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Monolithic concrete beams with external prestressing cables: an analysis under moving loads

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Abstract

Bridges are exposed to various loading conditions, and influence lines have important applications for the design of beams that are subjected to moving loads. The main purpose of this study was to evaluate the applicability of a transfer matrix method (TMM)-based static analysis of monolithic concrete beams with external prestressing cables subjected to moving loads. This approach is important for helping engineers in the preliminary design and is consistent with the design prescriptions of modern codes. The results, in terms of the load deflections and load increase of cable stress, are presented and discussed. The effects of friction at the deviators on the behavior of the externally prestressed concrete beams were also examined.

Key Words: External cables, moving loads, influence lines, transfer matrix method, linear elastic support, deviators, friction effect

1. Introduction

External prestressing has been extensively used in various engineering structures (Virlogeux, 1983; Hoang and Pasquignon, 1985; Jartoux, 1986; Virlogeux, 1990). This technique is considered an important tool in new constructions as well as in strengthening and repairing existing bridges (Foure and Hoang, 1993; Mutsuyoshi et al., 1995; Miyamoto and Nakamura, 1997; Abdunur and Godart, 1998; Lebet and Utz, 2005; Nordin, 2005; Fernàndez Ruiz et al., 2006), since it provides many advantages, such as extension of elastic behavior to higher loads, increase of ultimate capacity, and decrease of deflection under service loads. Many experimental studies (Harajli, 1993; Tan and Ng, 1997; Tan et al., 2001; Aparicio et al., 2002; Harajli, 2002; Tan and Tjandra, 2003; Aravinthan, 2005) have been conducted on the behavior of externally prestressed members. Some numerical analyses (Harajli, 1999; Ariyawardena and Ghali, 2002; Wu and Lv, 2003; El-Ariss, 2004) have also been undertaken. Although finite element models are capable of analyzing nonlinear behavior and incorporating

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complex phenomena such as slip, friction between external tendons and deviators, and material and geometrical nonlinearities, the analytical procedure is too complicated for these structures and the number of elements is extremely large. The present study is focused on the computation of influence line forces developed in an externally prestressed monolithic concrete beam, considering the displacement compatibility and friction effects at the elastic supports (deviator points) using the transfer matrix method (TMM) (Gery and Calgaro, 1973). This approach is simple and important in helping engineers in the design process. In addition, the proposed approach is consistent with the prescriptions of modern codes (Eurocode (EC2), 2004; AASHTO-LRFD, 2004) and represents an alternative for the preliminary design process, leaving the more complex and nonlinear analysis to the final structural verification. To this end, a Fortran 77 computer program was developed to perform the computations required. The TMM technique is proposed to evaluate both the bending moment and shear forces produced by the influence lines at some locations (deviator points and anchorage ends) of an externally prestressed concrete beam. The analytical equations used to calculate forces produced by the moving loads and external prestressing cables are reviewed hereafter. This analytical review gives insight into the elegance of the proposed method. With the proposed method, the problem can be efficiently handled by using a personal computer or undertaken by hand calculations with minimal computation effort. The steps are also simple, and they can be used for any arbitrary loading and extended to multi-span beams.

2. Method of analysis

Figure 1 shows a layout scheme of a simply supported beam prestressed by external cables. At equal distances along the beam, 2 external cables with a polygonal profile and 4 deviator points were provided. In this example, the prestressing load is transferred to the concrete beam through the deviator points and anchorage ends. It is assumed that the bearing supports are absolutely rigid and the intermediate supports at the deviator locations are linear elastic, with a spring constant only (neglecting rotational effects). As can be noted, the external prestressing cables extend horizontally in the inner part of the concrete beam, and one of the cables undergoes deviations from the horizontal line at the first and forth deviators and at the second and third deviators for the second cable. The deviators are mounted symmetrically about the median section of the beam and cables are anchored at both ends of the beam. According to EC2, spacers are used to reduce the vibration effects where the length of the unsupported external tendons is too long ($\geq$8 m). Another modern code, namely the AASHTO-LRFD, specifies a similar procedure.

2.1. Input data, modeling of the beam, and calculation assumptions

The data describing the dimensions, material, and loading of the beams taken as examples in this study, together with the data describing the bearing concepts at the deviator points, should be available for calculations (such data are useful for analytical or numerical examples). The actual structure was modeled as a statically indeterminate system with multiple span elements, and the bearings were modeled as absolutely stiff supports (at anchorage ends and bearing supports of the beam) and linearly elastic supports (at intermediate locations, where deviators are mounted). A general span element model is shown in Figure 2, with the main general loading that can be applied on each element, including the self-weight.

All calculations were performed in the vertical plane, where the moving concentrated loads are supposed to be acting, and deviator forces with the types of supports should be taken into consideration within the loading of the model. In addition, all calculations were based on real beam dimensions and the following 2 assumptions.

