

## Effect of Particle Size and Loading on Development Region in Two-Phase Flows

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### Abstract

An experimental study was conducted on the development length of a two-phase flow produced by the loading of solid particles by means of a particle feeder into air flowing through a horizontal pipe. Development length was defined as the sufficient distance downstream of the particle feeder in which a homogeneous particle distribution in air sensed with the variation of local friction factors was attained. The static pressure gradients  $dP/dx$  were measured along the test pipe to determine the local friction factors to estimate the development length for a variety of air-solid particle suspensions. Crushed wheat and semolina particles of different size, shape and apparent density were used to determine the influence of physical particle characteristics on the extent of development region. The measurements were conducted in air flow Reynolds number range of  $51500 \leq Re \leq 109000$  at particle loading ratios of  $M_p/M_a$ ,  $5\% \leq M_p/M_a \leq 30\%$ . In the covered ranges of the variables, development length was found to be a strong function of  $Re$  such that an increase in  $Re$  caused a decrease in the development length while loading ratio seemed to be of secondary importance. Although the particle size by itself was found to be of not much importance for the size range covered in the experiments, for the critical values of  $Re$  (103000) and  $M_p/M_a$  (5%) particle size was found to be effective on the development length.

**Key Words:** Two-Phase Flow, Clean Air Flow, Loading Ratio, Development Length, Friction Factor.

## İki-Fazlı Akımlarda Parçacık Büyüklüğünün ve Yüklemesinin Akım Gelişim Bölgesine Etkisi

### Özet

Yatay boru boyunca akan hava içerisine katı parçacıkların bir parçacık besleyici ile yüklenmesinden elde edilen iki-fazlı akımın gelişme bölgesi uzunluğuna yönelik deneysel bir çalışma yapıldı. Gelişme bölgesi uzunluğu, parçacık besleyiciden itibaren akış yönünde, hava içerisinde yerel sürtünme katsayısının değişimi ile ifade edilen homojen parçacık dağılımına ulaşılan yeterli mesafe olarak tanımlandı. Test borusu boyunca çeşitli hava-katı parçacık karışımlarında gelişim bölgesini tahmin etmek için statik basınç değişimleri  $dP/dx$ , sürtünme katsayılarını belirlemek için ölçüldü. Fiziksel parçacık özelliklerinin gelişme bölgesi uzunluğu üzerine etkisini belirlemek için farklı büyüklük, şekil ve göreceli yoğunluktaki bulgur ve irmik parçacıkları kullanıldı. Ölçümler  $51500 \leq Re \leq 109000$ , hava akış Reynolds sayısı aralığında ve  $5\% \leq M_p/M_a \leq 30\%$ , parçacık yükleme oranlarında yapıldı. Gelişme bölgesi uzunluğunun Reynolds sayısının güçlü bir fonksiyonu olduğu ve Reynolds sayısı arttıkça gelişme bölgesi uzunluğunun azaldığı, parçacık yükleme oranının ise ikincil derecede önemli olduğu bulundu. Parçacık büyüklüğünün deneylerde çalışılan büyüklükler için çok fazla önemli

olmadığı belirlenmiş olmasına karşın, kritik Reynolds sayısı (103000) ve parçacık yükleme oranı (5%) için parçacık büyüklüğünün gelişme bölgesi uzunluğunda etkin olduğu bulundu.

