

Effect of electric field on electromechanical response of a functionally graded piezoelectric material hollow cylinder

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Abstract: In this study, a general solution is developed to obtain the effect of nonaxisymmetric electric potential on a functionally graded piezoelectric material (FGPM) hollow cylinder. The material properties along the thickness of the cylinder are assumed to vary with the power law function. The solution technique is based on the complex Fourier series, and by using the separation of variables method the Navier equation is solved and the exact solutions for electric displacements and stresses in the FGPM hollow cylinder are obtained. Moreover, the effects of electric potential on the stresses and displacements of the FGPM hollow cylinder are investigated. The results show that with the specific electric potential, the mechanical and electrical properties of the FGPM hollow cylinder can be controlled.

Key words: Two-dimensional, electric field, piezoelectric, functionally graded materials, cylinder

1. Introduction

Functionally graded piezoelectric material (FGPM) is a kind of piezoelectric material with material composition and properties varying continuously along certain directions. This property makes FGPM suitable for many specific applications, such as actuators and sensors, and for use in smart structures. Jabbari et al. (2003) depicted an analytical solution for thermal and mechanical stresses in a functionally graded material hollow cylinder due to 2-dimensional steady-state loads. They used the complex Fourier series to solve the Navier equation and heat conduction. By using the Laplace transforms, Tianhu et al. (2004) depicted a nonaxisymmetric thermal shock problem for a half-space in the thermomagnetomechanical behavior of half-space geometries. By using the finite difference methods, the magnetothermomechanical response of multilayered conical shells subjected to magnetic and vapor fields was investigated by Lee (2009). The effect of Lorentz force on the 2-dimensional thermomechanical behavior of a functionally graded hollow cylinder was investigated by Fesharaki et al. (2011). Asghari and Ghafoori (2010) depicted the elasticity solution for 3-dimensional functionally graded rotating disks. Based on the 3-dimensional equations of mechanical and piezoelectricity, Chen et al. (2002) showed the solution of a piezoceramic hollow sphere analytically. Their numerical results were calculated for different boundary conditions imposed on the spherical surfaces. Shi and Chen (2004) investigated the general solution for a cantilever beam made of FGPM subjected to different loadings. They proposed and determined a pair of stress and induction functions in the form of polynomials. Moreover, based on these functions, they showed a set of general solutions for the beam under various loadings. Ootao and Tanigawa (2007) investigated the transient piezothermomechanical behavior of a functionally graded piezoelectric hollow sphere subjected to

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thermal, mechanical, and electric fields. They considered the thermoelastic and piezoelectric property parameters of a hollow sphere as power functions of the radial coordinate and depicted some numerical results for change in temperature, stresses, displacement, and electric potential distributions. Jafari Fesharaki et al. (2012) investigated the 2-dimensional solution for the electromechanical behavior of a functionally graded piezoelectric hollow cylinder. They used the Fourier series to solve the problem for a range of various boundary conditions.

In the present paper, a functionally graded piezoelectric hollow cylinder subjected to 2-dimensional mechanical load and electric field is considered. By using the complex Fourier series, the Navier equations for mechanical and electric potential are derived and solved. The effect of electric potential on the electromechanical behavior of the functionally graded piezoelectric hollow cylinder is then investigated.

2. Formulation of the problem

Consider a thick hollow cylinder made of FGPM subjected to 2-dimensional mechanical and electric fields as shown in Figure 1. The components of stresses and mechanical and electrical displacements in cylindrical coordinates (r, θ, z) are respectively expressed as:

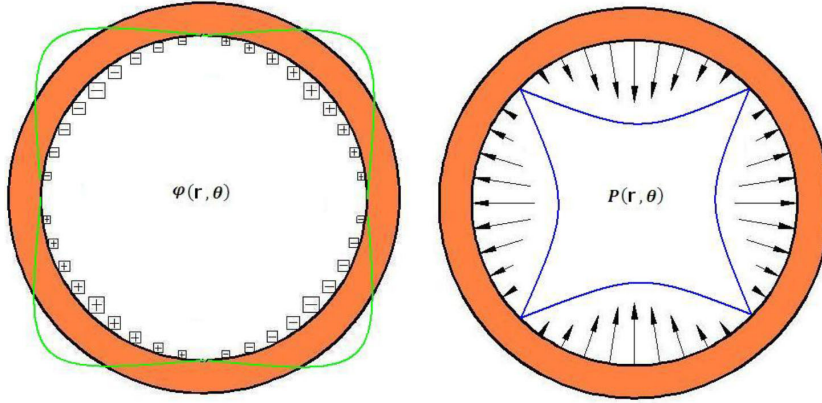


Figure 1. FGPM hollow cylinder due to 2-dimensional electromechanical loads.

