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Comment on "Exact solutions of incompressible Couette flow with constant temperature and constant heat flux on walls in the presence of radiation", R. C. Chaudhary, Preeti Jain [Turkish J. Eng. Env. Sci., 31(2007), 297-304]

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Abstract

This paper presents an appropriate title that reflects the mathematical model of the above paper. Correct solutions for velocity and skin friction variables are presented for the given mathematical model of the above paper. The corresponding numerical results for the velocity and skin friction are shown graphically.

Key Words: Radiation, transient free convection, vertical parallel plates, constant heat flux, constant temperature, correct solution.

The above paper investigates a closed form solution for the transient free convection flow of a viscous incompressible fluid between 2 infinite vertical parallel plates in the presence of radiation. The flow is set up due to free convective currents occurring as a result of application of constant heat flux at one wall and constant temperature on the other wall. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The governing partial differential equations are solved exactly using the Laplace-transform technique. The problem is interesting and challenging. However, there are errors in the above paper (Chaudhary and Jain, 2007), which are presented as below:

The mathematical model in a dimensionless form (Eqs. (18), (19) and (10) in p. 299) given in the above paper (Chaudhary and Jain (2007)) is

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta \tag{1}$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{3R+4}{3R\,\mathrm{Pr}}\right)\frac{\partial^2 \theta}{\partial y^2} \tag{2}$$

with the initial and boundary conditions as

$$\left.\begin{array}{lll} t \leq 0 : u = \theta = 0 & \text{for} & 0 \leq y \leq 1 \\ t > 0 : u = 0, \frac{\partial \theta}{\partial y} = -1 & \text{at} & y = 0 \\ u = 0, \theta = 0 & \text{at} & y = 1 \end{array}\right\} \tag{3}$$

From the boundary conditions it is very clear that both the plates at $y = 0$ and $y = 1$ are stationary. None of the plate is moving and then there is no question of Couette flow. Hence, the title of the problem does not reflect the mathematical model. The appropriate title that reflects the given mathematical model in the above paper (Chaudhary and Jain, 2007) may be either "Radiation effects on transient free convection flow between two vertical parallel plates with constant heat flux and constant temperature on walls" or "Exact solution of incompressible free convection flow with constant heat flux and constant temperature on walls in the presence of radiation".

Even for the given mathematical model in the above paper (Chaudhary and Jain, 2007), the solution for velocity variable (given in Eqs. (14) and (19), p. 300) is wrong and hence the expression for skin-friction (given in Eq. (20), p. 300). It is also mentioned that the limit $R \to \infty$ represents the case of absent radiation heat transfer effects and, in this case, their result is comparable with that reported by Paul et al. (1996), whose result is also wrong. The correct solution for the velocity variable in the presence of radiation is given by

$$\begin{aligned} u(y,t) \quad = \quad & \tfrac{1}{(s-1)} \tfrac{1}{\sqrt{s}} \sum_{n=0}^{\infty} \left[\{F_2(d,t) - F_2(c,t)\} + 2 \sum_{m=0}^{n} (-1)^m \{F_2(e,t) - F_2(f,t)\} \right. \\ & \left. +(-1)^n \{F_2(b,t) - F_2(a,t)\} \right] \ , \ R \neq 0 \end{aligned} \tag{4}$$

where

$$\begin{aligned} & s = \tfrac{3R\,\mathrm{Pr}}{(3R+4)}, \ a = (2n+y)\sqrt{s}, \\ & b = (2n+2-y)\sqrt{s}, \ c = 2n+y, \\ & d = 2n+2-y, \ e = 2m\left(\sqrt{s}-1\right)+c, \\ & f = 2m\left(\sqrt{s}-1\right)+d \end{aligned}$$

$$F_2(l,t) = \frac{1}{3}\sqrt{\frac{t}{\pi}}\left(4t + l^2\right)\exp\left(-\frac{l^2}{4t}\right) - l\left(t + \frac{l^2}{6}\right) erfc\left(\frac{l}{2\sqrt{t}}\right) \tag{5}$$

and l is a dummy variable.

In the limit $R \to \infty$, the results from Eq. (4) coincides with the results of Narahari et al. (2002).

The correct solution for velocity variable when $R \to 0$ is given by

$$\begin{aligned} u(y,t) \quad = \quad & \tfrac{1}{(\mathrm{Pr}-1)} \tfrac{1}{\sqrt{\mathrm{Pr}}} \sum_{n=0}^{\infty} \left[\{F_2(d,t) - F_2(c,t)\} + 2 \sum_{m=0}^{n} (-1)^m \{F_2(j,t) - F_2(r,t)\} \right. \\ & \left. +(-1)^n \{F_2(h,t) - F_2(g,t)\} \right] \end{aligned} \tag{6}$$

where

$$\begin{aligned} & g = (2n+y)\sqrt{\mathrm{Pr}}, \ h = (2n+2-y)\sqrt{\mathrm{Pr}}, \\ & j = 2m\left(\sqrt{\mathrm{Pr}}-1\right)+c, \ r = 2m\left(\sqrt{\mathrm{Pr}}-1\right)+d. \end{aligned}$$

