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# Investigation of momentum and kinetic energy correction coefficients in asymmetric compound cross-section flumes 

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#### Abstract

A series of laboratory experiments was performed in order to investigate momentum and kinetic energy correction coefficients in asymmetric rectangular compound cross-section channels. The Reynolds number varied between $14,348.9$ and $54,868.2$, as relative depth $\left(\mathrm{Y}_{r}\right)$ changed from 0.155 to 0.825 . Kinetic energy and momentum correction coefficients, $\alpha$ and $\beta$, were computed for 9 different models. As a result, the values of $\alpha$ and $\beta$ considering 106 data points related to 9 distinct cross-sections averaged 1.1525 and 1.1261, respectively.


Key words: Kinetic energy correction coefficient, momentum correction coefficient, open channels, compound crosssection, relative depth

## 1. Introduction

In open channel flow, normally the velocity distributions are not uniform over the cross-section; hence, the velocity head and the momentum flux are generally greater than the values computed by using the average velocity. These values may be corrected by using the so-called energy and momentum correction coefficients, which are always slightly greater than the limiting value of unity (Al-Khatib and Göğüs, 1999). Momentum and kinetic energy principles often are used in hydraulic problems (Seckin et al., 2009). Momentum and kinetic energy correction coefficients, $\beta$ and $\alpha$, are often assumed to be unity when the momentum and energy principles are used in the computations as presented by many authors (Chow, 1959; Streeter and Wylie, 1979; French, 1987; Massey, 1989; Chen, 1992; Roberson and Crowe, 1998; Seckin et al., 2009). On the other hand, since it is not possible to have one-dimensional open channel flow, the true kinetic energy at a cross-section is not necessarily equal to the spatially averaged energy. To account for this, the kinetic energy and momentum correction factors are introduced, and the $\alpha$ and $\beta$ generally are greater than unity.

Different theoretical expressions for $\alpha$ and $\beta$ based on different assumptions and conditions have been derived by many authors (Rouse, 1965; Golubtsov, 1976; Benedict, 1980; Fox and McDonald, 1985; Chen, 1991). Due to the limited data available in the literature especially for compound channels, $\beta$ and $\alpha$ are often assumed to be unity in the computations for uniform flow.

## 2. Theoretical considerations

As mentioned previously, the velocity distribution is nonuniform in open channel flow. As a result, the velocity head of an open channel flow is generally greater than the value computed by using the average velocity.

[^0]Therefore, the true velocity head may be expressed as $\propto \frac{U^{2}}{2 g}$, where $\propto$ is known as the kinetic energy correction coefficient, U is the cross-sectional mean velocity, and g is the gravitational acceleration. By definition, then the kinetic energy correction coefficient $\propto$ is as defined in Eq. (1).

$$
\begin{equation*}
\propto=\frac{1}{U^{3} A} \int_{A} u^{3} d A \tag{1}
\end{equation*}
$$

where $u$ is the point velocity at each point in the cross-section, $A$ is the flow area, and dA is the differential area in the whole flow area.

In the same way, in applying the momentum equation to open-channel flows with simple or compound cross-sections, the true momentum flux of an incompressible fluid passing a cross-section is given by the integral indicated by Eq. (2).

$$
\begin{equation*}
\int_{A} \rho u^{2} d A=\rho \beta U^{2} A \tag{2}
\end{equation*}
$$

where $\rho$ is the density of the fluid. The momentum correction coefficient, $\beta$, can be derived from Eq. (2) to yield the value presented in Eq. (3).

$$
\begin{equation*}
\beta=\frac{1}{\rho U^{2} A} \int_{A} \rho u^{2} d A \tag{3}
\end{equation*}
$$

Typically, the values of $\propto$ and $\beta$ for various open channel cross-sections may be determined by either graphical integration or numerical methods using the measured velocity distributions as defined by Eqs. (1) and (3). Energy and momentum correction coefficients are often used in computer models for the determination of water surface profiles considering open channels with compound cross-sectional area, such as HEC-2 1991 and HEC-RAS 2002 (Seckin et al., 2009).

