o N(U - U_p) + \beta^* g(T - T_o) - \sigma_o/\rho_o B^2 U = 0 \tag{11}$$

where β^* is the volumetric expansion coefficient. The linear momentum equation of the fluid phase, Eq. (7), will be replaced now by Eq. (11) in the governing equations.

Each of Eqs. (8) and (11) requires 2 boundary conditions to solve them completely. These can be written as

$$\partial_r U(0) = U(R) = 0, \partial_r T(0) = 0, T(R) = T_w \tag{12a-d}$$

where R is the pipe radius and T_w is the pipe wall temperature. Equations (12a) and (12b) indicate a symmetry condition and a no slip condition for the fluid phase in the pipe. Equation (12c) indicates a temperature symmetry condition and Eq. (12d) indicates that the fluid temperature at the wall of the pipe is some constant value T_w.

The formulation of the problem under consideration is now complete. It is convenient to non-dimensionalize the governing equations and conditions. This can be accomplished by using the following parameters:

$$r = R\eta, U = (/\rho R)u, U_p = (/\rho R)u_p,\ T = (T_w - T_o)\theta + T_o,\ T_p = (T_w - T_o)\theta_p + T_o \tag{13}$$

where η is the dimensionless transverse coordinate. u and u_p are the dimensionless fluid- and particle-phase velocities, respectively. θ and θ_p are the dimensionless fluid- and particle-phase temperatures, respectively. After performing the mathematical operations, the resulting dimensionless governing equations can be written as

$$D^2u + (1/\eta)Du - \alpha\kappa(u - u_p)Gr\theta + M^2u + \kappa H = 0 \tag{14}$$

$$(\frac{1}{Pr})[D^2\theta + (\frac{1}{\eta})D\theta] + \kappa\gamma\varepsilon(\theta_p - \theta) \mp \phi\theta = 0 \tag{15}$$

$$\alpha(u - u_p) - H = 0 \tag{16}$$

$$\varepsilon(\theta_p - \theta) = 0 \tag{17}$$

where D and D^2 denote a first- and a second-order ordinary derivative operators with respect to η, respectively. Furthermore, $\alpha = R^2N\rho/\mu$, $\kappa = \rho_p/\rho$, $Gr = g\beta^* R^3\rho(T_w - T_o)/\mu$, $M^2 = \sigma^2h^2/\mu$, $H = gR^3\rho^2/\mu^2$, $Pr = \mu c/k$, $\gamma = c_p/c$, $\varepsilon = \rho N_T R^2/\mu$ and $\phi = Qh^2/(\mu c)$ are the momentum inverse Stokes number, the particle loading, the Grashof number, square of Hartmann number, buoyancy parameter, the Prandtl number, the specific heat ratio, temperature inverse Stokes number, and the heat generation or absorption parameter, respectively. The $\pm\phi$ stands for the problem with a heat generation effect (positive sign) and a heat absorption effect (negative sign).

The dimensionless boundary conditions become

$$Du(0) = 0, u(1) = 0, D\theta(0) = 0, \theta(1) = 1 \tag{18}$$

Analytical Results and Discussion

Combining Eqs. (15) and (17) and then solving for the fluid-phase temperature θ subject to the corresponding boundary conditions yields the following fluid-phase temperature profile:

$$\theta(\eta) = I_o(\sqrt{\phi Pr}\eta)/I_o(\sqrt{\phi Pr}) \tag{19}$$

for a heat generating source and

$$\theta(\eta) = J_o(\sqrt{\phi Pr}\eta)/J_o(\sqrt{\phi Pr}) \tag{20}$$

for a heat absorbing sink.

The I_o and J_o are the modified Bessel function of the first kind of order zero and the Bessel function of the first kind of order zero, respectively.

According to Eq. (17), the particle-phase temperature profile will be the same as that of the fluid-phase.

Now, in order to solve for the fluid-phase velocity u, Eqs. (16) and (19) or (20) are substituted into Eq. (14) and then rearranged to give

$$D^2u + (1/\eta)Du - M^2u = -Gr\theta(\eta) \tag{21}$$

where $\theta(\eta)$ is given by Eq. (19) or (20) according to the sign before ϕ in Eq. (15).

The right side of the above equation shows that the differential equation is a non-homogeneous equation (the right side is different from zero). In other words, Eq. (21) is a non-homogeneous modified Bessel equation of order zero. The general solution of Eq. (21) can be shown to be

$$u(\eta) = Gr[I_o(M\eta)/I_o(M) - \theta\eta)]/(\pm\phi r - M^2) \tag{22}$$

where θ (η) is given by Eq. (19) for + ϕ or Eq. (20) for - ϕ.