**Anahtar Sözcükler:** İki-Fazlı Akım, Temiz Hava Akımı, Yükleme Oranı, Gelişme Bölgesi Uzunluğu, Sürtünme Katsayısı

## Introduction

Since Boothroyd (1966), there has been continuing research on the modelling of two-phase flows with special emphasis paid on the associated pressure drop and drag reduction. Some of the theoretical and experimental investigations conducted on the flow of gas-solid suspensions are given by Vaseski (1973) Radin et al (1975), Yang (1978), Özbelge (1983, 1984) Rizk and Elghobashi (1989), Kennedy and Kollmann (1993) Özbelge (1997). However it is known that the complete understanding of the flow of gas-solid suspensions is not reached yet due to the complexity of flow and particularly the lack of a well-defined, widely accepted consistent method of analysis applicable for a variety of conditions used in the cited literature. The basic parameters influencing the particulate flows are the shape and size of the particles, particle loading ratio,  $M_p/M_a$ , Reynolds number, Re of air flow, and flow direction being either horizontal or vertical. Radin (1974), Pfeffer and Kane (1974) reported the theoretical and experimental studies on gas-solid flows directed towards the drag reduction. The drag reduction with fine spherical particles was observed in a Re range of  $10000 \leq Re \leq 300000$ . Similarly Rossetti and Pfeffer (1972) reported drag reduction upto approximately 75% in a vertical upward tube flow of gas-solid suspensions while Garner and Kerekes (1980) mentioned a 6% reduction in drag in a Re range of;  $150000 \leq Re \leq 300000$  with a particle loading ratio of  $M_p/M_a=21\%$  for flow of wood pulp fibers in air. Coughran (1988) studied the influence of  $M_p/M_a$  and particle shape on the pressure drop in a horizontal flow of gas-solid suspensions in a Re range of  $61000 \leq Re \leq 114000$  considering the particles with diameters d, in the range of  $8 \mu m \leq d \leq 20 \mu m$ . Michaelides and Roy (1987) evaluated the available correlations used for the prediction of pressure drop in particulate flows emphasizing the need for further investigation on this subject. In this paper, a somewhat different feature of particulate flows, development length concept is discussed. Solid particles of considerably greater size than those utilized in the cited literature, were introduced into air directing

also towards the simulation of pneumatic conveying of granular particles in pipelines (Çarpınlioğlu and Gündoğdu, 1998). In the available literature, development length of a two-phase flow field can be determined either using the velocity profiles (Lodes and Mierka, 1990) or wall static pressure gradients (Obot et al, 1993). Here, the development length which was treated to be the necessary distance for the attainment of homogeneous particle distribution in air was estimated by evaluating the variation of local friction factors calculated from measurements of wall static pressure gradients. The influence of particle loading ratio, particle size and Re of clean air flow was determined using suspensions of crushed wheat and semolina particles loaded at different  $M_p/M_a$  in the range  $5\% \leq M_p/M_a \leq 30\%$  and for Re range of  $51500 \leq Re \leq 109000$ .

## Experimental Set up and Measurements

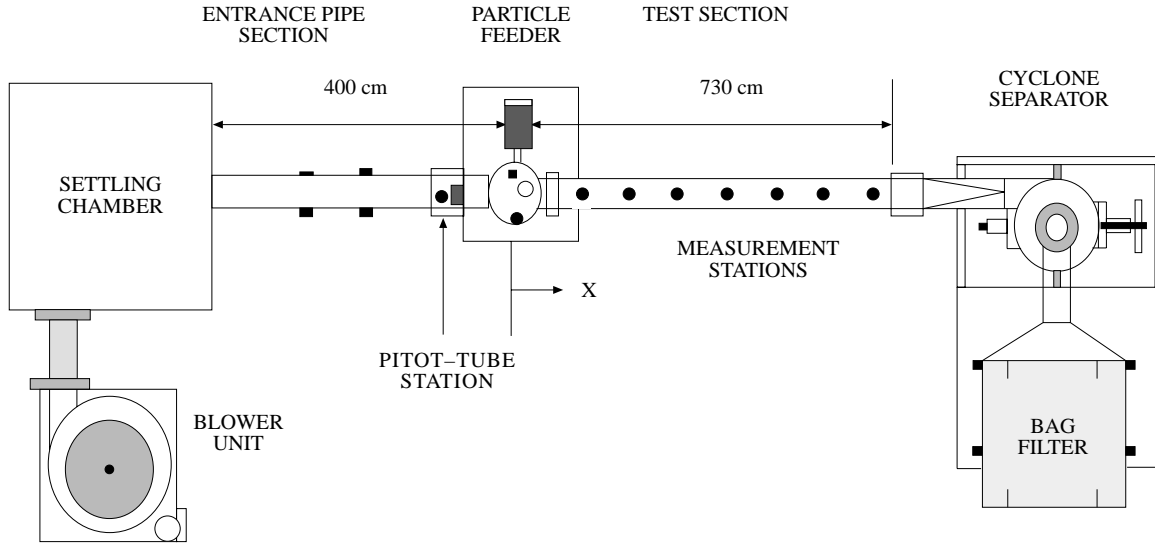
### Set up

The investigation was conducted in an open circuit blower type horizontal flow test set-up shown in Figure 1. The set-up consisted of a blower unit, a particle feeder and a pipe system connected to a cyclone separator-bag filter assembly. The blower is a centrifugal fan coupled to a variable speed control unit. A settling chamber was located at the discharge side of the blower to obtain a uniform air flow free from the disturbances induced by the blower. A (PVC) tube of inner diameter,  $D=106$  mm was used in the piping. The initial 400 cm of the pipe is called the entrance pipe section which is providing a sufficient length to have a fully-developed turbulent flow in the test section with clean air flow. The test section along which the pressure drops were taken had a length of 730 cm downstream of the particle feeder. Solid particles were collected in the cyclone separator following the test section. A reference pitot tube located at 30 cm upstream of the particle feeder was used to measure the mean flow velocity of air in the piping.