First, the calculations were performed with conventional friction because of its unknown extent, and because the real value of the friction coefficient depends on many factors and can only be determined by experimental investigations. Second, the deflection of the external cable does not follow the beam deflection, except at the deviator points when the beam is deformed by the externally prestressing cables and does not meet exactly the proposed method of analysis at every cross section; however, it was assumed that the accuracy of the outcomes was strongly dependent upon the initial conditions of the beam.

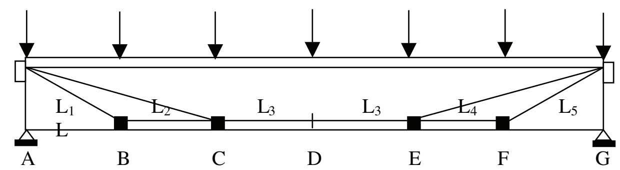

Figure 1. Typical layout scheme of an externally prestressed concrete beam.

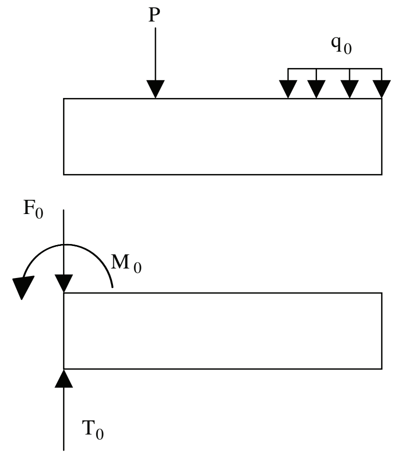

Figure 2. Model of an element with external loading.

2.2. Selection of the transfer matrix method

The most appropriate modeling and calculation procedures are the method of initial parameters in matrix form, that is, the so-called transfer matrix method (TMM) and the finite element method (FEM). Practically, equivalent results may be obtained by means of either of these 2 methods. However, the TMM was chosen and preferred, as it requires linear systems of significantly smaller ranges to be solved. Particularly, the FEM requires solving 2m equations (where m is the number of elements), while the TMM requires only z+2 equations (where z is the total number of rigid supports between the end supports of the beam). In addition, the TMM is purely analytical, implementing the solution to differential equations for beams in bending and shears. Furthermore, FEM results (solutions) are valid in the nodes only, whereas the TMM allows the user to obtain deflections, slopes, bending moments, and shear forces along the element itself on the basis of the calculated results in the nodes. The TMM also produces a system of equations that are simpler in comparison to those produced by the FEM. Consequently, the calculation model based on TMM was chosen and is further described hereafter.

2.3. Element transfer matrices and selection of initial parameters

In this section, a simplified TMM technique is reviewed to evaluate the state vectors at each cross section of a prestressed beam. The TMM uses a mixed form of the element force-displacement relationship and transfers the structural behavior parameters (state array) from one section to the other.

For calculation purposes, the beam is modeled as a system of multi-span beam elements, supported on rigid (absolute stiff) and/or linearly elastic supports (deviators). Each beam element is assumed to have a uniform cross section and all calculations are performed in the vertical plane. The basic object of the TMM is to determine the state vector (v_i in each section of the whole beam and at the supports.

Figure 3 illustrates the positive directions of the internal forces and associated displacements, i.e. the element state vector. A state vector of a beam at an arbitrary section has 4 components, which are the displacement, the rotation, the bending moment, and the shear force. These components are written, in the same order, in the following array:

$$v_i = \begin{bmatrix} w_i & \theta_i & M_i & Q_i & 1 \end{bmatrix}^T, \tag{1}$$

where w_i is the displacement or deflection components, θ_i is the slope of the element, Q_i is the internal shear force, and M_i is the internal bending moment.

Let us consider the beam element (j+1) between 2 nodes, (i) and (i+1), shown in Figure 4. The relation of the state vector (v_{i+1}) at the left side of node (i+1) and the state vector (v_i) at the right side of node (i) can be written, considering the span transfer matrix L_i, in the following form:

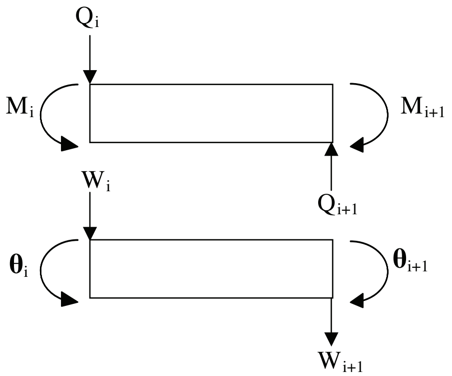

Figure 3. Positive direction of internal forces and element state vector.