$$\sigma_{rr} = q_{11} \frac{\partial U(r, \theta)}{\partial r} + q_{12} \left(\frac{U(r, \theta)}{r} + \frac{1}{r} \frac{\partial V(r, \theta)}{\partial \theta} \right) + e_{11} \frac{\partial \phi}{\partial r}. \quad (1)$$

$$\sigma_{\theta\theta} = q_{12} \frac{\partial U(r, \theta)}{\partial r} + q_{11} \left(\frac{U(r, \theta)}{r} + \frac{1}{r} \frac{\partial V(r, \theta)}{\partial \theta} \right) + e_{21} \frac{\partial \phi}{\partial r}. \quad (2)$$

$$\sigma_{r\theta} = q_{22} \left(\frac{1}{r} \frac{\partial U(r, \theta)}{\partial \theta} - \frac{V(r, \theta)}{r} + \frac{\partial V(r, \theta)}{\partial r} \right) + e_{31} \frac{1}{r} \frac{\partial \phi}{\partial \theta}. \quad (3)$$

$$D_r = e_{11} \frac{\partial U(r, \theta)}{\partial r} + e_{21} \left(\frac{U(r, \theta)}{r} + \frac{1}{r} \frac{\partial V(r, \theta)}{\partial \theta} \right) - q_{41} \frac{\partial \phi}{\partial r}, \quad (4)$$

$$D_\theta = e_{31} \left(\frac{1}{r} \frac{\partial U(r, \theta)}{\partial \theta} - \frac{V(r, \theta)}{r} + \frac{\partial V(r, \theta)}{\partial r} \right) - q_{51} \frac{1}{r} \frac{\partial \phi}{\partial \theta}, \quad (5)$$

where $\sigma_{ij}(i, j = r, \theta)$ are the stresses and ϕ is the electric potential. q_{ij} and e_{ij} ($i, j = 1, 2, 3$) are elastic and piezoelectric coefficients, respectively. $D_i(i = r, \theta)$ and $q_{i1}(i = 4, 5)$ are the electric displacements and dielectric

constants, respectively. The properties of the material, except for the Poisson ratio, are assumed to be varied along the thickness of the cylinder by a power law function in the radius direction as shown below.

$$\mu = \mu_0 \cdot r^\beta \quad (6)$$

Here, μ is the material properties, β is the power law index, and the subscripted 0 denotes the corresponding values of material at the outer surface of the FGPM hollow cylinder. U and V are the radial and circumferential displacements along the thickness of the cylinder, respectively.

The 2-dimensional equilibrium equations in the radial and circumferential directions and the charge equation of electrostatics are expressed as:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \quad (7)$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{2\sigma_{r\theta}}{r} = 0 \quad (8)$$

$$\frac{\partial D_r}{\partial r} + \frac{1}{r} \frac{\partial D_\theta}{\partial \theta} + \frac{D_r}{r} = 0. \quad (9)$$

Substituting Eqs. (1) through (5) into 2 mechanical equilibrium equations and the charge equation of electrostatics in cylindrical coordinates given in Eqs. (7) through (9) yields the following 3 Navier equations:

$$\frac{\partial^2 U}{\partial r^2} + \frac{g_1}{r} \frac{\partial V}{\partial r} + \frac{g_2}{r^2} U + \frac{g_3}{r^2} \frac{\partial^2 U}{\partial \theta^2} + \frac{g_4}{r} \frac{\partial^2 V}{\partial r \partial \theta} + \frac{g_5}{r^2} \frac{\partial V}{\partial \theta} + g_6 \frac{\partial^2 \phi}{\partial r^2} + \frac{g_7}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{g_8}{r} \frac{\partial \phi}{\partial r} = 0 \quad (10)$$