In the present study, numerical integration was used for the calculation of the kinetic energy and momentum correction coefficients for asymmetrical rectangular compound cross-sections. The cross-sectional area (A) of the channel was divided into $(N)$ number of elementary areas. For each elementary area $\left(\Delta A_{i}\right)$, the corresponding average velocity $\left(u_{i}\right)$ was determined from the measured velocities. The cross-sectional average velocity $(\mathrm{U})$ and the correction coefficients ( $\alpha$ and $\beta$ ) were calculated using Eqs. (4) to (6).

$$
\begin{align*}
& U=\frac{\sum_{i=1}^{N} u_{i} \Delta A_{i}}{A}  \tag{4}\\
& \beta=\frac{\sum_{i=1}^{N} u_{i}^{2} \Delta A_{i}}{U^{2} A}  \tag{5}\\
& \propto=\frac{\sum_{i=1}^{N} u_{i}^{3} \Delta A_{i}}{U^{3} A} \tag{6}
\end{align*}
$$

According to Seckin et al. (2009), for the case where the velocities are unidirectional but nonuniform across the section, Jaeger in 1956 found that the kinetic energy correction coefficient $(\propto)$ can be determined using Eq. (7).

$$
\begin{equation*}
\propto-1=3(\beta-1)+\frac{1}{A} \int_{A}\left[\frac{u-U}{U}\right] d A \tag{7}
\end{equation*}
$$

In order to investigate the variation in $\propto$ and $\beta$ in asymmetrical compound cross-sectional-shaped flumes, the present study was designed to obtain velocity distribution data. For this aim, a series of laboratory experiments were conducted in a compound channel flume. In addition, a general equation was derived to determine the relation between $\propto$ and $\beta$ using the entire data obtained from the 9 tested models.

## 3. Materials and methods

A series of laboratory experiments were done in an asymmetrical compound channel flume at the Fluid Mechanics Laboratory of Birzeit University, Palestine, to investigate the kinetic energy and momentum correction coefficients, $\propto$ and $\beta$ respectively. As can be seen from Figure 1, the flume consisted of a main channel and its asymmetrical floodplain.


Figure 1. Definition sketch of the flume used in the experiments.

The flume was glass-walled, 7.5 m long, 0.30 m wide, and 0.3 m deep, with a bottom slope of 0.0025 . Discharge was measured volumetrically with a flow meter with 0.1-L accuracy. A point gauge was used along the centerline of the flume for head measurements. All depth measurements were done with respect to the bottom of the flume. A pitot tube of circular section with external diameter of 8 mm was used to measure the static and total pressures, which were used to estimate the velocities and shear stresses at required points in the experiments conducted throughout this study.

Models of asymmetric rectangular compound cross-sections were fabricated from Plexiglas and placed at about the mid-length of the laboratory flume. Figure 1 shows the plan view and cross-section of the models with symbols designating important dimensions of model elements. Dimensions of various models used in the experiments are given in Table 1. In this study, the models tested are denoted by $\mathrm{B}_{i} \mathrm{Z}_{j}\left(\mathrm{~B}_{i}=10,15,20 \mathrm{~cm}\right.$; $\left.\mathrm{Z}_{j}=2,4,6 \mathrm{~cm}\right)$. The B and Z dimensions represent the width and step height of the main channel of the asymmetric compound cross-section, respectively.