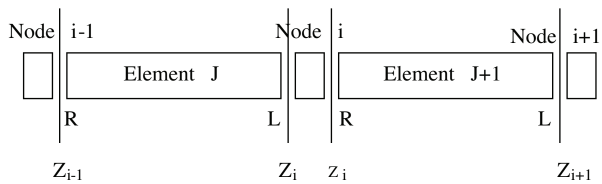

Figure 4. State vectors of beam element and nodes.

$$v_{i+1}^{left} = L_j \cdot v_i^{right} \quad . \tag{2}$$

Once more, considering the span (j+1), state vector (v_{i+1}) at the right side of node (i+1) is related to the state vector (v_{i+1}) at the left side of the same node, (i+1), using the span length transfer matrix S_i, as follows:

$$v_{i+1}^{right} = S_{i+1} \cdot v_{i+1}^{left}. \tag{3a}$$

Using Eq. (2), this becomes:

$$v_{i+1}^{right} = L_j \cdot S_{i+1} \cdot v_i^{right}. \tag{3b}$$

Hence:

$$v_{i+1}^{right} = Z_j \cdot v_i^{right}. \tag{4}$$

$Z_j = L_j \cdot S_{i+1}$, denotes the total transfer matrix of element (j), including the support at its right end. The expanded form of this matrix, with all of the loads considered, is written as follows:

$$Z_j = \begin{pmatrix} 1 & a_i & \frac{a_i^2}{2EI_i} & \frac{a_i^3}{6EI_i} - \frac{\kappa_i a_i}{GA_i} & -\frac{a_i^4}{EI_i}\left(\frac{T_i}{2} + \frac{F_i a_i}{6} + \frac{q_i a_i^2}{24}\right) + \frac{\kappa_i a_i}{GA_i}\left(F_i + \frac{q_i a_i}{2}\right) \\ & 1 & \frac{a_i}{2EI_i} & \frac{a_i^2}{2EI_i} & -\frac{a_i}{EI_i}\left(T_i + \frac{F_i a_i}{2} + \frac{q_i a_i^2}{6}\right) \\ & & 1 & & -\left(T_i + F_i a_i + \frac{q_i a_i^2}{2}\right) \\ & & & 1 & -\left(F_i + q_i a_i + R_i\right) \\ & & & & 1 \end{pmatrix}, \tag{5}$$

where a_i is the considered element length, EI_i is the element bending stiffness, GA_i is the element shear stiffness, F_i is the external concentrated force, Z_j is the total transfer matrix of any element (j), q_i is the external uniformly distributed load, κ_i is the shear form factor, and R_i is the support reaction.

In the event that there is only an external concentrated force static load acting at an arbitrary node (i) with shear stiffness κ_i ignored, Eq.(5) may take the following matrix form:

$$L_j = \begin{pmatrix} 1 & a_i & \frac{a_i^2}{2EI_i} & \frac{a_i^3}{6EI_i} & -\frac{a_i^4}{6EI_i}F_i \\ & 1 & \frac{a_i}{2EI_i} & \frac{a_i^2}{2EI_i} & -\frac{a_i}{2EI_i}F_i \\ & & 1 & & -a_i F_i \\ & & & 1 & -\left(F_i + R_i\right) \\ & & & & 1 \end{pmatrix}. \tag{6}$$

In such a case, the displacements and the moment are equal on both sides of the node, while the shear force is changed with the value of the concentrated load. The relations of the state vector at the right side of node (i) and the state vector at the left side of the same node (i) can be written in matrix form as:

$$\begin{pmatrix} w_i \\ \theta_i \\ M_i \\ Q_i \\ 1 \end{pmatrix}^{right} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -R_i \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} w_i \\ \theta_i \\ M_i \\ Q_i \\ 1 \end{pmatrix}^{left} \tag{7}$$

It should be noted that, in case there is no support at the considered node, (i), the transfer matrix S_i is obtained from Eq. (7) by taking $R_i = 0$.

The TMM scheme is accomplished by modeling the actual beam with external prestressing cables with different cross sections by dividing it into (n) beam elements with uniform values of loads and stiffnesses. The properties of each element, such as moment of inertia or modulus of elasticity, are calculated as mean values. Applying the TMM to a beam with (n) elements and (n+1) supports, the relation between the state vectors at both the utmost left and right ends is:

$$v_{n+1}^{uttermost-right} = Z_n \cdot Z_{n-1} \cdots Z_1 . v_1^{uttermost-left} \tag{8}$$

or

$$v_{n+1}^{uttermost-right} = H_i \cdot v_1^{uttermost-left}. \tag{9}$$

The effect of intermediate conditions such as rigid or elastic support can be taken into account when the overall transfer matrix is being calculated. After the transfer matrix scheme is completed and the overall transfer matrix has been computed, the boundary conditions at both utmost ends of the beam must be applied to obtain the unknown state vector elements at both ends. The initial parameters to be selected are the unknowns at the whole span's utmost left end. These are finally determined from the known parameters at the utmost right end of the whole beam, together with the reactions at the deviators. For the case of an intermediate rigid support, there is an internal unknown discontinuity (the reaction R) corresponding to this unknown, and the displacement is restrained. Applying this condition at the support, one of the initial unknowns of the state vector is eliminated and the new unknown reaction is introduced.