The particle-phase velocity can be found by substituting Eq. (22) into Eq. (16) to obtain

$$u_p(\eta) = Gr[I_o(M\eta)/I_o(M) - \theta\eta)]/(\pm\phi r - M^2) - H/\alpha \tag{23}$$

where $\theta\eta)$ is given as in Eq. (22).

Figures 2 and 3 display the changes in the fluid-phase velocity (u) and the particle-phase velocity (u_p) for various values of Hartmann number M, respectively. These are obtained by numerically evaluating Eqs. (22) and (23). The transverse magnetic field normal to the flow direction creates a drag-like force that acts in the direction opposite to the flow direction. This has the tendency to decrease the velocity of the fluid, which decreases the velocity of the particulate phase as well.

The effects of the Prandtl number Pr are shown in Figures 4 through 6 for a vertical pipe. Figure 4 shows the temperature profiles of both phases. In the presence of heat generation, as Pr increases, the temperature for both phases decreases. The velocity profiles of the fluid and particle phases are shown in Figures 5 and 6, respectively. The increase in Pr in the presence of heat generation has the tendency to decrease the magnitude of both the fluid- and the particle-phase velocity profiles. This behavior arises as a result of the temperature behavior discussed above.

Some results for u, u_p, θ and θ_p based on the closed-form solutions for the flow through a vertical pipe in the presence of a heat generation (source) or a heat absorption (sink) term $\pm\phi$ are presented in Figures 7 through 9. Figure 7 presents temperature profiles for both the fluid and particle phases (θ and θ_p) for different values of ϕ. In the absence of heat generation or absorption effects (ϕ=0), the temperature profile of both phases in the pipe is linear. However, as ϕ increases the temperature decreases and the profiles become nonlinear. On the other hand, as ϕ decreases the temperature profile becomes nonlinear again and increases as depicted in Figure 7. The velocity profiles of both phases (u and u_p) are shown in Figures 8 and 9. Increases in the values of ϕ have a tendency to decrease the buoyancy effects as explained above. This produces a reduction in the fluid- and particle-phase velocities there as clearly depicted in Figures 8 and 9.

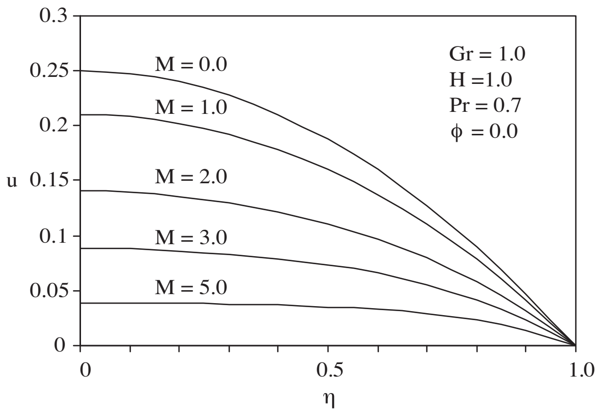

Figure 2. Effect of M on fluid-phase velocity profiles.

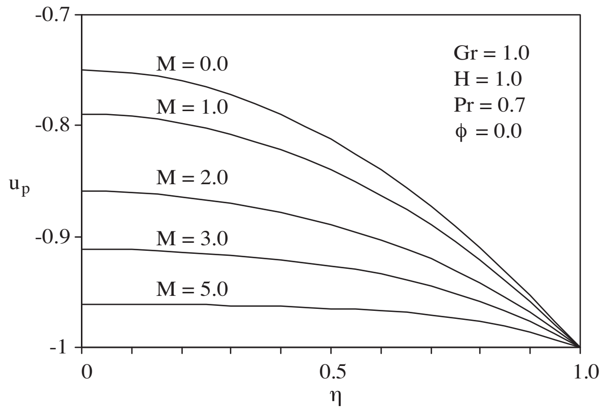

Figure 3. Effects of M on particle-phase velocity profiles.

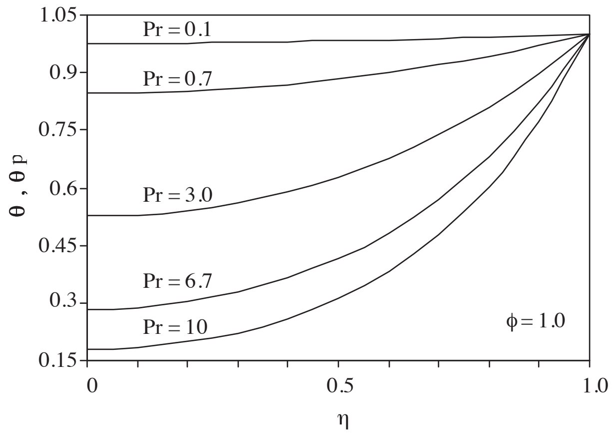

Figure 4. Effects of Pron fluid- and particle-phase temperature profiles.

