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The Prediction of the Wall Bounded Turbulent Flows Using the Code PASSABLE

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Abstract

Turbulent flows for flat plate and circular pipes were solved using the two equation k-ε model and the algebraic stress model (ASM). The computations were carried out using the parabolic solver PASSABLE code which uses a finite-difference and finite-volume approach with a forward maching technique. The results obtained with the code were compared with the results described in the literature and a good agreement was found.

Key Words: k-ε turbulence model, algebraic stress model, flat plate flow, pipe flow

PASSABLE Programı ile Duvarla Sınırlı Türbülanslı Akışların Hesaplanması

Özet

Düzlem levha ve borudaki türbülanslı akışlar iki denklemli k-ε ve cebrik gerilme türbülans modelleri kullanılarak çözülmüştür. Hesaplamalar ileriye doğru çözüm tekniği ile sonlu fark ve sonlu hacim yaklaşımı kullanan parabolik tipte PASSABLE bilgisayar programı ile gerçekleştirilmiştir. Bu bilgisayar programı ile elde edilen sonuçlar mevcut deneysel sonuçlarla karşılaştırılmış ve iyi bir uyum sağlanmıştır.

Anahtar Sözcükler: k-ε türbülans modeli, cebrik gerilme modeli, düzlem levha akışı, boru akışı

1. Introduction

The predictions of turbulent flows, which are of great practical importance, can be obtained using turbulence models. The main distinction between turbulence models is whether turbulence properties are related directly to the mean flow or obtained from a transport equation such as Reynolds transport equation. A wide variety of flows have been predicted with the k-ε and algebraic stress with reasonable accuracy.

Ng and Spalding (1972) have predicted the boundary layer flows near walls by means of a turbulence model which employs the turbulence energy, and the product of this energy with a length scale. Jones and Launder (1973) have calculated the fully-developed turbulent flow in a pipe with the k-ε model. A simplified algebraic stress model was used by Lemos and Sesonske (1985) to investigate the turbulent flow of mercury in a pipe. Direct numerical simulations and experiments on fully-developed turbulent pipe flow for low Reynolds numbers have been

carried out by Eggles et al. (1994). Spalart (1988) has studied the direct simulation of devoloping a turbulent boundary layer on a flat plate. Direct and large-eddy simulations of turbulence have been reviewed by Rogallo and Moin (1984). Kim and Chung (1988) and Yule et al. (1993) have applied the standard k-ε model and the algebraic stress model to turbulent jet flow. They have concluded that the algebraic stress model gave more accurate predictions than the k-ε model in terms of velocity components.

2. Turbulence models

The two-equation k-ε model, which specifes the turbulent viscosity, and the algebraic stress model (ASM), which takes into account the stress transport, were used in this study. Both models are based on modelled versions of exact transport equations for the Reynolds stresses and the rate of dissipation of turbulence energy.

k-ε

The Jones and Launder (1972) model, also known as the k-ε model, relates the distributions of the turbulence kinetic energy k and its rate of dissipation ε to an eddy viscosity. For the steady, incompressible flow, the distributions of k and ε were obtained from the solution of the following transport equations:

$$U_j\frac{\partial k}{\partial x_j}=\frac{\partial}{\partial x_j}\left[(\nu+\frac{\nu_t}{\sigma_k})\frac{\partial k}{\partial x_j}\right]+\nu_t(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i})\frac{\partial U_j}{\partial x_j}-\varepsilon \quad (1)$$

$$U_j\frac{\partial \varepsilon}{\partial x_j}=\frac{\partial}{\partial X_j}[(\nu+\frac{\nu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x_j}]+C_{\varepsilon 1}\frac{\varepsilon}{k}\nu_t(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i})\frac{\partial U_i}{\partial x_j}-C_{\varepsilon 2}\frac{\varepsilon^2}{k} \quad (2)$$

where $v_t=C_\mu k^2/\varepsilon$ and $\varepsilon=k^{3/2}/l$. l is the turbulence length scale. The model constants are σ_k=1, σ_ε=1.25, $C_{\varepsilon 1}$=1.44, $C_{\varepsilon 2}$=1.92 and C_μ=0.09 (Launder and Spalding, 1974).