The correct expression for skin-friction (τ_0) at the plate $y = 0$ in the presence of radiation is given by

$$\begin{aligned}\tau_0 \quad = \quad & \tfrac{1}{(s-1)}\tfrac{1}{\sqrt{s}} \sum_{n=0}^{\infty} \left[\{F_3(n,t) + F_3(n+1,t)\} - 2\sum_{m=0}^{n} (-1)^m \{F_3(w,t) + F_3(w+1,t)\}\right. \\ & \left. +(-1)^n \left\{F_4\left(n\sqrt{s},t\right) + F_4\left((n+1)\sqrt{s},t\right)\right\}\right]\end{aligned} \qquad (7)$$

where

$$w = m\left(\sqrt{s}-1\right) + n$$

$$F_3(l,t) = (t + 2l^2)erfc\left(\frac{l}{\sqrt{t}}\right) - 2l\sqrt{\frac{t}{\pi}}\exp\left(-\frac{l^2}{t}\right) \qquad (8)$$

$$F_4(l,t) = \left(t\sqrt{s} + 2l^2\right) erfc\left(\frac{l}{\sqrt{t}}\right) - 2l\sqrt{s\left(\frac{t}{\pi}\right)}\exp\left(-\frac{l^2}{t}\right) \qquad (9)$$

The correct velocity profiles for air and water are shown in Figures 1 and 2, respectively, and the correct skin friction variation with time at the plate $y = 0$ is shown in Figure 3.

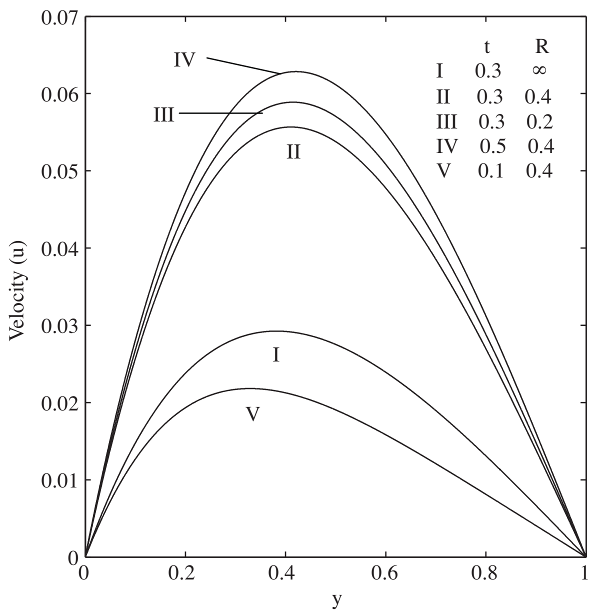

Figure 1. Velocity profiles of air (Pr = 0.71).

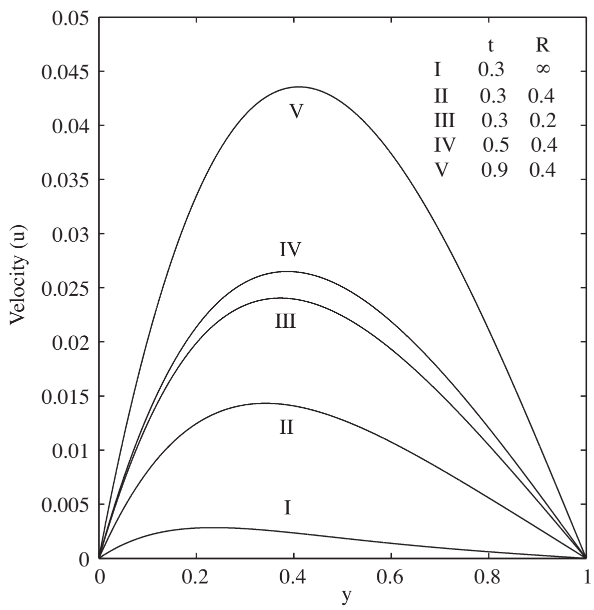

Figure 2. Velocity profiles of water (Pr = 7.0).

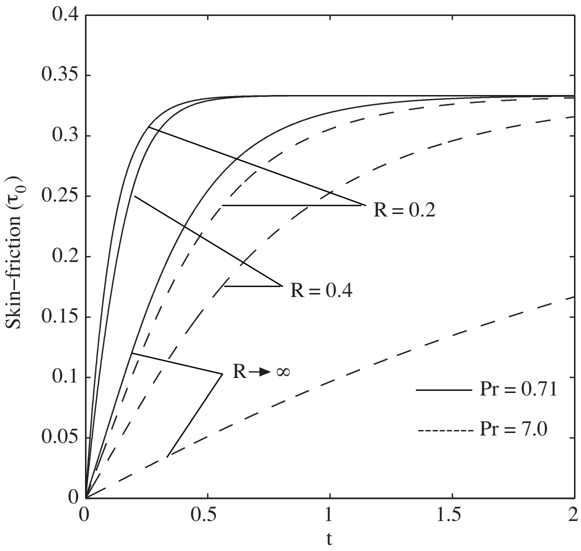

Figure 3. Skin-friction profiles at the plate y = 0.

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