Table 1. Geometrical properties of the asymmetric compound channel models.

| Model | B <br> $(\mathrm{cm})$ | Z <br> $(\mathrm{cm})$ | $\mathrm{B}_{f}$ <br> $(\mathrm{~cm})$ | $\mathrm{B}_{O}$ <br> $(\mathrm{~cm})$ | $\Theta_{1}$ <br> $($ degrees $)$ | $\Theta_{2}$ <br> $($ degrees $)$ | $\mathrm{B}_{O} / \mathrm{B}_{f}$ <br> $(-)$ | $\mathrm{B}_{O} / \mathrm{Z}$ <br> $(-)$ | $\mathrm{B}_{O} / \mathrm{B}$ <br> $(-)$ | $\mathrm{B}_{f} / \mathrm{Z}$ <br> $(-)$ | $\mathrm{B}_{f} / \mathrm{B}$ <br> $(-)$ | $\mathrm{B} / \mathrm{Z}$ <br> $(-)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{B}_{10} \mathrm{Z}_{2}$ | 10 | 2 | 20 | 30 | 26.57 | 153.43 | 1.50 | 15.00 | 3.00 | 10.00 | 2.00 | 5.00 |
| $\mathrm{~B}_{10} \mathrm{Z}_{4}$ | 10 | 4 | 20 | 30 | 26.57 | 153.43 | 1.50 | 7.50 | 3.00 | 5.00 | 2.00 | 2.50 |
| $\mathrm{~B}_{10} \mathrm{Z}_{6}$ | 10 | 6 | 20 | 30 | 26.57 | 153.43 | 1.50 | 5.00 | 3.00 | 3.33 | 2.00 | 1.67 |
| $\mathrm{~B}_{15} \mathrm{Z}_{2}$ | 15 | 2 | 15 | 30 | 26.57 | 153.43 | 2.00 | 15.00 | 2.00 | 7.50 | 1.00 | 7.5 |
| $\mathrm{~B}_{15} \mathrm{Z}_{4}$ | 15 | 4 | 15 | 30 | 26.57 | 153.43 | 2.00 | 7.50 | 2.00 | 3.75 | 1.00 | 3.75 |
| $\mathrm{~B}_{15} \mathrm{Z}_{6}$ | 15 | 6 | 15 | 30 | 26.57 | 153.43 | 2.00 | 5.00 | 2.00 | 2.50 | 1.00 | 2.5 |
| $\mathrm{~B}_{20} \mathrm{Z}_{2}$ | 20 | 2 | 10 | 30 | 26.57 | 153.43 | 3.00 | 15.00 | 1.50 | 5.00 | 0.50 | 10.00 |
| $\mathrm{~B}_{20} \mathrm{Z}_{4}$ | 20 | 4 | 10 | 30 | 26.57 | 153.43 | 3.00 | 7.50 | 1.50 | 2.50 | 0.50 | 5.00 |
| $\mathrm{~B}_{20} \mathrm{Z}_{6}$ | 20 | 6 | 10 | 30 | 26.57 | 153.43 | 3.00 | 5.00 | 1.50 | 1.67 | 0.50 | 3.33 |

The required experiments were first conducted in the models of smallest $B(=10 \mathrm{~cm})$ with varying $Z$ values $(=2 \mathrm{~cm}, 4 \mathrm{~cm}$, and 6 cm$)$ and then B was increased to 15 cm at the required amount of $\mathrm{Z}(=2 \mathrm{~cm}, 4$ cm , and 6 cm ), and finally for $\mathrm{B}=20 \mathrm{~cm}$ with the same 3 values of Z . The entrance angles, $\theta_{1}$ and $\theta_{2}$, were $26.565^{\circ}$ and $153.35^{\circ}$, respectively. The transition length was twice the floodplain width, $\mathrm{B}_{f}$.

In order to obtain the velocity distribution in the 9 different asymmetric rectangular compound crosssections, the channel cross-section was divided into a number of vertical velocity profiles normal to the direction of flow. Then the total and static heads were measured at several points along these normal lines by the use of a pitot (Preston) tube. More points were taken close to the channel boundary. Towards the free surface, the distances between the points where the velocities measured were increased. Figure 2 shows a definition sketch for vertical lines over which velocity measurements were made in models $\mathrm{B}_{i} \mathrm{Z}_{j}\left(\mathrm{~B}_{i}=10,15,20 \mathrm{~cm} ; \mathrm{Z}_{j}=2\right.$, $4,6 \mathrm{~cm}$ ).