$$\frac{\partial^2 V}{\partial r^2} + \frac{g_9}{r} \frac{\partial V}{\partial r} - \frac{g_9}{r^2} V + \frac{g_{10}}{r^2} \frac{\partial^2 V}{\partial \theta^2} + \frac{g_{11}}{r} \frac{\partial^2 U}{\partial r \partial \theta} + \frac{g_{12}}{r^2} \frac{\partial U}{\partial \theta} + \frac{g_{13}}{r^2} \frac{\partial \phi}{\partial \theta} + \frac{g_{14}}{r} \frac{\partial^2 \phi}{\partial r \partial \theta} = 0 \quad (11)$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{g_{15}}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{g_9}{r} \frac{\partial \phi}{\partial r} + g_{16} \frac{\partial^2 U}{\partial r^2} + \frac{g_{17}}{r} \frac{\partial U}{\partial r} + \frac{g_{18}}{r^2} U + \frac{g_{19}}{r^2} \frac{\partial^2 U}{\partial \theta^2} + \frac{g_{20}}{r} \frac{\partial^2 V}{\partial r \partial \theta} + \frac{g_{21}}{r^2} \frac{\partial V}{\partial \theta} = 0. \quad (12)$$

Here, the newly defined coefficients g in Eqs. (10) through (12) are the constants along the radial direction.

3. Solving the problem

Eqs. (10) through (12) must be solved simultaneously, and for this purpose, the complex Fourier series are assumed for mechanical displacement components U and V and electric potential ϕ as follows below.

$$U(r, \theta) = \sum_{n=-\infty}^{\infty} u_n(r) e^{in\theta} \quad (13)$$

$$V(r, \theta) = \sum_{n=-\infty}^{\infty} v_n(r) e^{in\theta} \quad (14)$$

$$\phi(r, \theta) = \sum_{n=-\infty}^{\infty} \phi_n(r) e^{in\theta} \quad (15)$$

Here, $u_n(r)$, $v_n(r)$, and $\phi_n(r)$ are the coefficients of the Fourier series. Substituting Eqs. (13) through (15) into Eqs. (10) through (12) and considering the boundary conditions yields a system of equations that has only the general solution.

The boundary conditions can be expanded in the complex Fourier series as given below.

$$H(r, \theta) = \sum_{n=-\infty}^{\infty} h_n(r) e^{in\theta} \quad (16)$$

Here, $h_n(r)$ is the coefficient of the Fourier series for boundary conditions. These boundary conditions may be either the given pressures or the displacements at the inner or outer surface of the FGPM hollow cylinder.

4. Results and electric field effects

Consider a thick functionally graded piezoelectric hollow cylinder with inner radius $a = 0.7$ m and outer radius $b = 1$ m. Consider the PZT-4 for material properties; the material constants were reported by Alashti and Khorsand (2011). The inside boundary is traction-free and the outside boundary as assumed to be fixed as:

$$U(1, \theta) = V(1, \theta) = 0. \quad (17)$$

To investigate the electric field effects on the response of the piezoelectric hollow cylinder, consider 2 different cases for boundary conditions:

$$\text{Case1} : \sigma_{rr}(0.7, \theta) = 0, \sigma_{rr}(1, \theta) = 0, \phi(0.7, \theta) = 30 \cos 2\theta, \phi(1, \theta) = 0, \quad (18)$$

$$\text{Case2} : \sigma_{rr}(0.7, \theta) = 0, \sigma_{rr}(1, \theta) = 0, \phi(0.7, \theta) = 60 \cos 2\theta, \phi(1, \theta) = 0, \quad (19)$$

Considering the power of material inhomogeneity β in Eq. (6) to be equal to 2, Figures 2 and 3 show the radial and circumferential mechanical displacements in the functionally graded piezoelectric hollow cylinder. It can be seen that the boundary conditions are satisfied and the electric potential has a significant effect on increasing and decreasing the radial and circumferential displacements.