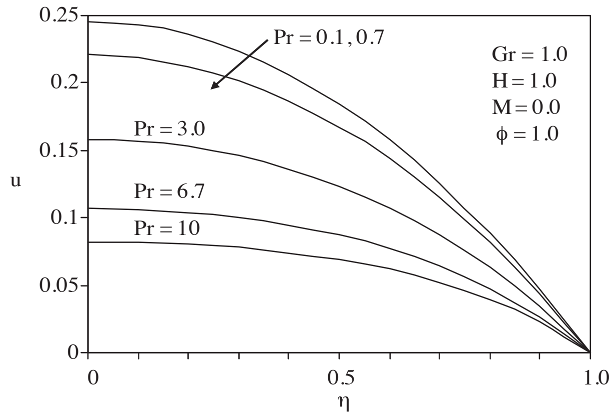

Figure 5. Effects of Pr on fluid-phase velocity profiles.

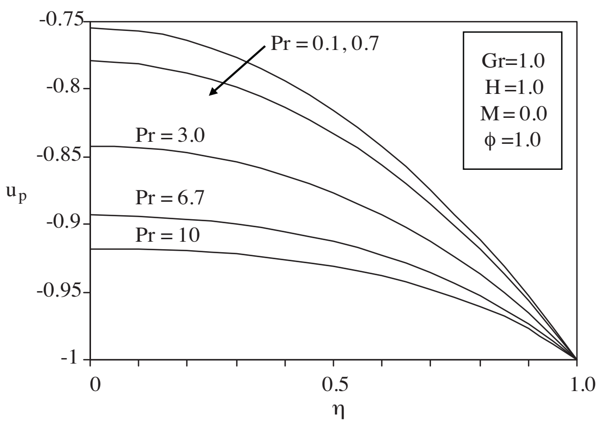

Figure 6. Effect of Pr on particle-phase velocity profiles.

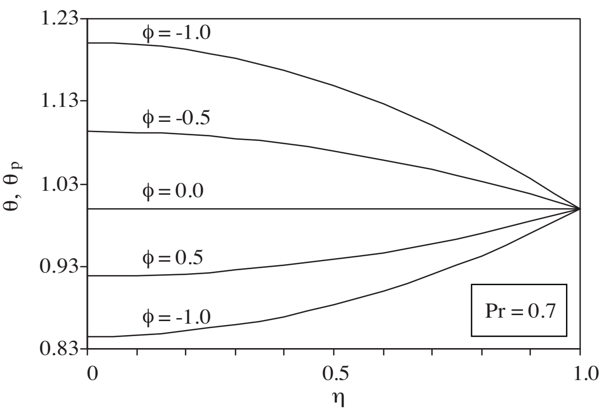

Figure 7. Effect of ϕ on fluid- and particle-phase temperature profiles.

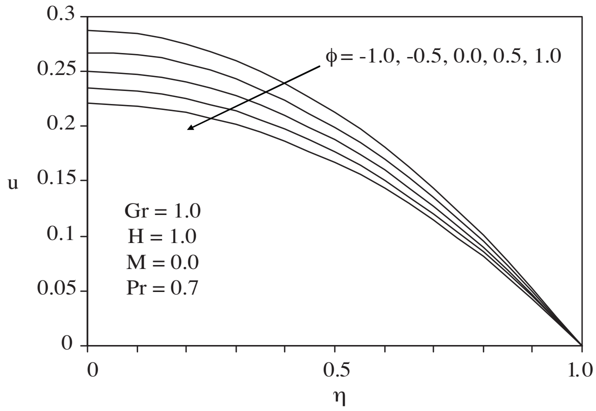

Figure 8. Effects of ϕ on fluid-phase velocity profiles.