## 4. Results and discussion

For different relative depth values, the computed values of the kinetic energy and momentum correction coefficients of the 9 models are given in Table 2. They are related to the relative depth ( $\mathrm{Y}_{r}$ ) because it reflects the geometry effect on the discharge distribution in compound cross-section channels as $\mathrm{Y}_{r}$ is defined as the floodplain depth to the total depth associated with the compound cross-section $\left(Y_{f} / h\right)$, which is a dimensionless quantity.

There is a large difference in the velocity distribution between main channel and floodplains for compound channels. That is why for the 9 asymmetrical compound channels tested in this study, the values of $\alpha$ and $\beta$ averaged 1.1525 and 1.1261, respectively, while they averaged 1.0604 and 1.0222 , respectively, for single channels (Blalock and Sturm, 1981, 1983). This means that the average values of $\alpha$ and $\beta$ for channels with simple cross-sections are lower than their corresponding values associated with compound channels. In a study conducted by Seckin et al. (2009) considering a symmetrical compound channel, it was determined that the
average values of the kinetic energy and momentum correction coefficients $\alpha$ and $\beta$ were 1.1309 and 1.0458, respectively. This also means the ( $\alpha$ and $\beta$ ) average values for symmetric rectangular channels are lower than their corresponding values for asymmetric compound channels.


Figure 2. Definition sketch for vertical lines over which velocity measurements were made for the different models (dimensions are in cm ).

As it can be noted from Eq. (7), the last term is often small because the integral always changes sign (Seckin et al., 2009). Thus, it makes sense to seek a linear regression between $(\alpha-1)$ and $(\beta-1)$. For the 9 different asymmetric compound channels, the values of $(\alpha-1)$ versus the values of $(\beta-1)$ are plotted in Figure 3, which shows a strong linear correlation between the 2 plotted values. Therefore, a simple linear single-variable regression model is formulated using all 106 data points derived from the 9 tested asymmetric cross-sections. The generated model is presented in Eq. (8), which can be used to predict the ( $\alpha-1$ ) value as a function of $(\beta-1)$ value for an asymmetrical rectangular compound cross-section. This regression model is associated with a very high correlation coefficient value $(\mathrm{R}$-squared $=0.998)$.

$$
\begin{equation*}
(\propto-1)=0.990(\beta-1)+0.027 \tag{8}
\end{equation*}
$$

In addition to the relationship between $(\alpha-1)$ and $(\beta-1)$, the relationship between $\alpha$ and $\beta$ is also shown in
Table 2. Experimental data obtained from the asymmetrical compound channel flume.