The whole elastic beam is described by means of a whole beam matrix (H) and a whole beam vector (b). Both of them are assembled on the basis of the boundary conditions at each support and at the rightmost end, by means of the span transfer matrices. For each span, the transfer matrices are simply matrix products of transfer matrices that relate the state vector in the section of any support (deviator elastic support or the leftmost end rigid support) to the next one, as in Eqs. (8) and (9). The displacements and forces at each node of the beam are then calculated using the matrix multiplication scheme. The calculation is performed for the different loading cases taken all together to determine the state vectors at the end of the beam.

The beam ends may be free, simply supported, or fixed. Any case can be analytically considered; however, the most common situation is that both of the ends are simply supported. In the case of a beam with simply supported ends, the unknown initial parameters are θ_o and Q_o, the beam's leftmost end slope and shear force, respectively. These parameters, together with all of the reaction forces in existing supports R_i, are determined from the known boundary conditions at the rightmost end of the beam (the total number of equations to be solved is thus (z+2) only). The vector of unknowns consists then of the 2 initial parameters (θ_o and Q_o) and of the reaction forces at the deviators (R_i), as follows:

$$b_o = \begin{pmatrix} \theta_o & Q_o & R_1 & \cdots & R_z \end{pmatrix}. \tag{10}$$

Using Eq. (9), the following relation is obtained:

$$v^{uttermost-right} = H \cdot v^{uttermost-left}, \tag{11}$$

where H is the overall transfer matrix, which can be easily calculated by multiplying the span and support transfer matrices.

In the particular case of a beam with simply supported ends, as shown in Figure 1, Eq. (11) can be written, in the expanded form, as follows:

$$v_5 = H \cdot b_o + (H_1 + H_2 + H_3 + H_4 + H_5) \cdot b, \tag{12}$$

where:

$$H_1 = L_1, \tag{13a}$$

$$H_2 = L_1 \cdot S_B \cdot L_2, \tag{13b}$$

$$H_3 = L_1 \cdot S_B \cdot L_2 \cdot S_C \cdot L_3 \cdot S_D \cdot L_3, \tag{13c}$$

$$H_4 = L_1 \cdot S_B \cdot L_2 \cdot S_C \cdot L_3 \cdot S_D \cdot L_3 \cdot S_E, \tag{13d}$$

$$H_5 = L_1 \cdot S_B \cdot L_2 \cdot S_C \cdot L_3 \cdot S_D \cdot L_3 \cdot S_E \cdot L_4, \tag{13e}$$

and

$$H = L_1 \cdot S_B \cdot L_2 \cdot S_C \cdot L_3 \cdot S_D \cdot L_3 \cdot S_E \cdot L_4 \cdot S_F \cdot L_5. \tag{13f}$$

Once the components of vector b_o are known, the state vectors in each section of the beam may be easily found by a simple matrix multiplication, beginning from the known state vector at the leftmost end of the beam. This is the beam bending and shear response in terms of deflection, slope, bending moment, and shear force at both ends of each element.

2.4. Linear elastic support reaction

In a concrete beam under flexure, the strain induced at every cross section varies according to the bending moment diagram. In the case of a beam prestressed with external cables, the unbounded cable freely moves in a relative change of the beam deformation, under external loadings. Therefore, the cable strain is basically different from the concrete strain at every cross section and, as stated above, the deflection of the external cable does not follow the beam deflection, except at the deviator points as the beam is deformed. One of the major difficulties concerning beams prestressed with external cables is in calculating the cable strain. In this section, an analytical method based on the deformation compatibility and friction at the deviators is reviewed (Aparicio, 2002; Harajli, 2002).

In Figure 5, F_i and F_{i+1} are tensile forces in the cable segments (i) and (i+1) at any deviator and θ_i and θ_{i+1} are cable angles, respectively. Thus, the force equilibrium in the horizontal direction, taking into account the friction effect (Aparicio, 2002), can be expressed as:

$$F_{i+1} \cdot \cos\theta_{i+1} = F_i \cdot \cos\theta_i + (-1)^{\lambda_i} \cdot \mu \cdot (F_i \cdot \sin\theta_i + F_{i+1} \cdot \sin\theta_{i+1}), \tag{14}$$

where μ is the friction factor at the deviator, and λ_i is the coefficient that depends on the slipping direction and has a value of $\lambda_i = 1$ if $F_i \cdot \cos\theta_i > F_{i+1} \cdot \cos\theta_{i+1}$, and $\lambda_i = 2$ if $F_i \cdot \cos\theta_i < F_{i+1} \cdot \cos\theta_{i+1}$.

Eq. (18) can be rewritten in terms of increments of tensile forces, where ΔF_i and ΔF_{i+1} are the increments of tensile forces at both sides of the deviator:

$$\Delta F_{i+1} \cdot \cos\theta_{i+1} = \Delta F_i \cdot \cos\theta_i + (-1)^{\lambda_i} \cdot \mu \cdot (\Delta F_i \cdot \sin\theta_i + \Delta F_{i+1} \cdot \sin\theta_{i+1}). \tag{15}$$

At this stage, it should be pointed out that the friction between the cable and the deviator is expressed by the coefficient μ, the actual value of which depends on many factors and can only be determined by experimental investigations. It should also be kept in mind that friction coefficients are not easy to find in the literature from a qualitative point of view. The available values cannot be true and no comparison could be made with any experimental results; for the purpose of simplicity, many of the previous studies (Pisani, 1996; Aparicio et al. 2002) considered only 2 extreme cases, namely free slip and perfectly fixed.