The algebraic stress model (ASM)

The ASM is a procedure for solving transport equations for all individual Reynolds stresses. Launder et al. (1975) have assumed proportionality between the net transport of each stress and the corresponding transport of k, which yields

$$\frac{u_i\bar{u}_j}{k}(P_k-\varepsilon)=(1-C_2)P_{ij}+\frac{2}{3}C_2\delta_{ij}P_k-C_1\frac{\varepsilon}{k}u_i\bar{u}_j+\frac{2}{3}(C_1-1)\delta_{ij}\varepsilon \quad (3)$$

where the production rate of k is defined by $P_k=-u_i\bar{u}_j(\partial U_i/\partial x_j)$.$P_{ij}$ is the rate of production tensor of $u_i\bar{u}_j$ and δ_{ij} is a Kronecker delta. The model constants are C_1=1.8 and C_2=0.6 (Gibson, 1978).

Both models used in this study are valid for high Reynolds number flows, away from viscous near-wall regions only, ignoring the effects of fluid viscosity on turbulence. It thus follows that wall-bounded flows require special treatment near the wall to account for viscous-sublayer effects. In particular, the wall shear stress, the near-wall generation and dissipation of turbulence energy need to be considered. As a description of the wall function (Schlichting, 1979), the non-dimensional wall distance was defined $y^+=yU_\tau/\nu$ where U_τ is a friction velocity. In our computation, it was assumed that at $y^+\geq 30$ viscous effects were negligible and that within $y^+<30$ momentum transport could be ignored.

3. Numerical Procedure

The general equations for steady axisymmetric flow of the boundary-layer type reduce to the following forms:

continuity:

$$\frac{\partial \rho U}{\partial x}+\frac{1}{r}\frac{\partial \rho r V}{\partial r}=0 \quad (4)$$

momentum:

$$\frac{\partial \rho U^2}{\partial x}+\frac{1}{r}\frac{\partial r\rho UV}{\partial r}=-\frac{\partial P}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}(r\mu\frac{\partial U}{\partial r}-r\rho\overline{uv}) \quad (5)$$

$$\frac{\partial \rho U\phi}{\partial x}+\frac{1}{r}\frac{\partial r\rho V\phi}{\partial r}=-\frac{1}{r}\frac{\partial}{\partial r}(-r\Gamma_\phi\frac{\partial \phi}{\partial r}+r\rho\overline{\phi v}+S_\phi) \quad (6)$$

Equation 6 can represent Equations 1 or 2 by choosing ϕ=k and $\phi=\varepsilon$, respectively, and inserting gradient diffusion models for $\overline{kv}$ and $\overline{v\varepsilon}$.

The PASSABLE code of Leschziner (1981) was used in this study. The code solved the governing equations in a forward marching manner. At any stage of the solution, upstream conditions were used to obtain the flow conditions on a single downstream

plane. Discretization was based on the fully conservative finite-volume approach. Second-order central differences were used to approximate the cross-flow convection and diffusion terms. The formulation was implicit in the streamwise direction. The code solved discretized versions of the partial differential equations governing the transport of momentum and any relevant intensive scalar properties over a domain whose width generally varied with downstream distance. The numerical mesh was such that a fixed number of mesh lines covered the width of the flow field whether this width was constant or variable with the streamwise direction. The mesh was non-orthogonal in general, reverting to a Cartesian or a polar cylindrical system when the width of the flow field was held constant. The method adopted in the code was to transform the governing equations from the (x,y) to an (ξ, η) system, as the flow over a flat plate shown in Figure 1. Our procedure was forward marching one, advancing the solution from any upstream to a following downstream station. Figure 2 shows the numerical mesh and associated control volumes within a forward step. The main control volume relating to the axial velocity component U and any scalar (P. k and ε) is shown in Figure 2a. The faces η_+ and η_- bisect co-responding internodal distances. As shown in Figure 2b, the radial velocity component V was staggered midway between nodal locations P_j and P_{j-1}, which had advantages for the formulation of the continuity equation from which V was obtained.The hybrid difference scheme (HDS) was used for the momentum equations (Patankar, 1980).