| Model |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{B}_{10} \mathrm{Z}_{2}$ |  |  |  | $\mathrm{B}_{10} \mathrm{Z}_{4}$ |  |  |  | $\mathrm{B}_{10} \mathrm{Z}_{6}$ |  |  |  | $\mathrm{B}_{15} \mathrm{Z}_{2}$ |  |  |  | $\mathrm{B}_{15} \mathrm{Z}_{4}$ |  |  |  |
| $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ | $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ | $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ | $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ | $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ |
| 0.818 | 1.0278 | 1.0097 | 2.87 | 0.669 | 1.0358 | 1.0125 | 2.87 | 0.559 | 1.0315 | 1.0110 | 2.87 | 0.825 | 1.1304 | 1.1125 | 1.16 | 0.640 | 1.1199 | 1.1036 | 1.16 |
| 0.815 | 1.0366 | 1.0128 | 2.85 | 0.664 | 1.0581 | 1.0232 | 2.51 | 0.542 | 1.0301 | 1.0105 | 2.86 | 0.817 | 1.1581 | 1.1393 | 1.14 | 0.633 | 1.1494 | 1.1321 | 1.13 |
| 0.810 | 1.0391 | 1.0137 | 2.86 | 0.655 | 1.0311 | 1.0108 | 2.87 | 0.524 | 1.0323 | 1.0112 | 2.87 | 0.811 | 1.1683 | 1.1498 | 1.12 | 0.615 | 1.2341 | 1.2160 | 1.08 |
| 0.802 | 1.0387 | 1.0135 | 2.86 | 0.646 | 1.0338 | 1.0117 | 2.88 | 0.504 | 1.0319 | 1.0111 | 2.87 | 0.789 | 1.2297 | 1.2038 | 1.13 | 0.630 | 1.1563 | 1.1378 | 1.13 |
| 0.792 | 1.0434 | 1.0152 | 2.85 | 0.633 | 1.0433 | 1.0150 | 2.88 | 0.483 | 1.0461 | 1.0159 | 2.89 | 0.785 | 1.2757 | 1.2536 | 1.09 | 0.612 | 1.2613 | 1.2425 | 1.08 |
| 0.780 | 1.0455 | 1.0158 | 2.88 | 0.619 | 1.0444 | 1.0153 | 2.89 | 0.464 | 1.0459 | 1.0159 | 2.88 | 0.770 | 1.3462 | 1.3249 | 1.07 | 0.600 | 1.3015 | 1.2812 | 1.07 |
| 0.765 | 1.0487 | 1.0170 | 2.87 | 0.596 | 1.0460 | 1.0158 | 2.91 | 0.444 | 1.0471 | 1.0163 | 2.90 | 0.756 | 1.3979 | 1.3756 | 1.06 | 0.579 | 1.3611 | 1.3411 | 1.06 |
| 0.744 | 1.0514 | 1.0179 | 2.86 | 0.565 | 1.0424 | 1.0145 | 2.91 | 0.429 | 1.0424 | 1.0146 | 2.90 | 0.744 | 1.4182 | 1.3969 | 1.05 | 0.551 | 1.4010 | 1.3838 | 1.04 |
| 0.733 | 1.0449 | 1.0156 | 2.88 | 0.545 | 1.0431 | 1.0148 | 2.92 | 0.394 | 1.0526 | 1.0180 | 2.91 | 0.722 | 1.4878 | 1.4626 | 1.05 | 0.524 | 1.4567 | 1.4354 | 1.05 |
| 0.701 | 1.0475 | 1.0164 | 2.89 | 0.512 | 1.0511 | 1.0175 | 2.93 | 0.368 | 1.0512 | 1.0175 | 2.92 | 0.697 | 1.5771 | 1.5469 | 1.06 | 0.494 | 1.5209 | 1.4972 | 1.05 |
| 0.661 | 1.0435 | 1.0149 | 2.93 | 0.459 | 1.0501 | 1.0169 | 2.96 | 0.310 | 1.0490 | 1.0166 | 2.95 | 0.672 | 1.6587 | 1.6320 | 1.04 | 0.459 | 1.6208 | 1.5962 | 1.04 |
| 0.592 | 1.0255 | 1.0087 | 2.93 | 0.394 | 1.0424 | 1.0142 | 2.98 | 0.268 | 1.0411 | 1.0140 | 2.95 | 0.636 | 1.6820 | 1.6543 | 1.04 | 0.385 | 1.7473 | 1.7224 | 1.03 |