Since the stress in an external cable usually remains below the elastic limit up to the failure of the beam, it is possible to rewrite the force equilibrium at the deviator in terms of cable strain by dividing both sides of Eqs. (14) and (15) by EA, where E and A are the elastic modulus and the area of the cable, and $\Delta\varepsilon_i$ and $\Delta\varepsilon_{i+1}$ are the cable strains at both sides of the deviator. The force equilibrium can then be expressed as:

$$\Delta\varepsilon_{i+1} \cdot \cos\theta_{i+1} = \Delta\varepsilon_i \cdot \cos\theta_i + (-1)^{\lambda_i} \cdot \mu \cdot (\Delta\varepsilon_i \cdot \sin\theta_i + \Delta\varepsilon_{i+1} \cdot \sin\theta_{i+1}). \tag{16}$$

As stated above, the strain induced at every cross section of a concrete beam under flexure varies according to the bending moment diagram and is uniform over the length of the cable segment between any 2 successive deviators or anchorage ends. Furthermore, the cable friction obviously exists at the deviator points, resulting in a different level of strain increase between the 2 successive cable segments.

Since the elongation of an external cable is assumed to be uniform over the length of the cable segment between any 2 successive deviators or anchorage ends, Eq. (16) can alternatively be written in terms of cable displacements. Therefore, using Eq. (16), one can analytically obtain the cable displacements of each segment. The mathematical expression of the displacement at any arbitrary deviator can be expressed as:

$$l_{i+1} \cdot \varepsilon_{i+1} \cdot \cos\theta_{i+1} = l_i \cdot \varepsilon_i \cdot \cos\theta_i + (-1)^{\lambda_i} \cdot \mu \cdot (l_i \cdot \varepsilon_i \cdot \sin\theta_i + l_{i+1} \cdot \varepsilon_{i+1} \cdot \sin\theta_{i+1}) \tag{17}$$

or

$$\delta_{i+1} \cdot \cos\theta_{i+1} = \delta_i + (-1)^{\lambda_i} \cdot \mu \cdot (\delta_i + \delta_{i+1} \cdot \sin\theta_{i+1}). \tag{18}$$

It can be seen from Eq. (18) that the displacement variation within one external cable element depends mainly upon the friction at the deviator and cable angle.

Figure 6 shows the vertical force and displacement components in one external cable element. The vertical force and displacement components of the inclined cable are defined, respectively, as follows:

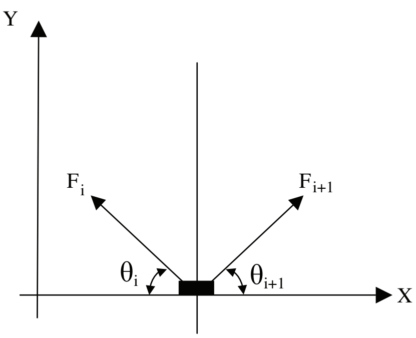

Figure 5. Force equilibrium at a deviator.

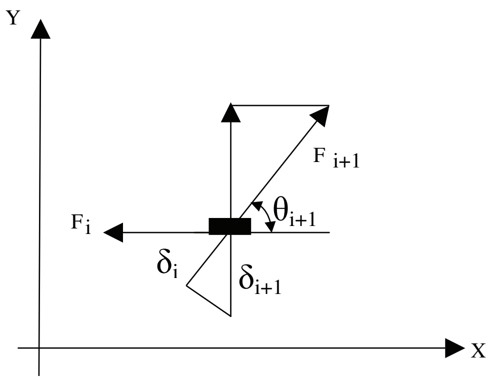

Figure 6. Force and displacement components in an external cable.

$$R_i = F_{i+1} \cdot \sin\theta_{i+1}, \tag{19}$$

$$v_{i+1} = \frac{\delta_{i+1}}{\sin\theta_{i+1}}. \tag{20}$$

By definition, the stiffness is the ratio of a force divided by the corresponding displacement, taken in the same direction. Hence, the elastic spring (cable) constant at the deviator point of the inclined cable segment is defined as follows:

$$k_{i+1} = -\frac{F_{i+1}}{\delta_{i+1}} \sin^2\theta_{i+1}. \tag{21}$$

The minus sign in Eq. (21) is introduced as the reaction acts naturally in the opposite direction of the applied force. The matrix transfer for a linear elastic support is then written in the following form:

$$Z_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ k_i & 0 & 0 & 1 \end{bmatrix}. \tag{22}$$

The vertical reaction at an arbitrary linear elastic support (deviator point) is proportional to the longitudinal deformation (elongation) and the friction effect, if any. It is computed using the following relation:

$$R_i = k_i \cdot v_i. \tag{23}$$

3. Validity of the proposed method and numerical results

To evaluate the performance of the method, static bending moments and shear forces of 2 simply supported beams, subjected to unit moving loads, were first calculated and the accuracy of the static solutions obtained by the present method was examined by comparing the issued results with those obtained by the analytical beam method. Verification of the model against experimental data is expected to be the matter of further work, as the implementation of the calculation procedure is in constant progress.