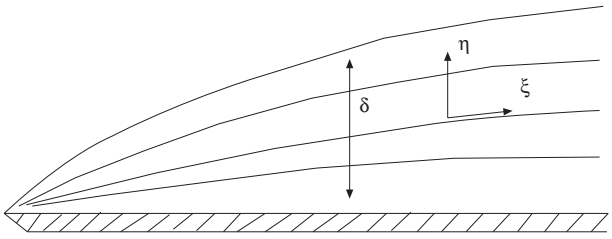

Figure 1. The ξ, η coordinate system

The solution of the momentum equation required knowledge of the pressure gradients. The treatment in the code initially used the pressure gradient of the upstream station. This resulted in a velocity distribution that would not necessarily satisfy overall mass conservation. The aim was therefore to iterate and reduce the amount of mass flux imbalance. The finite-difference equations for the variables U, k and ε were solved using the tridiagonal matrix algorithm. Approximately 16 nodes across the flow, from the centerline, were found to give grid-independent solutions.All computations for pipe flow were carried out using approximately 4450 steps downstream.

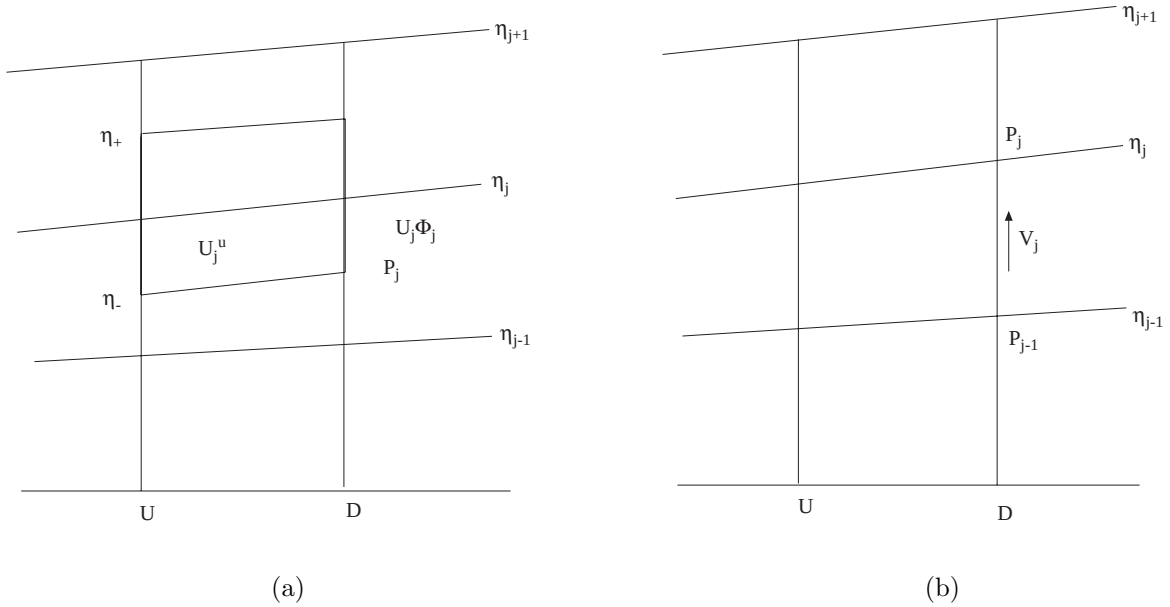

Figure 2. Numerical mesh and control volumes a) U-momentum and scalars; b) V-momentum