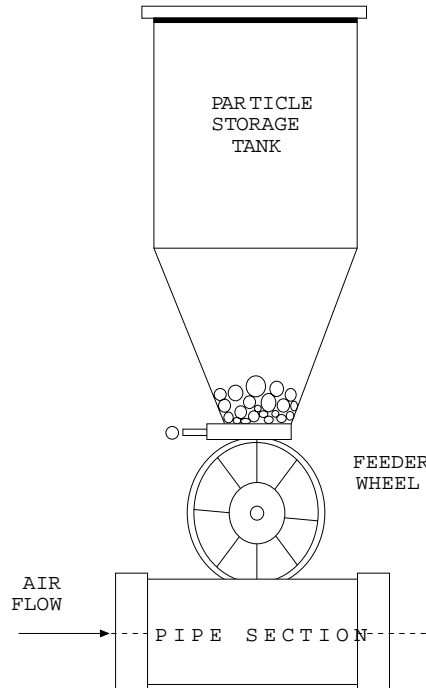
**Particle Feeder**

A celled wheel particle feeder designed and constructed by the second author Gündoğdu (1995) was used in this study. The schematic view of the particle feeder-storage tank is shown in Figure 2a. The loading of the solid particles was varied by adjust-

ing the rotational speed of the feeder wheel through a variable speed control unit such that a range of  $5\% \leq M_p/M_a \leq 30\%$  was covered. The calibration curves of the feeder wheel for the loadings of crushed wheat and semolina particles in terms of  $M_p$  and  $N$  are shown in Figure 2b.