To examine the accuracy of the method, analysis was carried out on a beam without prestressed reinforcements. A layout scheme of the simply supported beam, cross section, and loading arrangements are shown in Figure 7. For the unit moving load, 5 loading points were considered. The locations of the unit moving loads provided were at distances that varied from 0 to 28,000 mm with an increment of 7000 mm, starting from the left support. The results obtained by the proposed static solution transfer matrix method and those of analytical calculations, in terms of static bending moments and shear forces, are shown in Tables 1-5. All results are given in absolute values, as the signs of the bending moments or the shear forces may change. The bending moment and shear force values were both computed using numerical and analytical methods. Numerical and theoretical values with respective ratios are reported in each Table. As one can observe, columns 2 to 4 report the bending moments, and columns 5 to 7 those of shear forces. One can also observe the absence of bending moment values as the unit point load is applied to left and right supports, respectively, and for which cases the bending moment cannot be developed. However, the results show that the numerical bending moments and shear forces are equal to or very close to the values of the analytical method.

Table 1. Bending moments and shear forces for a beam without prestressing cables.

Unit Load $P = 1$ at A	M $_{nu}$ (kNm)	M_{th}	M_{nu}/M_{th} (kNm)	Q_{nu} (kN)	Q_{th} (kN)	Q_{nu}/Q_{th}
A	0.00	0.00	-	1.00	1.00	1.00
B	0.00	0.00	-	0.75	0.75	1.00
C	0.00	0.00	-	0.50	0.50	1.00
D	0.00	0.00	-	0.25	0.25	1.00
E	0.00	0.00	-	0.00	0.00	-

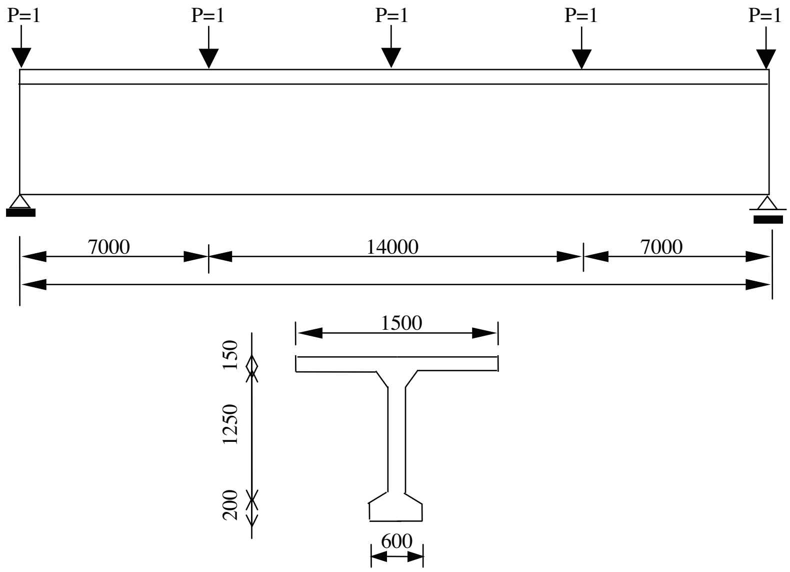

Figure 7. Layout scheme of a beam without externally prestressed cables.

Table 2. Bending moments and shear forces for a beam without prestressing cables.

Unit Load $P = 1$ at A	M $_{nu}$ (kNm)	M_{th}	M_{nu}/M_{th} (kNm)	Q_{nu} (kN)	Q_{th} (kN)	Q_{nu}/Q_{th}
A	0.00	0.00	-	0.00	0.00	-
B	4.35	4.50	0.97	1.00	1.00	1.00
C	2.90	3.00	0.97	0.50	0.50	1.00
D	1.45	1.50	0.97	0.25	0.25	1.00
E	0.00	0.00	-	0.00	0.00	-

Table 3. Bending moments and shear forces for a beam without prestressing cables.

Unit Load $P = 1$ at A	M $_{nu}$ (kNm)	M_{th}	M_{nu}/M_{th} (kNm)	Q_{nu} (kN)	Q_{th} (kN)	Q_{nu}/Q_{th}
A	0.00	0.00	-	0.00	0.00	-
B	2.98	3.00	0.99	0.22	0.25	0.88
C	5.95	6.00	0.99	1.00	1.00	1.00
D	2.98	3.00	0.99	0.27	0.25	1.08
E	0.00	0.00	-	0.00	0.00	-

Table 4. Bending moments and shear forces for a beam without prestressing cables.