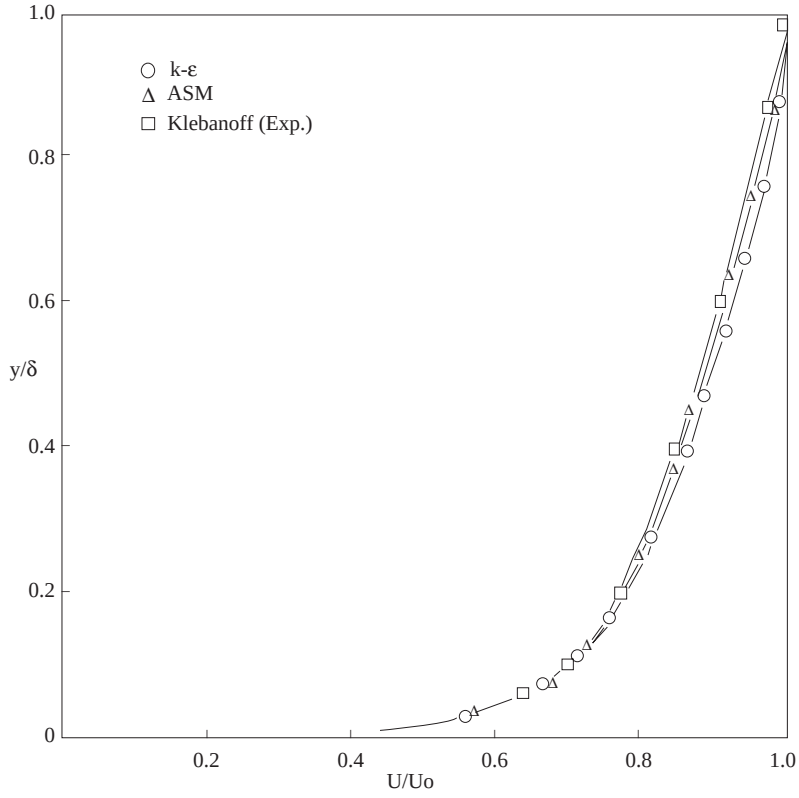

Figure 3. Axial mean velocity profiles on the flat plate

4. Results

The computations were performed with an algebraic stress model and the standard k-ε model for predictions of mean velocity and turbulence kinetic energy in the flow field of a flat and a circular pipe. The results were compared with Klebanoff's (from Ng and Spalding, 1972) and Laufer's (from Jones and Launder, 1973).

Flat plate

The initial values of the velocity and turbulence kinetic energy were uniform and equal to 10 m/s and 0.0001 m^2/s^2, respectively. The plate length was 6.5 m. The upper boundary of solution domain was expanded in the streamwise direction appropriate to the parabolic nature of the flow over the flat plate. The boundary layer thickness was 0.1133 m at x=6.294 m from the leading edge of the plate. The Reynolds number, based on free stream velocity and boundary layer thickness, was 7.5×10^4. This value also corresponded with a Reynolds number of 3.94×10^6 calculated from the free stream velocity and downstream distance. Figure 3 shows the comparison between the velocity profiles predicted with the k-ε model and ASM at x=6.294m for flow over a flat plate and shows the experimental data of Klebanoff. The agreement seems very good. The predictions of turbulence kinetic energy in the boundary layer along the flat plate are given in Figure 4, together with Klebonoff's. The prediction values at $y/\delta > 0.2$ for k are slightly larger than the experimental data. It can be seen that the results obtained from the k-ε model are close to the experimental data. Figure 5 shows the variation in the skin friction coefficient along the flat plate. Also shown are the results obtained by Coles and Hirst (1968) from the data of Wieghardt and the results of Prandtl's theoretical formula defined as C_f=$0.0592Re^{-0.2}$ (Schlichting, 1979). The predictions obtained with the k-ε model and ASM showed only small discrepancies. After x=3m, the predictions agree well with Prandtl's theoretical formula.