Table 2. Continued

| Model |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{B}_{15} \mathrm{Z}_{6}$ |  |  |  | $\mathrm{B}_{20} \mathrm{Z}_{2}$ |  |  |  | $\mathrm{B}_{20} \mathrm{Z}_{4}$ |  |  |  | $\mathrm{B}_{20} \mathrm{Z}_{6}$ |  |  |  |
| $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ | $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ | $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ | $\mathrm{Y}_{\mathrm{r}}$ | $\alpha$ | $\beta$ | $\frac{\alpha-1}{\beta-1}$ |
| 0.504 | 1.0925 | 1.0817 | 1.13 | 0.821 | 1.0534 | 1.0185 | 2.88 | 0.649 | 1.0470 | 1.0166 | 2.83 | 0.512 | 1.0469 | 1.0163 | 2.88 |
| 0.496 | 1.1163 | 1.1043 | 1.12 | 0.802 | 1.0474 | 1.0165 | 2.88 | 0.630 | 1.0590 | 1.0210 | 2.81 | 0.500 | 1.0457 | 1.0159 | 2.87 |
| 0.478 | 1.1363 | 1.1234 | 1.10 | 0.789 | 1.0419 | 1.0145 | 2.88 | 0.626 | 1.0584 | 1.0205 | 2.85 | 0.469 | 1.0423 | 1.0147 | 2.87 |
| 0.455 | 1.2089 | 1.1928 | 1.08 | 0.787 | 1.0392 | 1.0137 | 2.87 | 0.615 | 1.0559 | 1.0195 | 2.87 | 0.417 | 1.0456 | 1.0158 | 2.88 |
| 0.444 | 1.2446 | 1.2268 | 1.08 | 0.785 | 1.0393 | 1.0138 | 2.86 | 0.592 | 1.0443 | 1.0155 | 2.85 | 0.400 | 1.0437 | 1.0151 | 2.89 |
| 0.423 | 1.3028 | 1.2845 | 1.06 | 0.780 | 1.0339 | 1.0119 | 2.86 | 0.565 | 1.0448 | 1.0156 | 2.87 | 0.388 | 1.0492 | 1.0169 | 2.92 |
| 0.400 | 1.3555 | 1.3382 | 1.05 | 0.756 | 1.0419 | 1.0145 | 2.88 | 0.545 | 1.0530 | 1.0185 | 2.86 | 0.362 | 1.0336 | 1.0116 | 2.91 |
| 0.375 | 1.3868 | 1.3673 | 1.05 | 0.733 | 1.0375 | 1.0129 | 2.90 | 0.518 | 1.0489 | 1.0170 | 2.87 | 0.294 | 1.0444 | 1.0152 | 2.91 |
| 0.326 | 1.4100 | 1.3913 | 1.05 | 0.718 | 1.0414 | 1.0144 | 2.87 | 0.481 | 1.0564 | 1.0197 | 2.86 | 0.250 | 1.0513 | 1.0177 | 2.90 |
| 0.259 | 1.5244 | 1.4997 | 1.05 | 0.706 | 1.0559 | 1.0191 | 2.93 | 0.452 | 1.0507 | 1.0175 | 2.89 | 0.189 | 1.0574 | 1.0199 | 2.88 |
| 0.231 | 1.6336 | 1.6074 | 1.04 | 0.655 | 1.0350 | 1.0121 | 2.90 | 0.322 | 1.0590 | 1.0202 | 2.91 |  |  |  |  |
| 0.155 | 1.7569 | 1.7235 | 1.05 | 0.574 | 1.0595 | 1.0204 | 2.92 | 0.286 | 1.0557 | 1.0192 | 2.91 |  |  |  |  |

Figure 4 using all 106 data points generated from the 9 cross-sections with the corresponding regression model given by Eq. (9).

$$
\begin{equation*}
\alpha=0.990 \beta+0.037 \tag{9}
\end{equation*}
$$

Normally, the velocity distribution in the flumes is determined by the shape and roughness of the channel. All data presented in this study are limited to flumes with smooth surfaces and the possible effects of the surface roughness were not examined. Thus, the averaged values of $\propto$ and $\beta$ proposed herein are recommended for asymmetric compound smooth surface open channels.


Figure 3. Relation between $(\alpha-1)$ and $(\beta-1)$.


Figure 4. Relation between kinetic energy correction factor $(\alpha)$ and momentum correction factor $(\beta)$.

Values of the Reynolds number (Re) for the different tested models are estimated using Eq. (10) and are presented in Table 3.

$$
\begin{equation*}
R e=U D / \nu \tag{10}
\end{equation*}
$$

where D is the hydraulic depth, which is defined as the cross-sectional area of the water normal to the direction of flow in the channel divided by the width of the free surface; $\nu$ is the kinetic viscosity of water. As can be seen from Table 3, Re varied between $14,348.9$ and $54,868.2$, as relative depth $\left(\mathrm{Y}_{r}\right)$ changed from 0.155 to 0.825 .