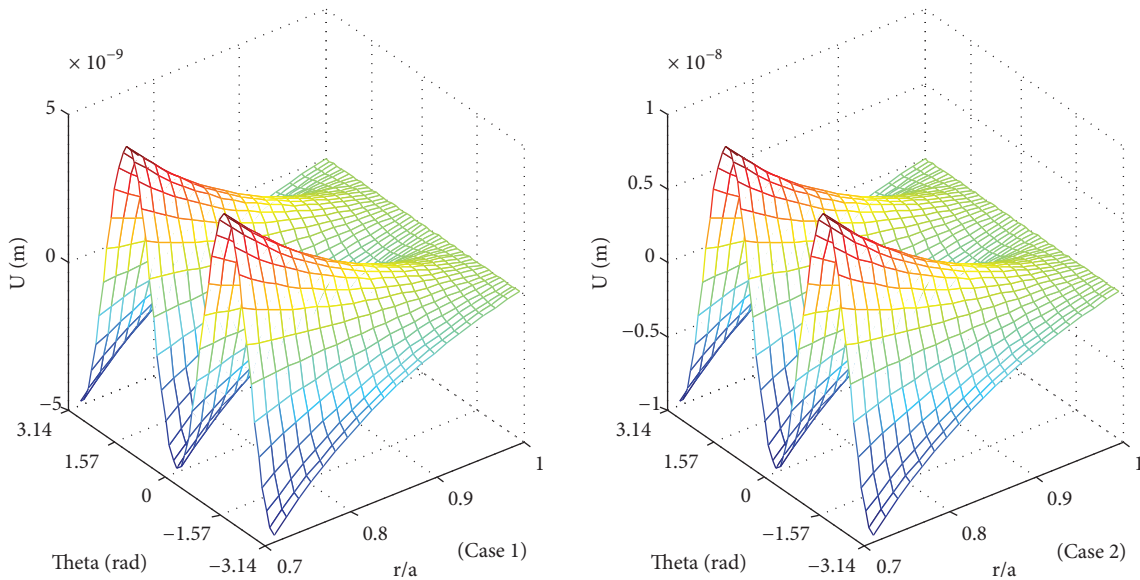


Figure 2. Radial displacements in FGPM hollow cylinder for cases 1 and 2.

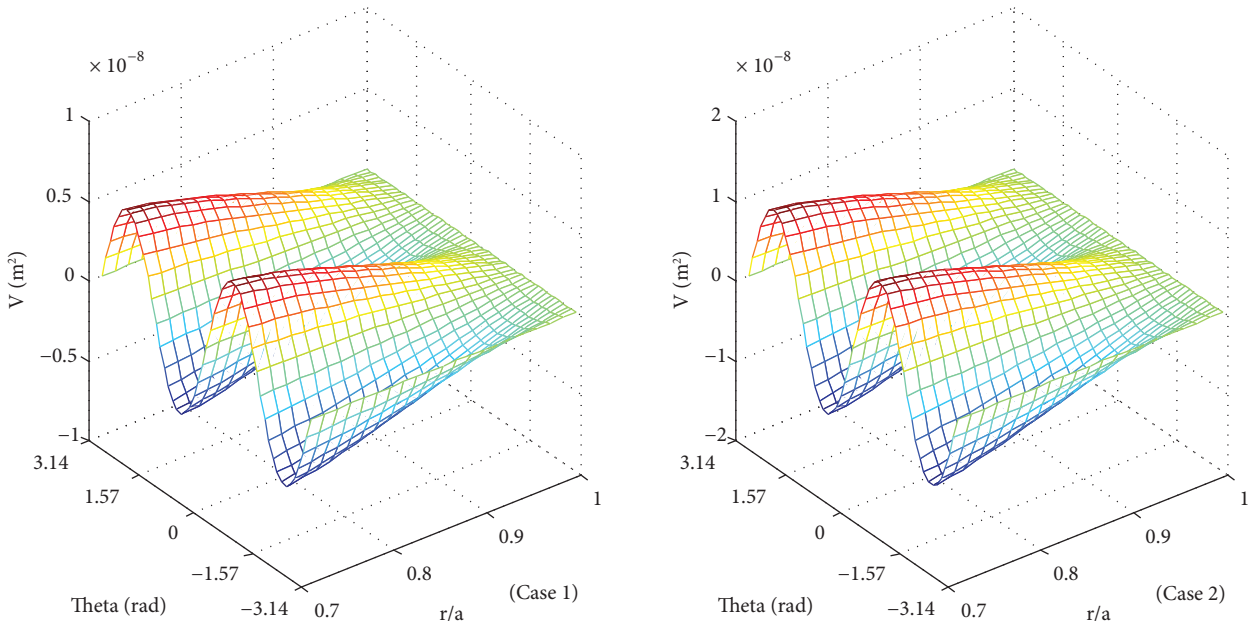


Figure 3. Circumferential displacements in FGPM hollow cylinder for cases 1 and 2.

Figures 4 and 5 show the radial and circumferential electric displacement distributions along the thickness of the functionally graded piezoelectric hollow cylinder for cases 1 and 2. It can be seen that the displacement distributions change with the changing of the electric potential and follow the pattern of the sinusoidal distribution at the inner surface of the cylinder. By concentrating on Figures 2–5, it can be seen that due to the given boundary condition at the outer surface of the cylinder, the mechanical and electrical displacements have zero value, and these values show that the outer surface of the cylinder is fixed.

