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A Chance-Constrained LP Model for Short Term Reservoir Operation Optimization

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Abstract

The effective use of water reservoirs is one of the most important problems of recent times. In particular, the optimization of the operation of water supply systems greatly affected by periods of drought is of principal importance in reducing the damage the users may face. The natural inflows which affect the result of this multistage decision problem have a random nature that must be taken into account. In this study a change-constrained LP model is proposed which takes this important factor into consideration. This model has been applied to the optimization of the monthly operation of a real water supply system. The results for different exceedance probabilities obtained by this model are compared with those obtained by a DP model.

Key Words: Reservoir operation, Optimization, Statistics

Kısa Süreli Hazne İşletme Optimizasyonu İçin Şans Kısıtlı LP Modeli

Özet

Biriktirme haznelerinin etkin kullanımı günümüzün en önemli problemlerinden biridir. Özellikle kurak dönemlerden oldukça etkilenen su temini sistemlerinin işletmelerinin optimizasyonu, kullanıcıların karşılaşabilecekleri zararları en aza indirmek açısından çok önem taşımaktadır. Bu çok adımlı karar verme probleminin sonucunu etkileyen tabii akımların rastgele karakterleri gözönüne alınmalıdır. Bu çalışmada, bu önemli davranışı gözönüne alan bir şans kısıtlı LP modeli sunulmaktadır. Bu model, gerçek bir su kaynakları sisteminin aylık işletmesinin optimizasyonuna uygulanmştır. Değişik aşılma olasılıkları için bu LP modeli ile elde edilen sonuçlar, DP modeli ile elde edilenlerle karşılaştırılmıştır.

Anahtar Sözcükler: Hazne işletmesi, Optimizasyon, İstatistik

Introduction

At present, the water supply of large cities generally comes from surface resources. Water resources and water needs are not always compatible and the regulation is carried out by storage reservoirs. Although water resources are called renewable, the average usable amount is constant. Furthermore, this amount diminishes because of pollution; thus their effective use is an important problem for all countries. The rise in population increases water need management and periods of drought affect the water supply systems. In those periods the management of water resources systems to minimize the damage to the users is of particular importance. Increases in both the number of objectives and in the relations between water resources systems, make reservoir management for the effective use of the resources a highly complex problem, especially during periods of drought. The most important uncertainties in this problem arise from the random nature of the inflows to the system. In this study, a chance-constrained linear programming (LP) model which takes this random nature into consideration was developed and the optimum management policy was sought. The reservoir planning and operation studies which have made progress in recent decades are based on Rippl's (1983) graphical method. The weakness of this method, which can only investigate a constant need, is overcome by Thomas and Burden's '(1963) sequent peak algorithm. The operation of water resource systems is a multistage decision problem. Dorfman (1962) first used the Linear Programming for the solution of this problem.

After Yehs, (1985) state of the art study, investigating the methods and practical application in reservoir operation, Dynamic Programming (DP) and Linear Programming (LP) methods are commonly used in the solution of this problem.

The principle of these optimization methods, is the determination of a set of decision variables maximizing or minimizing an objective function subject to a set of constraints. The objective function and the constraints are mathematical functions related to decision variables. In a reservoir operation problem it is convenient to designate the release or the ending storage as the decision variable. The constraits can be expressed as reservoir capacity constraints and continuity equation. The steps in this multistage decision problem are generally equal time periods. Although DP is the most convenient method for the solution of this problem, it requires a different formulation for each different system. Contrarily, LP can be taken as an easy method with its ready-to-use solution packages. The most significant difficulties in the use of this method are the linearity conditions of both the objective function and the constraints, as well as the rapid rise of the number of constraints with the stage number Yeh (1981) used the LP for short term decisions and the DP for long term decisions.