**Figure 1.** Top view of the set-up



**Figure 2a.** The Particle Feeder

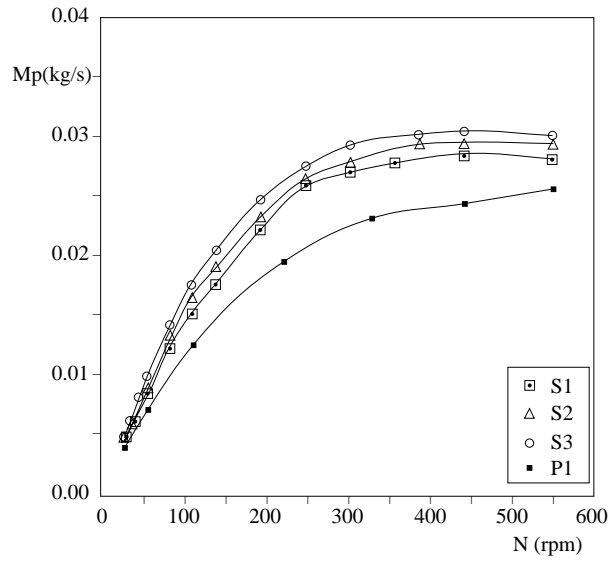


Figure 2b. Calibration Curves of the Feeder Wheel

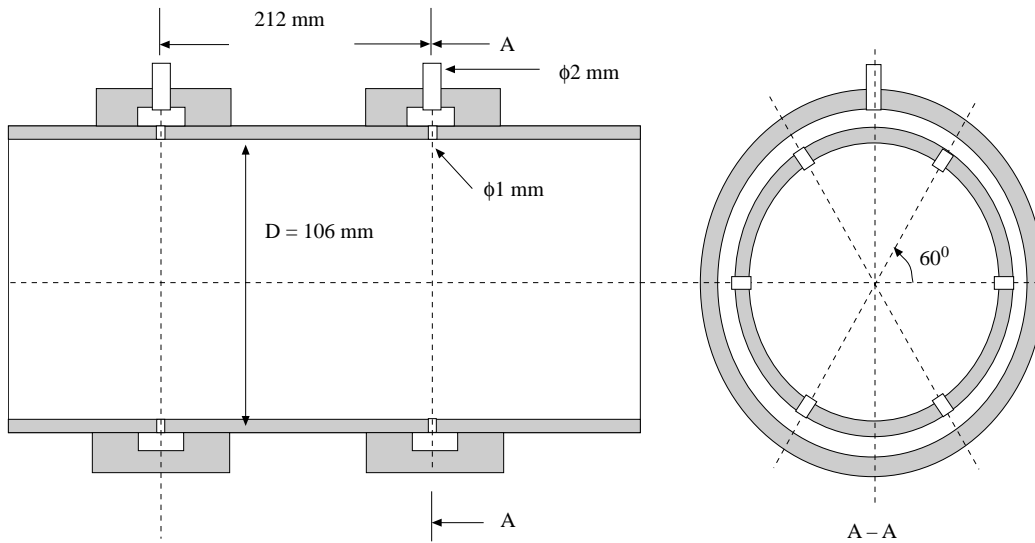


Figure 2c. Section Views of the Pressure Ring

### Solid Particles

Granular crushed wheat and three different sized semolina particles which were obtained by sieving semolina represented by letters  $P_1$  and  $S_1, S_2, S_3$  respectively, were used. The physical characteristics of tested solid particles are given in Table 1. The parti-

cle size distribution was determined by weighing the sieved quantities using Endecotts' EFL 2 MK 3 Test Sieve Shaker. The apparent densities of the particles,  $\rho_p$  were measured according to ASTM B212-76. The range of  $\rho_p$  studied in this work was  $643 \text{ kg/m}^3 \leq \rho_p \leq 746 \text{ kg/m}^3$ .

**Table 1.** Physical Characteristics of Solid Particles

Material	Material Code	Particle Size Range (microns)	Apparent Density (kg/m <sup>3</sup> )	Shape
Crushed Wheat	P1	200-1250	746.0	Irregular
Semolina	S1	600-900	673.2	Roughly Spherical
Semolina	S2	450-600	663.2	Spherical
Semolina	S3	300-450	643.0	Spherical

**Measurements**

The local static pressure gradient  $dP/dx$  on the pipe wall was determined by means of a pressure ring covering the pipe circumference installed according to BS 1042. The section views of pressure ring are given in Figure 2c. The measurements were conducted along the test section at distances from the particle feeder as  $x=3D$ ,  $x=12.5 D$ ,  $x=21.86 D$ ,  $x=31.19 D$ ,  $x=40.53 D$ ,  $x=49.87 D$  and  $x=59.16 D$ . Inclined leg alcohol-micromanometers were used in collaboration with the pressure rings. The measurements were evaluated at the standard atmospheric pressure and temperature of 100 kPa and 25°C respectively.

The pressure measurement sensitivity is such that the pressures as low as 0.35 Pa could be measured. Local magnitudes of flow friction factor,  $f$  were calculated using the well-known equality:

$$f = (dP/dx)(D/2\rho_a U^2) \tag{1}$$

The ranges of the experimental variables covered are given in Table 2. In the first part of the investigation suspensions of  $P_1$  were tested. The results obtained were then checked by using the suspensions of  $S_1, S_2, S_3$  to consider the influence of physical particle characteristics.

**Table 2.** Ranges of the Experimental Variables

Material Code	Re	U(m/s)	(Mp/Ma)%	Measurement Stations (x/D)
P <sub>1</sub>	51500	7.5	5, 10, 15, 20, 25, 30	3.0,12.5,21.86, 31.19,40.53, 49.87, 59.16
	68600	10.0	5,10,15,20	
	85830	12.5	5,10,15	
	103000	15.0	5,10,15	
	109000	16.0	5,10,15	
S1,S2,S3	51500	7.5	5,10,15,20	3.0,21.86, 40.53, 59.16
	68600	10.0	5,10,15,20	
	85830	12.5	5,10,15,20	
	103000	15.0	5,10,15	

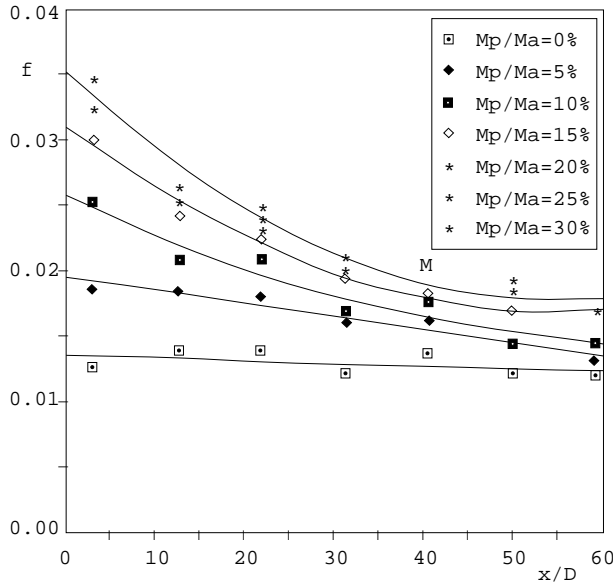
**Results and Discussion**

The local static pressure gradients  $dP/dx$  were measured along the test section for flows of clean air without particles and a variety of two-phase flows with  $5\% \leq M_p/M_a \leq 30\%$ . The sample data set for  $dP/dx$