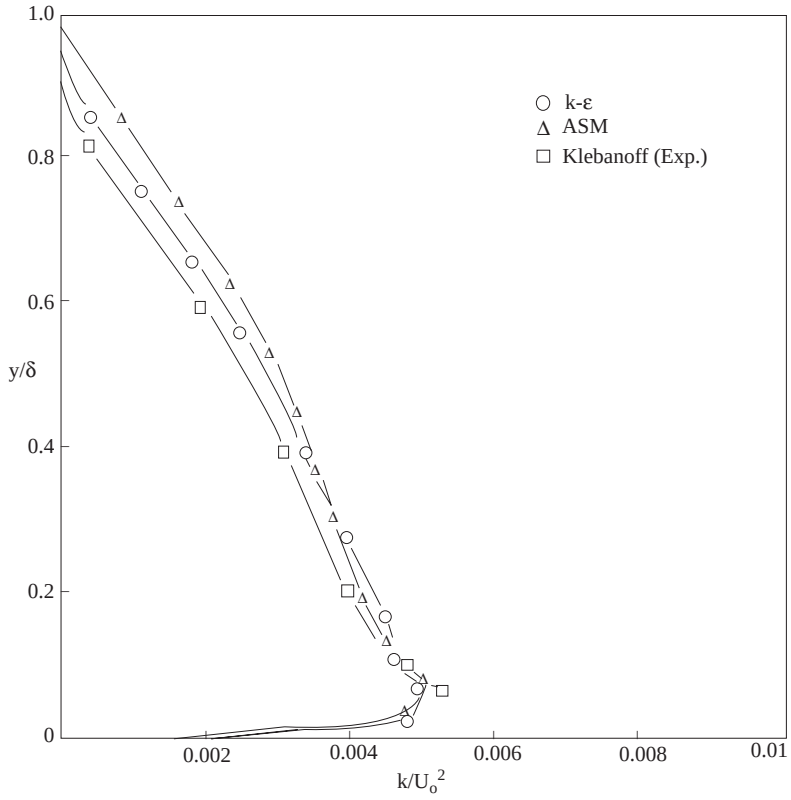

Figure 4. Kinetic energy profiles on the flat plate

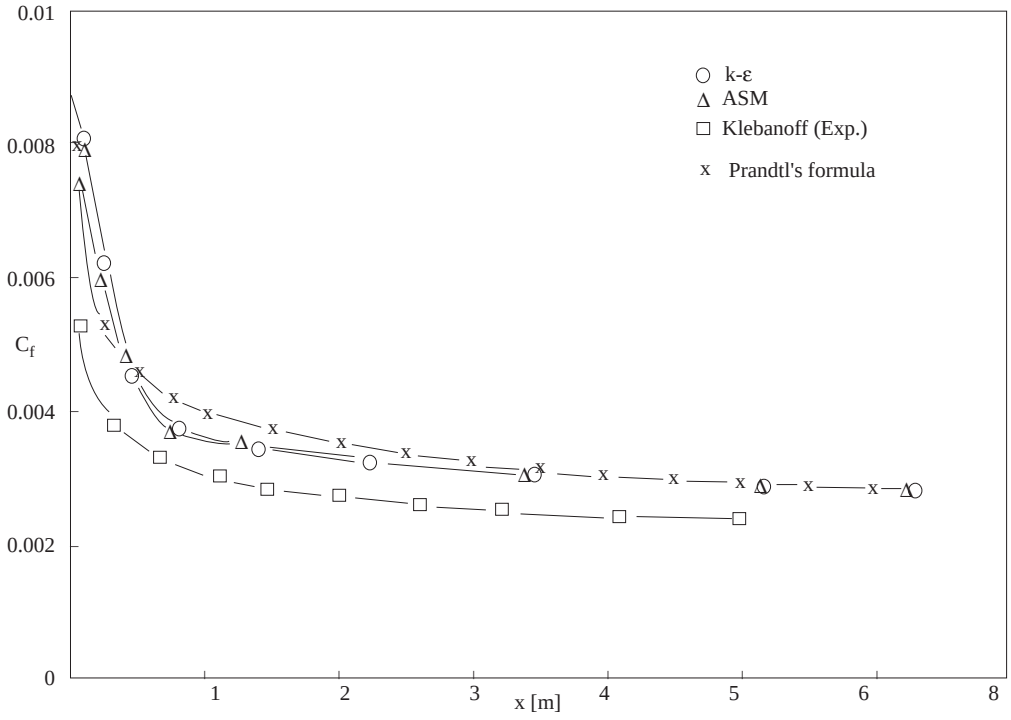

Figure 5. Distribution of skin friction coefficients along the flat plate

circular pipe

The length and diameter of the pipe were L=7.2 m and D=0.18 m. The inlet profiles for the velocity and turbulence kinetic energy were uniform. The inlet velocities were 4.2 m/s and 42 m/s for Re=50000 and Re=500000, respectively. The initial turbulence kinetic energy was 0.0001 m^2/s^2. The variation of the entrance length of the flow to the Reynolds number is given in Figure 6 for the k-ε model and ASM. As the Reynolds number increased, the entrance length also increased. It can be seen that the values predicted with ASM were slightly higher than those of the k-ε model.

Figures 7a and b show the velocity profiles predicted with the k-ε model and ASM for various Reynolds numbers in the developed flow region. These profiles compare with Laufer's results. For Re=50000, the predictions coincide with each other and agree with the experimenatl data (Figure 7a). The predictions of ASM are closer to the experimental data for Re=500000 (Figure 7b).

The kinetic energy profiles predicted with the k-ε model and ASM are compared in Figures 8a and b for Re=50000 and 500000, respectively. There were discrepancies between the predictions and the results, but there were only small discrepancies in the region near the wall. Furthermore, the k-ε model predicted lower values than the ASM for the region near the wall.

5. Conclusions

This research was primarily concerned with the accuracy and economics of the PASSABLE computer code which incorporated the turbulence models of standard k-ε and algebraic stress. The results for the considered flows in turbulence models in code had only small discrepancies and were in agreement with experiments except for the kinetic energy profiles. Because the code solve the governing equations in a forward marching manner which allows a one-dimensional storage of all dependent variables, it is highly economical.