1. Chance Constrained (CC) Formulation for LP

LP models are often used in water resource problems. The objective function

$$Max\ x_0 = \sum_{j=1}^{n} c_j x_j \qquad (1)$$

and the constraints

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \quad i = 1, 2 \ldots m \qquad (2)$$

$$x_j \geq 0 \qquad j = 1, 2, \ldots m \qquad (3)$$

are linear functions of decision variables (x_j). If one introduce S_t, Q_t, E_t and R_T as, respectively, the beginning storage, the inflow, the evaporation and the release in stage t, the linear operating rule can be written, for the operation period (N) thus:

$$R_t = S_t + Q_t - E_t - b_j \quad t = 1, 2, \ldots N, \quad j = 1, 2, \ldots 12 \qquad (4)$$

as a function of operation decision variable (b_j), where j denotes the months of the year. If (4) is introduced in the continuity equation

$$S_{t+1} = S_t + Q_t - E_t - R_t \quad t = 1, 2, \ldots N \qquad (5)$$

where S_{t+1} is the ending storage, the equality

$$b_j = S_{t+1} \qquad (6)$$

can be obtained for the decision variable b_j.

The net evaporation loss (E_t) is a linear function of the average storage

$$E_t = e_{ot} + e_t \frac{S_t + S_{t+1}}{2} t = 1, 2, \ldots, N \qquad (7)$$

where e_{ot} and e_t are fixed loss and loss per unit of storage volume, respectively. If (7) and (6) are introduced in (4) linear decision rule can be written as a function of only the decision variables:

$$R_t = Q_t - e_{ot} + (1 - \frac{e_t}{2}) b_{j-1} - (1 + \frac{e_t}{2}) b_j \qquad (8)$$
$$t = 1, 2, \ldots, N, j = 1, 2, \ldots, 12$$

Since the decision variables (b_j) in (8) are determined with respect to inflows (Q_t), the random (stochastic) nature of these inflows must also be considered. At any stage, the deficit ($0 \leq D_t \leq T$), can be expressed as the difference between the target (T) and the release (R_t).

$$D_t = T - R_t, R_t \leq T,\ t = 1, 2, \ldots, N \qquad (9)$$

Thus, the nonexceedance probability of the deficit for a particular percentage ($0 \leq f_t \leq 1$) of the target can be written as the chance constraint

$$P[D_t \leq f_t T] \geq p \qquad (10)$$

Writing the deficit (9) as a function of the operating rule (8) and introducing into the chance constraint (10) results in the relationship between the target (T) and the inflow (Q_t) yields

$$P\left[(1-f_t)T + e_{ot} - (1-\frac{e_t}{2})b_{j-1} + (1-\frac{e_t}{2})b_j \leq Q_t\right] \geq P \qquad (11)$$

Q_t is a random variable having a particular probability structure. Depending on its probability function, the exceedance probability of a certain Q_t^p value can be written thus:

$$P[Q_t \geq Q_t^p] = p \qquad (12)$$

and if one introduces (12) into (11), the chance constraint can be rewritten in a deterministic form:

$$(1-f_t)T + e_{ot} - (1-\frac{e_t}{2})b_{j-1} + (1+\frac{e_t}{2})b_j \leq Q_t^p \qquad (13)$$
$$t = 1, 2, \ldots N,\quad j = 1, 2, \ldots, 12$$

Thus, with the LP model, the decisions (b_j, j=1,2,...,12) maximizing the objective function

$$Max\ T_d,\ T_d = (1-f_t)T \qquad (14)$$

can be determined, subject to the constraints

$$T_d + e_{ot} - (1-\frac{e_t}{2})b_{j-1} + (1+\frac{e_t}{2})b_j \leq Q_t^p \qquad (15a)$$
$$S_{min} \leq b_j \leq K,\ t=1, 2, \ldots, N,\ j=1, 2, \ldots, 12 \qquad (15b)$$

where S_{min} and K denote the minimum reservoir storage and reservoir capacity, respectively.

2. Case Study

2.1. Formulation

This proposed method was applied to Büyükçekmece reservoir, which is one of the units of the Istanbul Water Supply System. Reservoir characteristics are $S_{min} = 20 \times 10^6 m^3, K = 182 \times 10^6 m^3$ and the area-volume relationship is shown in Fig.1 in two-step linearized form.