measurements is given in Table 3. One-phase flow of clean air was such that a constant  $dP/dx$  along the test section, independent of Re in the covered speed range, was observed. However,  $dP/dx$  along the test section with two-phase flows indicated a variation of  $dP/dx$  with length from the particle feeder. The non-

dimensional representation of the measurements was expressed in terms of  $f$  and  $x/D$ . The sample plots for the variation of  $f$  with  $x/D$  at different loading ratios of two-phase flows in the studied range of  $Re$  are given in Figures 3a,3b,3c,3d,3e. As a reflection of constant  $dP/dx$  with clean air,  $f_a$  does not vary with  $x/D$  indicating fully-developed turbulent air flow in the test section. However  $f_{p+a}$  in particulate flows decreases with  $x/D$  along the test section. In comparison with the behaviour of  $f_a$  with  $x/D$ , development length of particulate flows can be estimated by a critical value of  $x/D = (x/D)_c$  at which  $f_{p+a}$  ceases to decrease rather tending to a constant value not affected from an increase in  $x/D$  further. The inspec-

tion of the above-mentioned figures indicates that  $(x/D)_c$  is a function of  $Re$  for a specified solid particle suspension in air. In conformity with the suggestion of Lodes and Mierka (1990) the development length increases with a decrease in  $Re$ . However as a contradiction to their predictions, loading ratio does not seem to have a strong influence on the extent of the development region since the curves of  $f_{p+a}$  corresponding to different  $M_p/M_a$  reach their constant value almost at the same  $x/D$ . On the other hand, increase in the loading ratio is associated with an increase in the magnitudes of local  $f_{p+a}$  values particularly within the development region, for  $x/D < (x/D)_c$  for the considered particle sizes.



**Figure 3a.** Variation of  $f$  with  $x/D$  in  $P_1$  suspensions at  $Re=51500$ .

Furthermore with suspensions of  $P_1$  an interesting fact observed from Figure 3a is that the loading ratio seems to be not much important for  $M_p/M_a > 15\%$  since the data corresponding to  $M_p/M_a > 15\%$  can be represented by a single curve which describes the variation of  $f_{p+a}$  with  $x/D$  for  $M_p/M_a$  of 20%,  $M_p/M_a$  of 25% and  $M_p/M_a$  of 30%. Therefore, with suspensions of  $S_1, S_2, S_3$  higher loading ratios ( $M_p/M_a > 20\%$ ) were not tested. The maximum magnitude of  $f_{p+a}$  in the suspensions of  $P_1, S_1, S_2, S_3$  is usually seen at the nearest position to the particle feeder,  $x/D=3$  with a rapid drop in  $f_{p+a}$  further downstream  $x/D > 3$  for  $Re \leq 103000$ . However, with the suspensions of  $P_1$  at  $Re=109000$ ,  $f_{p+a}$  re-

tains its maximum magnitude at distances far downstream of the particle feeder pointing out a possible change in flow behaviour in the development region for  $Re > 103000$  as can be seen in Figure 3b. Furthermore, with the suspensions of  $S_2, S_3$  at  $M_p/M_a=5\%$  for the covered range of  $Re$ ,  $f_{p+a}$  does not vary with  $x/D$  as can be seen from the sample plots for  $Re=85830$  and  $Re=103000$  given in Figures 3d and 3e. Therefore, regardless of the magnitude of  $Re$  as the particle size is reduced ( $d < 600 \mu m$ ) and when the loading ratio is small enough ( $M_p/M_a=5\%$ ) two-phase flow is fully-developed likewise the fully-developed clean air flow. On the other hand with the suspension of  $S_1$  at the same critical loading ra-

ratio of  $M_p/M_a=5\%$  but only for  $Re=103000$ ,  $f_{p+a}$  is constant downstream of the particle feeder. Although the covered range of the variables are not extensive to suggest a general conclusion it can be

said that particle size, loading ratio and  $Re$  are interrelated factors influencing the development length altogether with the presence of critical values of  $Re$  and  $M_p/M_a$  with the following suggestions:

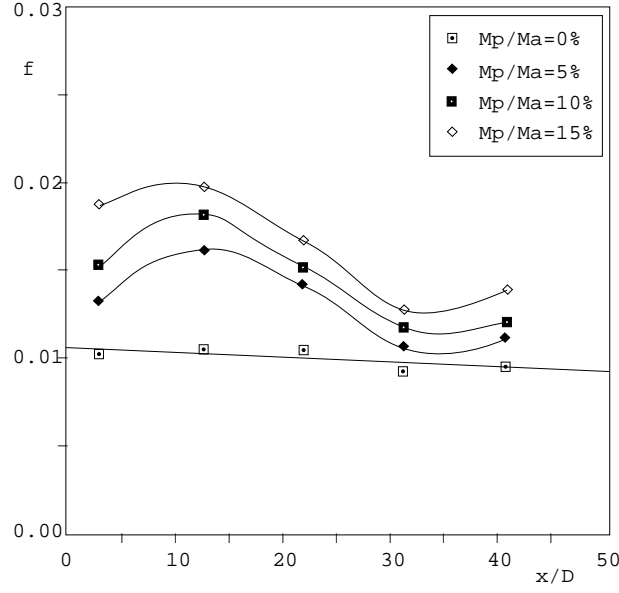


Figure 3b. Variation of  $f$  with  $x/D$  in P1 suspensions at  $Re=109000$ .

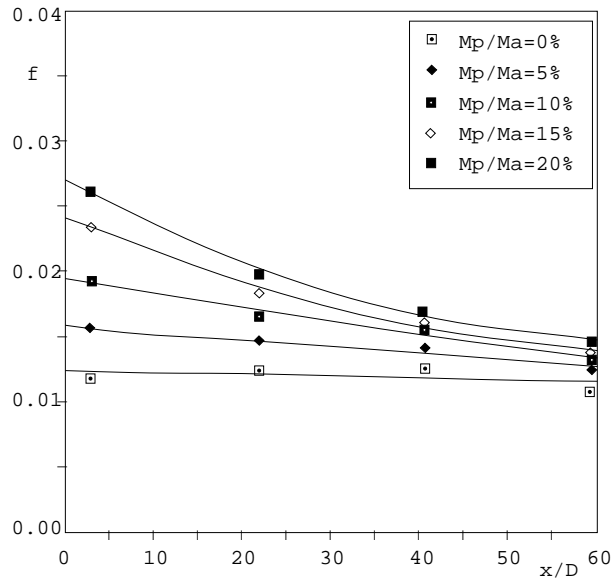


Figure 3c. Variation of  $f$  with  $x/D$  in S1 suspensions at  $Re=68600$ .

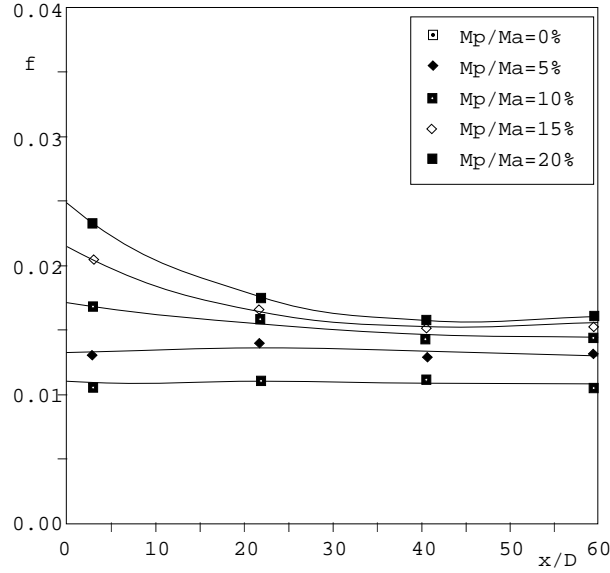


Figure 3d. Variation of  $f$  with  $x/D$  in S2 suspensions at  $Re=85830$ .