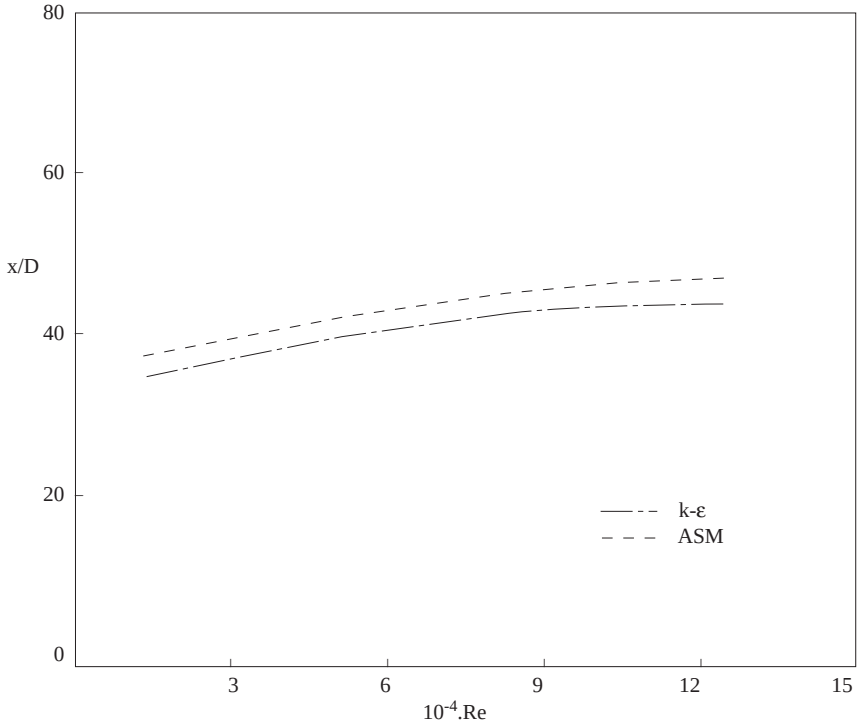

Figure 6. The entrance length in the pipe flow

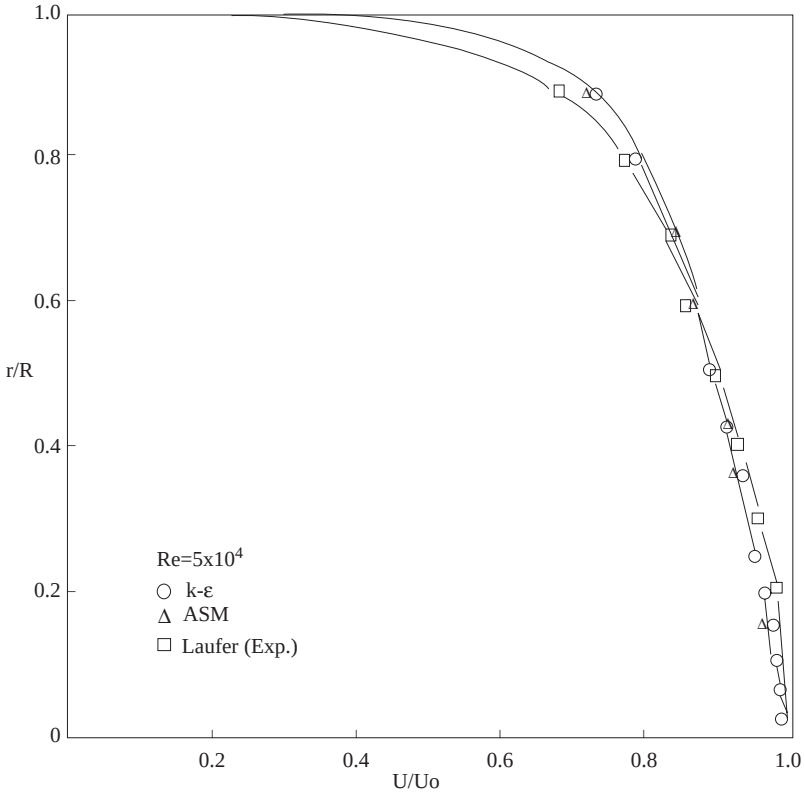

Figure 7a. Axial mean velocity profiles in the pipe flow a)Re=5×10^4

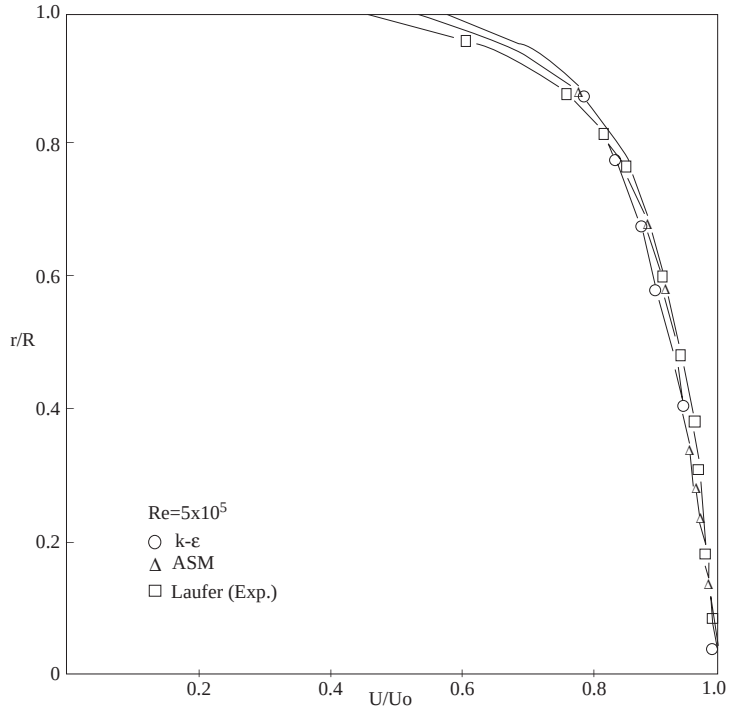

Figure 7b. Axial mean velocity profiles in the pipe flow a)Re=5×10^5

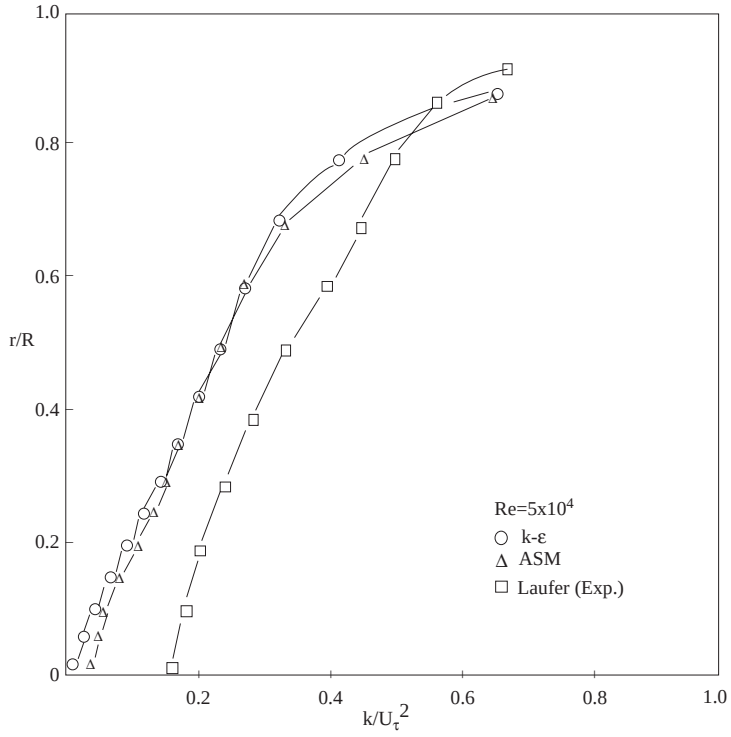

Figure 8a. Kinetic energy profiles in the pipe flow a)Re=5×10^4

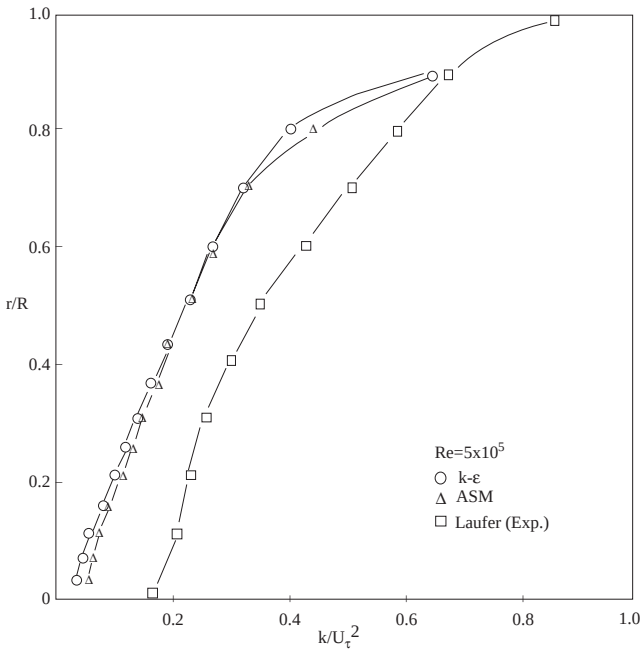

Figure 8b. Kinetic energy profiles in the pipe flow a)Re=5×10^5

C_f	:	Skin friction coefficient
D	:	Pipe diameter
k	:	Turbulent kinetic energy, $u_i\bar{u}_i/2$
l	:	Turbulence length scale
L	:	Length of the plate or pipe
r	:	Radial distance
R	:	Pipe radius
u_i	:	Fluctuating velocity component in x_i direction
U	:	Mean velocity component in axial direction
U_0	:	Free stream or centerline mean velocity
U_τ	:	Friction velocity
V	:	Mean velocity component in radal direction
x	:	Axial distance
y	:	Distance from wall
y^+	:	Dimensionless wall coordinate
σ	:	Boundary layer thickness
ε	:	Rate of dissipation of k
ν	:	Kinematic viscosity

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