Unit Load $P = 1$ at A	M $_{nu}$ (kNm)	M_{th}	M_{nu}/M_{th} (kNm)	Q_{nu} (kN)	Q_{th} (kN)	Q_{nu}/Q_{th}
A	0.00	0.00	-	0.00	0.00	-
B	1.45	1.50	0.97	0.25	0.25	1.00
C	2.91	3.00	0.97	0.50	0.50	1.00
D	4.35	4.50	0.97	1.00	1.00	1.00
E	0.00	0.00	-	0.00	0.00	-

Table 5. Bending moments and shear forces for a beam without prestressing cables.

Unit Load $P = 1$ at A	M $_{nu}$ (kNm)	M_{th}	M_{nu}/M_{th} (kNm)	Q_{nu} (kN)	Q_{th} (kN)	Q_{nu}/Q_{th}
A	0.00	0.00	-	0.00	0.00	-
B	0.00	0.00	-	0.25	0.25	1.00
C	0.00	0.00	-	0.50	0.50	1.00
E	0.00	0.00	-	0.75	0.75	1.00
D	0.00	0.00	-	1.00	1.00	1.00

The accuracy of the proposed method was also verified by computing the bending moments and shear forces of a simply supported beam with external prestressing cables and 5 moving loads. The dimension, span length, and loading arrangement are shown in Figure 8. At the distance of 8000 mm from each other and symmetrically located about the mid-span section, 4 deviators were provided. Numerical and theoretical values for bending moments and shear forces are reported in Tables 6 and 7, respectively. Values of bending moments and shear forces are indicated for each unit point load and at every support along the prestressed beam.

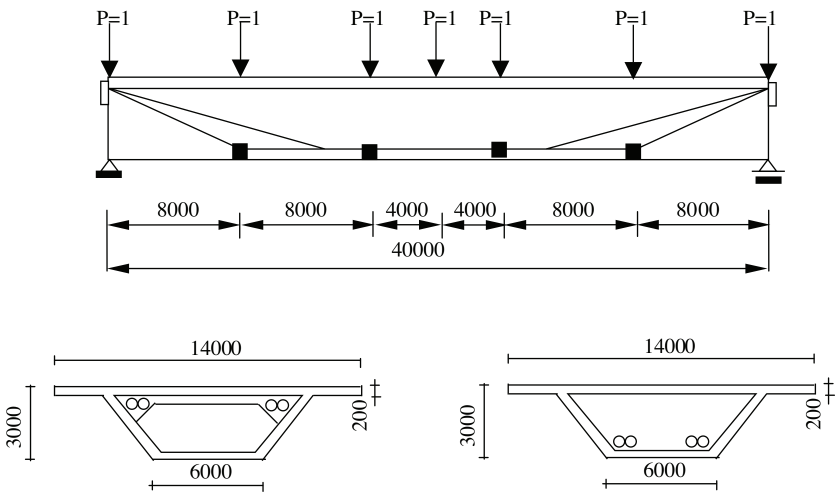

Figure 8. Layout scheme of a beam with externally prestressed cables.

Table 6. Numerical values of bending moments for a beam with prestressing cables.

Unit Load Locatins	${}^{A}Q_{nu}$ (kN)	${}^{B}Q_{nu}$ (kN)	${}^{C}Q_{nu}$ (kN)	${}^{D}Q_{nu}$ (kN)	${}^{E}Q_{nu}$ (kN)	${}^{F}Q_{nu}$ (kN)	${}^{G}Q_{nu}$ (kN)
P=1 at A	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P=1 at B	0.00	6.40	4.82	4.02	3.23	1.64	0.00
P=1 at C	0.00	4.80	9.62	8.02	6.42	3.23	0.00
P=1 at D	0.00	4.00	8.01	10.02	8.02	4.03	0.00
P=1 at E	0.00	3.21	6.41	8.00	9.61	4.81	0.00
P=1 at F	0.00	1.60	3.21	4.05	4.81	6.40	0.00
P=1 at G	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 7. Numerical values of shear forces for a beam with prestressing cables.

Unit Load Locatins	${}^{A}Q_{nu}$ (kN)	${}^{B}Q_{nu}$ (kN)	${}^{C}Q_{nu}$ (kN)	${}^{D}Q_{nu}$ (kN)	${}^{E}Q_{nu}$ (kN)	${}^{F}Q_{nu}$ (kN)	${}^{G}Q_{nu}$ (kN)
P=1 at A	1.00	0.83	0.67	0.50	0.33	0.17	0.00
P=1 at B	0.00	0.99	0.60	0.45	0.30	0.18	0.00
P=1 at C	0.00	0.19	0.99	0.45	0.35	0.15	0.00
P=1 at D	0.00	0.12	0.38	0.99	0.45	0.25	0.00
P=1 at E	0.00	0.19	0.32	0.40	1.00	0.35	0.00
P=1 at F	0.00	0.05	3.21	4.05	4.81	6.40	0.00
P=1 at G	0.00	0.17	0.33	0.50	0.67	0.83	1.00

Finally, as the presented method is proved to be reliable, it can be used to generate information on some aspects relative to the characteristic responses of the concrete beam with external prestressing cables and linear elastic supports, such as static load versus displacement, bending moments versus displacement, and stress versus mid-span displacement relationships, for the sake of analytical purposes, taking into account various conditions and parameters such as the locations of the unit moving loads, the contribution of linear elastic supports at deviators, and friction effects.