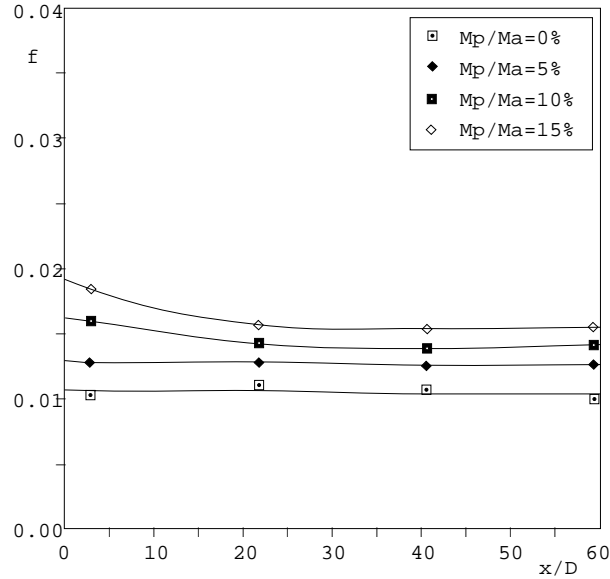


Figure 3e. Variation of  $f$  with  $x/D$  in S3 suspensions at  $Re=103000$ .

- 1) fully-developed two-phase flow is obtained with large sized particles ( $d < 600\mu m$ ) and loading ratio is at its suggested critical value ( $M_p/M_a=5\%$ );
- 2) fully-developed two-phase flow is obtained with large sized particles ( $900\mu m > d > 600\mu m$ ) at a critical loading ratio of  $M_p/M_a=5\%$  only with

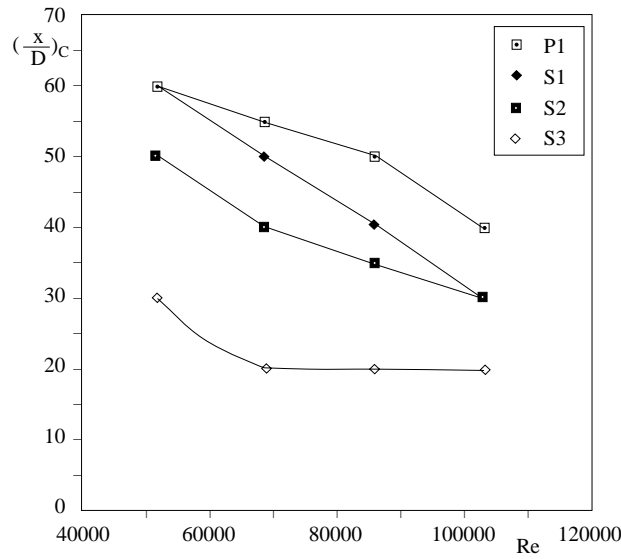
$Re$  greater than the suggested critical value of  $Re=103000$ ;

- 3) in fully-developed two-phases flows, always there exists an apparent increase in  $f_{p+a}$  over  $f_a$  due to the drag of the particles.



**Table 3.** Sample Data Set of  $dP/dx$  (Pa/m) for P1 Suspensions

Re	x/D	Mp/Ma						
		0%	5%	10%	15%	20%	25%	30%
51500	3.000	3.980	5.891	8.100	9.572	10.309	11.045	11.045
	12.500	4.430	5.907	6.645	7.705	8.122	8.457	8.457
	21.860	4.430	5.773	6.696	7.173	7.931	7.720	7.540
	31.189	3.889	5.126	5.445	6.223	6.696	6.521	6.521
	40.530	4.414	5.150	5.671	5.885	6.621	6.553	6.543
	49.868	3.882	4.658	4.658	5.435	6.211	5.913	
	59.160	3.900	4.206	4.658	5.435	5.435	5.435	
109000	3.000	14.727	19.145	22.090	27.245			
	12.500	15.186	23.627	26.581	28.796			
	21.860	15.432	20.674	22.150	24.365			
	31.189	13.592	15.558	17.113	18.669			
	40.530	13.798	16.185	17.656	20.599			



**Figure 4.** Variation of  $(x/D)_c$  with Re in suspensions of P1,S1,S2,S3.

In order to estimate the overall influence of particle size, variation of  $(x/D)_c$  with Re in the suspensions  $P_1, S_1, S_2, S_3$  is given in Figure 4. As Re increases  $(x/D)_c$  decreases in suspensions of  $P_1, S_1, S_2$  with greater the size of the particles greater the development length. The dependency of  $(x/D)_c$  on Re is such that with the finest particles of  $S_3$ ,  $(x/D)_c$  does not vary much with Re rather tending to a constant value of 20 for  $Re > 65000$ . At the end of the development length,  $(x/D)_c$ , two-phase flow is fully developed free from the loading effects and the magnitude of  $f_{p+a}$  is defined as  $(f_{p+a})_c$ . The variation of  $(f_{p+a})_c$  with Re in particulate flows with  $M_p/M_a=10\%$  is given as a sample in Figure 5 together with the data