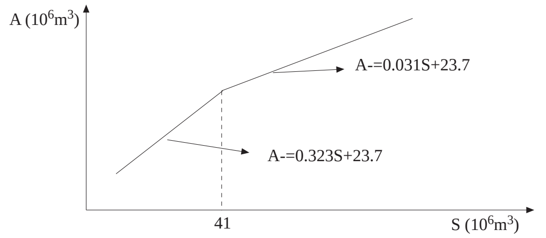

Figure 1. Area-Volume Relationship of Büyükçekmece Reservoir

In Table 1, mean monthly precipitation (P) and mean monthly evaporation (E) values are given. The evaporation volume can be calculated as follows:

Table 1. Evaporation Values for Büyükçekmece Reservoir

	Month	O	N	D	J	F	M	A	M	J	J	A	S
	P(cm)	5.1	7.5	8.4	6.7	4.5	4.4	4.5	2.9	2.5	2.5	1.8	2.4
	E(cm)	9.5	4.9	2.5	1.8	2.2	5.3	9.6	14.9	20.1	25.4	24	16.9
	ϵ_t(cm)	4.4	-2.6	-5.9	-4.9	-2.3	0.9	5.1	12.0	17.6	22.9	22.2	14.5
	e_{ot}	0.5	-0.3	-0.7	-0.55	-0.25	0.1	0.6	1.4	2.05	2.65	2.55	1.65
S<41	$10^2 e_t$	1.42	-0.84	-1.90	-1.58	-0.74	0.29	1.65	3.88	5.68	7.40	7.17	4.68
	e_{ot}	1.05	-0.6	-1.4	-1.15	-0.55	0.2	1.2	2.85	4.2	5.4	5.25	3.45
S<41	$10^2 e_t$	0.14	-0.08	-k0.18	-0.15	-0.07	0.03	0.16	0.37	0.55	0.71	0.69	0.45

$$E_t = 10^{-2}\epsilon_t \times A = 10^{-2}\epsilon_t(bS+a) = e_{ot}+e_t S \quad (16)$$

where ϵ_t is the monthly net evaporation rate and $e_{ot} = 10^{-2}\epsilon_t.a, e_t = 10^{-2}\epsilon_t.b$. The values for e_{ot} and e_t are shown in Table 1.

Since the purpose here is short-term optimal operation policy determination, the operation period was taken to be one year (N=12), considering the periodicity in both inflows and needs. Thus, the objective function of this LP formulation can be written

$$Max\ T_d \quad (17)$$

and the constraints

$$T_d - (1-\frac{e_t}{2})b_{j-1} + (1+\frac{e_t}{2})b_j \leq Q_t^p - e_{ot} \quad (18a)$$
$$t, j = 1, 2, \ldots, 12$$
$$b_j \leq 182 \times 10^6, j = 1, 2, \ldots, 12 \quad (18b)$$
$$-b_j \leq -20 \times 10^6, j = 1, 2, \ldots, 12 \quad (18c)$$

where the decision parameters (b_j) are the monthly ending storages. Since the operation period is one year (12 months), there will be no difference between the t and j subscripts.

3. Solution

In the application phase the exceedance probability value, p, and thus the values for Q_t^p must be determined. Here two approaches can be introduced. First, the determination of Q_t^p values for each month of the year as the values corresponding to a certain exceedance probability p, according to (12). The low flows and mean flows corresponding, respectively, to $p = 0.8$ and $p = 0.5$ are taken into account. The Q_t^p values for these probabilities are given Table 2 as 1 and 2, respectively. The sequential nature of the inflows has importance in reservoir operation and target value. Then, the second approach is to determine the Q_t^p values from the 20 years long observed flow series. Thus the most critical year, the second most critical yera and least critical year (wet year) were taken into account corresponding, respectively, to exceedance probabilities p=0.95, p=0.90 and p=0.05. In the observed series, these correspond to the years 1991, 1973, 1982, respectively, and the Q_t^p values are given in Table 2 as No 3,4,5. An important problems is the determination of the firm yield in planning and operation phases of the storage reservoirs. Thus the percentage (f_t) in (10) and (13) equals zero ($f_t = 0$) and then the objective function (17) becomes $Max\ T_d = Max\ T$. The critical period determines the firm yield. The definition of the critical period depends on the nature of the problem. Here, it is defined as the time period where the inflows are lowest for a particular water use (need) in a particular water resource system. The critical period was determined to be two years (24 months), from 1990 to 1991, shown in Table 2 as No 6.