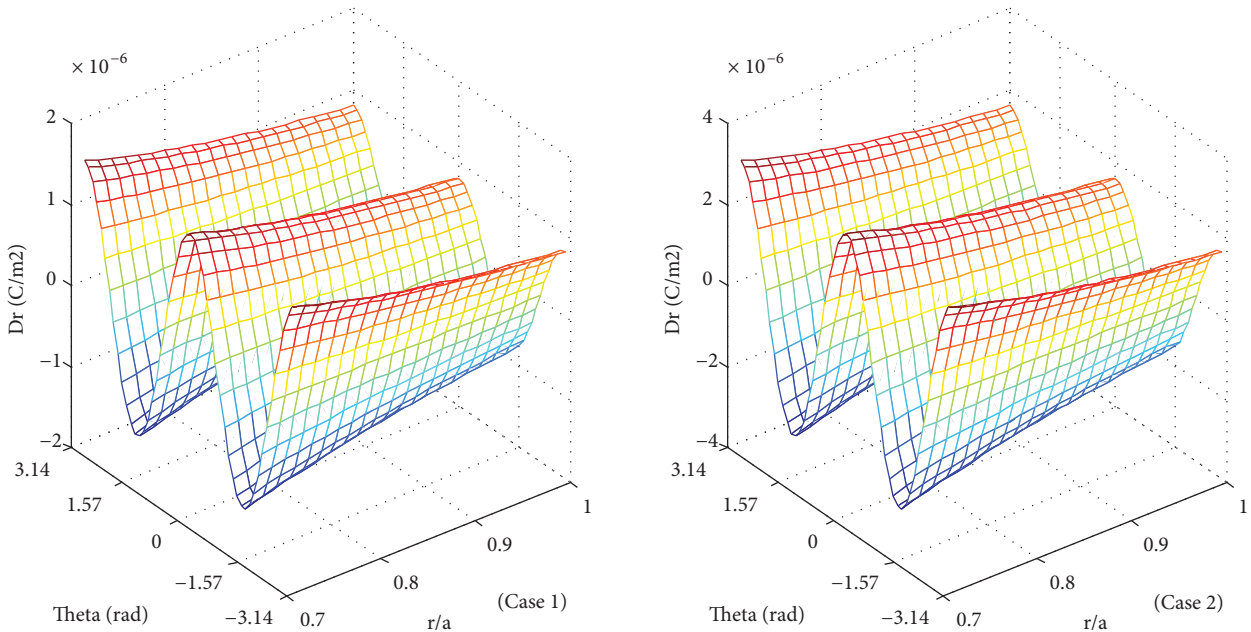


Figure 4. Radial electric displacements in FGPM hollow cylinder for cases 1 and 2.