of clean air  $(f_a)_c$ . As expected for the limited range of Re,  $(f_a)_c$  stays almost constant at 0.012 with a maximum scatter of 11% in clear air. For the particulate flows, the magnitudes of  $(f_{p+a})_c$  are greater than  $(f_a)_c$  with the approximate values of 0.0131, 0.0147, 0.0156 and 0.0164 with the suspensions of  $P_1, S_1, S_2$  and  $S_3$  respectively. The maximum scatter of the data from the above-mentioned values without no considerable change with Re are seen to be 13.8%, 13.6%, 15.4% and 16.2% for the suspensions of  $P_1, S_1, S_2$  and  $S_3$  respectively. Therefore, roughly constant  $(f_{p+a})_c$  values in the covered range of Re are treated to be a confirmation on the attainment of fully-developed flow at  $(x/D)_c$ . As the particle

size decreases, the magnitudes of  $(f_{p+a})_c$  increases due to the increase in the number of particles at the same loading ratio. In this study drag reduction has not been observed since there is no sufficient length downstream of  $(x/D)_c$ . Therefore it can be said that for drag reduction to be observed in particulate flows, flow development downstream of the particle loading should be maintained as it was done by Coughran (1988) who used a distance of approximately 120D for the attainment of fully-developed two-phase flow. In fact Coughran seems to use this distance to be safe enough because there was no data related to the development length in two-phase flow fields at that time. However it can be said that a distance much less than 120 D could be sufficient as an estimate based on the results reached in this study, since the particles he used are very small ( $d=8 \mu\text{m}$  and  $d=20 \mu\text{m}$ .)

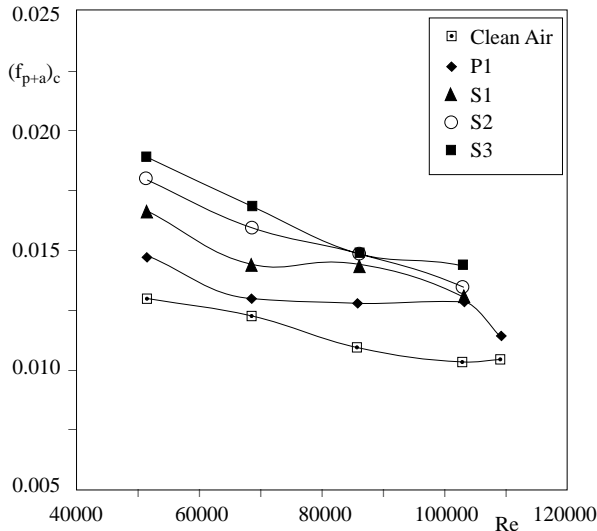


Figure 5. Variation of  $f_c$  with Re with 10% loadings of P1,S1,S2,S3.

### Concluding Remarks

In the covered range of the experimental parameters it can be said that the development length of a particulate flow is mainly influenced by the Reynolds number based on clean air flow, Re and the size of the particles. The loading ratio by itself does not cause a significant variation in the development length. However as Re increases, development length decreases and there exists critical values for the loading ratio and Re determined to be 5% and 103000

respectively for which the influence of the particle size on the development length becomes significant. As a recommendation for further study the influence of the particle shape should be investigated experimentally in the covered range of the variables cited herein.

### Acknowledgement

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### Nomenclature

d	Diameter of solid particles, $\mu\text{m}$
D	Inside pipe diameter, m
$dP/dx$	Local static pressure gradient, Pa/m
f	Friction factor, $(dP/dx)(D/2\rho_a U^2)$
$f_a$	Clean air flow friction factor
$(f)_c$	Flow friction factor at $x/D=(x/D)_c$
$f_{p+a}$	Particle-air flow friction factor
$(f_a)_c$	Clean air flow friction factor at $x/D=(x/D)_c$
$(f_{p+a})_c$	Particle-air flow friction factor at $x/D=(x/D)_c$
$M_a$	Mass flow rate of air, kg/s
$M_p$	Mass flow rate of solid particles, kg/s
$M_p/M_a$	Particle loading ratio
N	Rotational speed of the feeder wheel, rpm
P	Local wall static pressure, $\text{N}/\text{m}^2$
Re	Airflow Reynolds number, $UD/\nu$
U	Mean air velocity, m/s
x	Axial distance measured from the particle feeder, m
$\nu$	Kinematic viscosity of air, $\text{m}^2/\text{s}$
$\rho_a$	Density of air, $\text{kg}/\text{m}^3$
$\rho_p$	Apparent density of solid particles, $\text{kg}/\text{m}^3$

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