It can be clearly seen from Table 3 that as the discharge decreases ( $\mathrm{Y}_{r}$ values decrease) the values of Re decrease, which is true for all tested models.

The value ranges associated with $\alpha$ and $\beta$ as presented in this study (Table 2) can be utilized for practical purposes considering the geometric conditions (Table 1) under which velocity measurements were conducted in asymmetrical rectangular compound cross-sections and the same range of Re. The estimated $\alpha$ and $\beta$ value ranges can be used for models of the same geometry and the same range of Re in order to calculate the energy and momentum flux of the flow for a particular cross-section.

Table 3. Reynolds number for various types of models.

| Model |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{B}_{10} \mathrm{Z}_{2}$ |  | $\mathrm{~B}_{10} \mathrm{Z}_{4}$ |  | $\mathrm{~B}_{10} \mathrm{Z}_{6}$ |  | $\mathrm{~B}_{15} \mathrm{Z}_{2}$ |  | $\mathrm{~B}_{15} \mathrm{Z}_{4}$ |  |  |  |
| $\mathrm{Y}_{r}$ | Re | $\mathrm{Y}_{r}$ | Re | $\mathrm{Y}_{r}$ | Re | $\mathrm{Y}_{r}$ | Re | $\mathrm{Y}_{r}$ | Re |  |  |
| 0.818 | $50,694.1$ | 0.669 | $44,620.4$ | 0.559 | $45,608.7$ | 0.825 | $53,326.0$ | 0.640 | $47,576.8$ |  |  |
| 0.815 | $49,302.6$ | 0.664 | $42,975.4$ | 0.542 | $42,991.3$ | 0.817 | $50,527.9$ | 0.633 | $45,713.1$ |  |  |
| 0.810 | $47,414.4$ | 0.655 | $42,235.1$ | 0.524 | $40,965.1$ | 0.811 | $49,117.3$ | 0.615 | $40,959.5$ |  |  |
| 0.802 | $44,741.0$ | 0.646 | $40,500.9$ | 0.504 | $37,839.5$ | 0.789 | $43,402.8$ | 0.630 | $45,145.3$ |  |  |
| 0.792 | $41,329.5$ | 0.633 | $39,356.7$ | 0.483 | $33,786.8$ | 0.785 | $41,033.0$ | 0.612 | $39,762.2$ |  |  |
| 0.780 | $36,889.5$ | 0.619 | $36,413.8$ | 0.464 | $30,928.3$ | 0.770 | $36,931.6$ | 0.600 | $37,615.2$ |  |  |
| 0.765 | $33,464.7$ | 0.596 | $32,423.7$ | 0.444 | $28,206.9$ | 0.756 | $33,975.2$ | 0.579 | $34,377.2$ |  |  |
| 0.744 | $29,700.9$ | 0.565 | $28,465.3$ | 0.429 | $26,264.3$ | 0.744 | $32,153.4$ | 0.551 | $31,420.1$ |  |  |
| 0.733 | $27,690.5$ | 0.545 | $27,001.0$ | 0.394 | $23,859.4$ | 0.722 | $28,755.9$ | 0.524 | $28,694.9$ |  |  |
| 0.701 | $24,671.4$ | 0.512 | $23,878.2$ | 0.368 | $21,728.9$ | 0.697 | $25,241.0$ | 0.494 | $25,915.7$ |  |  |
| 0.661 | $19,249.2$ | 0.459 | $19,380.8$ | 0.310 | $17,962.8$ | 0.672 | $22,304.1$ | 0.459 | $22,744.4$ |  |  |
| 0.592 | $14,348.9$ | 0.394 | $14,835.3$ | 0.268 | $15,067.2$ | 0.636 | $19,982.9$ | 0.385 | $18,300.9$ |  |  |

Table 3. Continued.