The computed results of the beam in terms of loads versus displacements and bending moments versus displacement relationships are shown in Figures 9 and 10, respectively. One can observe that all beams behave essentially in the same manner for the different locations of the unit loads. Figure 11 shows the computed results of the beam in terms of increase of cable loads versus mid-span displacements. One can also observe that the increase of cable load exhibited essentially similar curves. This means that the stress in the external cables increased by a small amount in the range of analysis considered. It is also interesting to point out, by comparing the curves of loads versus displacements (Figure 9) and the increase of cable load versus mid-span displacements (Figure 10), that the 2 curves are very similar in shape, indicating the close relationship between the deflections and the load increase in the external cables. Therefore, close relationships can then be drawn by a linear relationship in terms of load variations in the external cable versus the mid-span deflection responses.

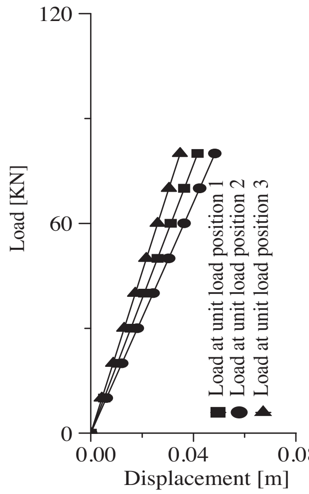

Figure 9. Load versus mid-span displacement of the externally prestressed beam.

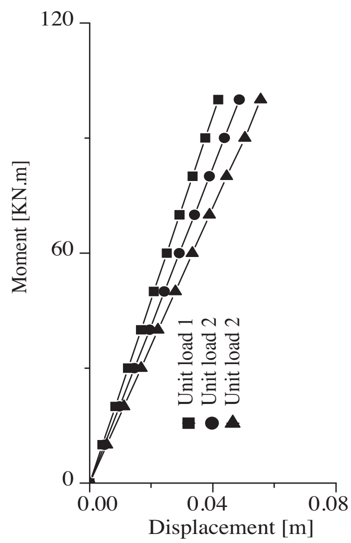

Figure 10. Moment versus mid-span displacement of the externally prestressed beam.

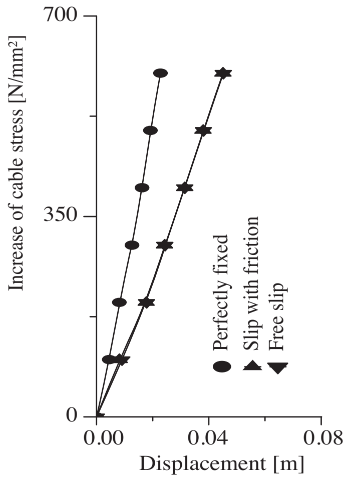

Figure 11. Increase of cable stress versus mid-span displacement.

In the range of analysis, which generally corresponds to the conditions under service load, the displacements of the beam are very small and induce a small amount of tensile load in each external cable, leading inevitably to an extremely small unbalanced force at the deviator location. As a result, the cable slip generally cannot occur at this stage. That is, the friction at the deviators has a negligible effect on the deflection responses. The beam deflection responses with consideration of free slip, slip with friction of 0.20, and perfectly fixed states could be more or less identical.

4. Conclusion

This paper described details of a calculation procedure promoting the advantage of the "somewhat forgotten" transfer matrix method. The method was used to study the static behavior of a concrete beam with external prestressing cables subjected to a unit moving load. Computed results of bending moments and shear forces were presented. Comparisons between computed numerical and analytical theory results highlighted the effectiveness and the degree of accuracy of the proposed model. Further results related to loads versus displacements, bending moment versus displacements, and increase of cable load versus mid-span displacement relationships were also analyzed. However, verifications of the model against experimental data were not made. A proposal of this kind is expected to be the matter of a forthcoming paper. It is to be pointed out once again that this paper presented only the basic information related to concrete beams with external prestressing cables computations, in order to understand the usefulness of the proposed method.

Nomenclature

a_i	element length [m]
F_e	external concentrated force [N]
EI_i	element bending stiffness [Nm^2]
GA_i	element shear stiffness [N]
F_i	external concentrated force [N]
L_j	total transfer matrix of the element (j)
M_e	external concentrated moment [Nm]
M_i	internal bending moment [Nm]
q_e	external uniformly distributed load [N/m]
q_i	external uniformly distributed load [N/m]
Q_i	internal shear force [N]
R_i	support reaction [N]
S_j	support transfer matrix of the element (j)
T_i	external concentrated bending moment [Nm]
Z_i	extended support transfer matrix
w_i	displacement or deflection components [m]
κ_i	shear form factor
θ_i	slope of the element [m/m]

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