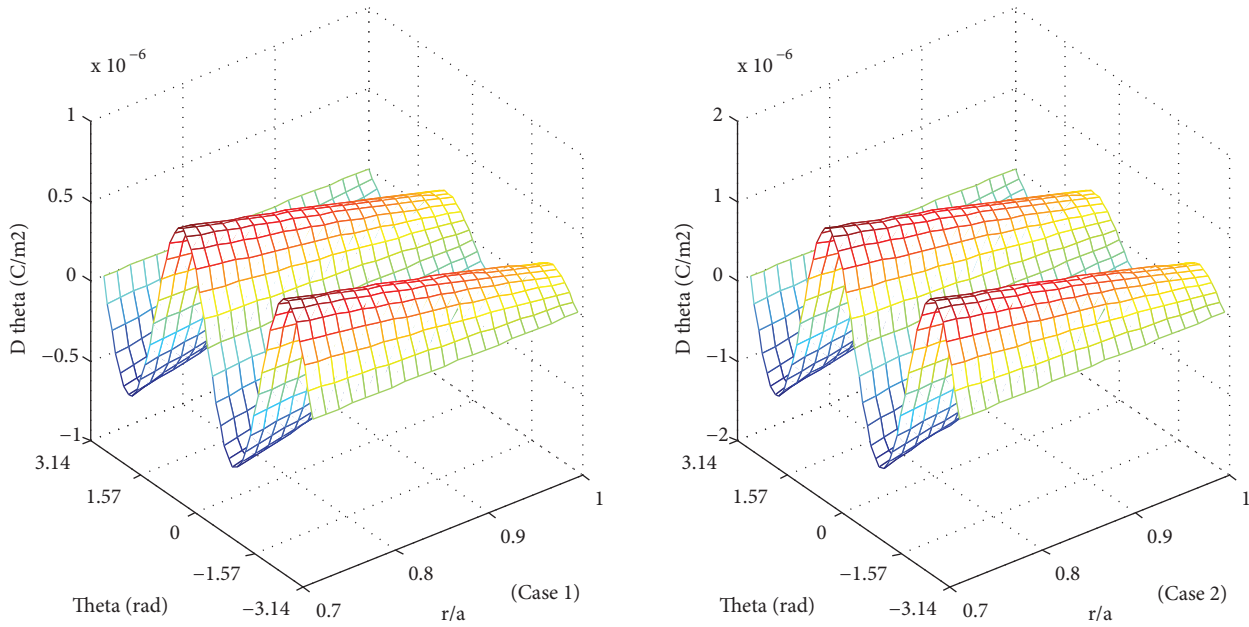


Figure 5. Circumferential electric displacements in FGPM hollow cylinder for cases 1 and 2.

Figure 6 shows the distributions of electric potential in the thickness of the piezoelectric hollow cylinder that satisfies the given boundary conditions. It can be seen that the electric potential satisfied the given boundary conditions, and it changed sinusoidally at the inner surface of the cylinder due to the boundary conditions and decreased along the radial direction of the cylinder.

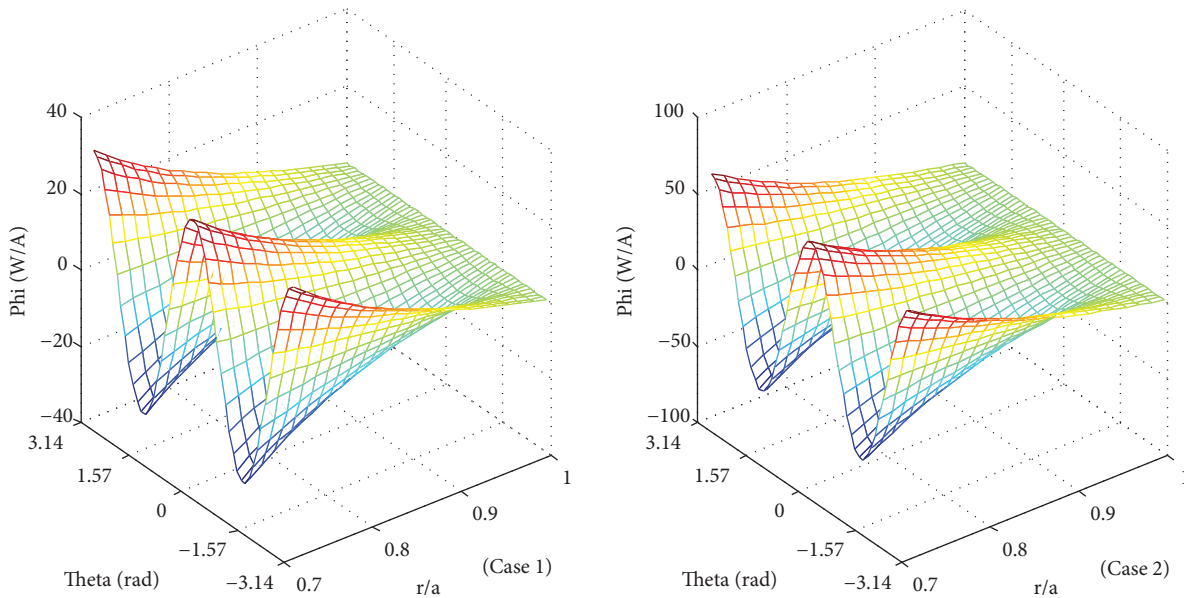


Figure 6. Electric potential in FGPM hollow cylinder for cases 1 and 2.

5. Conclusion

In this paper, the effect of electric potential on the nonaxisymmetric electromechanical behavior of a functionally graded piezoelectric hollow cylinder was obtained. The material properties along the thickness of the cylinder

were assumed to vary with the power law function. First, by using the Fourier series, the technique of the solution was developed. Next, for investigation of the effect of electric potential on the behavior of a functionally graded piezoelectric hollow cylinder, 2 examples were solved. The results show that with the specific electric potential, the mechanical and electrical property of the FGPM hollow cylinder can be controlled.

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