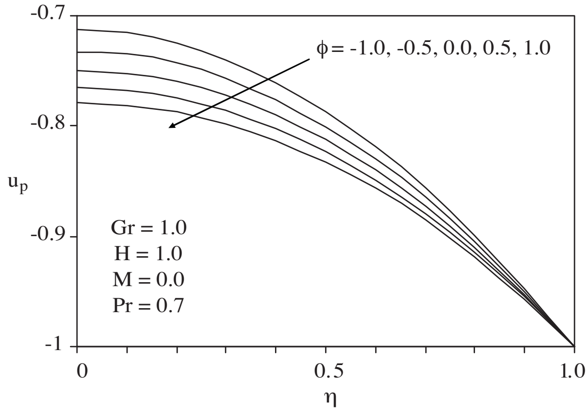

Figure 9. Effects of ϕ on particle-phase velocity profiles.

Conclusions

The mathematical modeling of natural convection flow of a particulate suspension was formulated in its general form by stating the conservation laws of mass, linear momentum, and energy for both the fluid and particle phases. The general formulation took into account the effects of the magnetic field and the possible presence of heat generation or absorption effects. The governing equations were non-dimensionlized, solved analytically, and closed-form solutions were obtained. Representative results were plotted to illustrate the influence of the physical parameters on the solutions. An increase in the values of the Hartmann number has the tendency to decrease the velocity of the fluid, which decreases the velocity of the particulate phase as well. In the presence of heat generation, as Pr increases, the temperature for both phases decreases. As a result of this, the velocity profiles of the fluid and particle phases have the tendency to decrease their magnitude. In the absence of heat generation or absorption effects, the temperature profile of both phases in the pipe is linear. However, as ϕ increases the temperature decreases and the profiles become nonlinear. On the other hand, as ϕ decreases the temperature profile becomes nonlinear again and increases. Increases in the values of ϕ have a tendency to decrease the buoyancy effects, producing a reduction in the fluid- and particle-phase velocities.

Nomenclature

B	magnetic induction vector
B	transverse magnetic induction
c	fluid-phase specific heat at constant pressure
c_p	particle-phase specific heat at constant pressure
g	gravitational acceleration vector
g	gravitational acceleration
Gr	Grashof number
h	channel width
H	dimensionless buoyancy parameter
k	fluid-phase thermal conductivity
M	Hartmann number
N	interphase momentum transfer coefficient
N_T	interphase heat transfer coefficient
P	fluid-phase hydrostatic pressure
Pr	fluid-phase Prandtl number
Q	heat generation or absorption coefficient
T	fluid-phase temperature
T_p	particle-phase temperature
u	fluid-phase dimensionless vertical velocity
u_p	particle-phase dimensionless vertical velocity
U	fluid-phase vertical velocity
U_p	particle-phase vertical velocity
V	fluid-phase velocity vector
$\mathbf{V}_p$	particle-phase velocity vector
x, r	polar coordinates

Greek Symbols

α	velocity inverse Stokes number
γ	specific heat ratio
ε	temperature inverse Stokes number
η	dimensionless y-coordinate

θ dimensionless fluid-phase temperature
θ_p dimensionless fluid-phase temperature κ particle loading
μ fluid-phase dynamic viscosity
ρ fluid-phase density
ρ_p particle-phase density
σ fluid-phase electrical conductivity
ϕ dimensionless heat generation or absorption coefficient
$\boldsymbol{\nabla}$ gradient operator vector

References

Chamkha, A.J., "Hydromagnetic Two-Phase Flow in a Channel", International Journal of Engineering Science, 33, 437-446, 1995a.

Chamkha, A.J., "Time Dependent Two-Phase Channel Flow Due to an Oscillating Pressure Gradient", Fluid/Particle Separation Journal, 8, 196-203, 1995b.

Chamkha, A.J. and Ramadan, H., "Analytical Solutions for the Two-Phase Free Convection Flow of a Particulate Suspension Past an Infinite Vertical Plate", International Journal of Engineering Science, 36, 49-60, 1998.

Drew, D.A., "Mathematical Modeling of Two-Phase Flow", Annual Review of Fluid Mechanics, 15, 261-291, 1983.

Farlow, S.J., Partial Differential Equations for Scientists and Engineers. John Wiley & Sons, In., New York, 1982.

Marble, F.E., "Dynamics of Dusty Gases", Annual Review of Fluid Mechanics, 2, 397-447, 1970.

Ramadan, H. and Chamkha, A.J., "Two Phase Free Convection Flow over an Infinite Permeable Inclined Plate with Non-Uniform Particle-Phase Density", International Journal of Engineering Science, 37, 1351-1367, 1999.

Ritter, J.M., "Two Phase Fluid Flow in Pipes and Channel", M.S. Thesis, Tennessee Technological University, 1976.

Ritter, J.M. and Peddieson, J., "Transient Two-Phase Flows in Channels and Circular Pipes", Proceeding of the Sixth Canadian Congress of Applied Mechanics, 1977.