| Model |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{B}_{15} \mathrm{Z}_{6}$ |  | $\mathrm{~B}_{20} \mathrm{Z}_{2}$ |  | $\mathrm{~B}_{20} \mathrm{Z}_{4}$ |  | $\mathrm{~B}_{20} \mathrm{Z}_{6}$ |  |  |
| $\mathrm{Y}_{r}$ | Re | Y | R | Re | $\mathrm{Y}_{r}$ | Re | $\mathrm{Y}_{r}$ |  |
| 0.504 | $46,749.0$ | 0.821 | $54,868.2$ | 0.649 | $51,290.7$ | 0.512 | $50,827.1$ |  |
| 0.496 | $45,120.5$ | 0.802 | $50,258.7$ | 0.630 | $47,898.9$ | 0.500 | $48,922.9$ |  |
| 0.478 | $42,997.7$ | 0.789 | $47,109.4$ | 0.626 | $46,122.9$ | 0.469 | $46,011.7$ |  |
| 0.455 | $38,846.9$ | 0.787 | $45,448.3$ | 0.615 | $44,258.9$ | 0.417 | $40,639.9$ |  |
| 0.444 | $37,111.8$ | 0.785 | $43,779.5$ | 0.592 | $40,719.9$ | 0.400 | $38,396.8$ |  |
| 0.423 | $34,158.0$ | 0.780 | $42,657.6$ | 0.565 | $37,479.0$ | 0.388 | $36,848.1$ |  |
| 0.400 | $31,510.3$ | 0.756 | $36,540.0$ | 0.545 | $35,663.3$ | 0.362 | $33,758.6$ |  |
| 0.375 | $29,551.6$ | 0.733 | $33,748.3$ | 0.518 | $32,694.1$ | 0.294 | $29,206.8$ |  |
| 0.326 | $26,721.8$ | 0.718 | $30,712.2$ | 0.481 | $29,507.3$ | 0.250 | $26,309.0$ |  |
| 0.259 | $22,171.4$ | 0.706 | $27,590.9$ | 0.452 | $27,263.4$ | 0.189 | $23,170.2$ |  |
| 0.231 | $19,724.1$ | 0.655 | $22,185.8$ | 0.322 | $20,339.8$ |  |  |  |
| 0.155 | $16,210.9$ | 0.574 | $16,843.7$ | 0.286 | $18,454.9$ |  |  |  |

## 5. Conclusions

In the present study, a series of laboratory experiments was conducted to investigate the momentum and kinetic energy coefficients in channels with asymmetric rectangular compound cross-sections. Two linear single-variable regression models for estimating $(\alpha-1)$ as a function of $(\beta-1)$ and $\alpha$ as a function of $\beta$ were formulated using all 106 data points obtained from the 9 tested cross-sections. This study explored the practical average values of $\propto$ and $\beta$ and found them to be 1.1525 and 1.1261 , respectively, considering the 9 different straight asymmetrical compound channel flumes. The range of kinetic energy coefficient and momentum correction coefficient values can be used for practical purposes for models of the same geometry and the same range of Re.

## Nomenclature

A whole flow area
B bottom width of the approach channel
$B_{f}$ floodplain channel width

## $\mathrm{B}_{O}$ bottom width of the upstream channel

D hydraulic depth
dA an elementary area in the whole flow area
g gravitational acceleration

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h main channel water depth
$\mathrm{N} \quad$ number of small areas, $\Delta \mathrm{Ai}$
Q average full cross-sectional discharge point velocity at each point in the cross-section cross-sectional mean velocity
r correlation coefficient
$Y_{f} \quad$ floodplain water depth
$Y_{m c} \mathrm{~h}$, main channel water depth
$Y_{r} \quad$ relative depth, which equals the $Y_{f} / Y_{m c}$ ratio
Z step height
$\propto \quad$ kinetic energy correction coefficient
$\beta$ momentum correction coefficient
$\nu \quad$ kinetic viscosity of water
$\theta_{1}, \theta_{2}$ entrance angles
$\rho \quad$ density of the fluid

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