The first 5 series corresponding to various p values were used in the LP model described in (17) and (18) and the results are given in Table 3. The ending storage values maximizing the objective function for these series are in Table 4. Similarly, the resulting T_d value for the series 6 is in Table 5 and the linear operating rule monthly ending storages ($b_j, j = 1, 2, \ldots, 12$) are in Table 6.

Table 2. $Q_t^p (10^6 m^3/\text{month})$ Values for Different p Values

No	Month	O	N	D	J	F	M	A	M	J	J	A	S
1	$Q^{0.8}$	1.3	4.0	6.4	8.9	8.0	6.4	3.8	2.0	2.0	1.9	2.0	2.1
2	$Q^{0.5}$	6.3	10.7	17.1	19.7	15.1	16.8	8.0	5.0	3.1	2.3	2.2	2.4
3	1991	2.0	5.6	6.8	5.8	5.1	3.5	3.4	3.6	3.1	1.6	2.3	3.1
4	1973	0.8	7.7	18.4	5.5	10.0	3.4	3.8	5.0	0.6	0.5	0.4	0.5
5	1982	3.5	7.0	8.3	63.8	32.3	19.1	6.1	5.6	2.9	2.5	2.2	2.5
6	Critical	1.3	14.9	63.2	15.6	13.1	1.4	1.6	1.7	1.9	1.9	1.4	1.5
	period	2.0	5.6	6.8	5.8	5.1	3.5	3.4	3.6	3.1	1.6	2.3	3.1

Table 3. T_d Values from LP and DP Models for Various Q_t^p Series

Flow series / Model	1	2	3	4	5
LP	2.5	6.9	2.1	3.1	11.2
DP	2.6	7.2	2.0	3.1	11.4

4. Solution With Linear Programming (LP)

The problem can be investigated, deterministically, with given inflow values, using the continuity equation (5). With LP formulation (1-3), using (6) and (7), the objective function can be written as follows:

Table 4. Monthly Reservoir Ending Storages for Different Q_t^p Series Obtaiend With LP and DP Models

Flow series	Month / Model	O	N	D	J	F	M	A	M	J	J	A	S
1	CC LP	20	22	27	34	40	44	44	41	36	30	25	22
	DP	20	22	27	35	41	45	45	42	37	31	25	22
2	CC LP	20	24	35	50	29	69	67	60	49	38	29	21
	DP	20	24	35	49	58	69	68	63	54	44	33	22
3	CC LP	20	24	30	35	38	40	40	38	35	30	25	21
	DP	20	25	31	36	40	42	42	40	37	31	27	21
4	CC LP	20	25	42	45	53	53	52	51	44	36	29	23
	DP	20	25	42	46	53	54	53	52	45	37	29	23
5	CC LP	25	22	20	74	96	105	98	89	76	62	47	34
	DP	25	22	20	74	96	105	98	89	76	62	47	34

Table 5. T_d Values Obtained in Critical Period With LP and DP Models

Model	CC LP	LP	DP
T_d	1.54	4.8	4.8

$$MaxT \tag{19}$$

and the constraints thus:

$$R_t(1-\frac{e_t}{2})b_{t-1}+(1+\frac{e_t}{2})b_t \leq Q_t - e_{ot} \tag{20a}$$
$t=1,2,\ldots,N$

$$-R_t+(1-\frac{e_t}{2})b_{t-1}-(1+\frac{e_t}{2})b_t \leq -Q_t-e_{ot} \tag{20b}$$
$t=1,2,\ldots,N$

$$T-R_t \leq 0 \quad t=1,2,\ldots,N \tag{20c}$$

$$S \leq 182.10^6 \quad t=1,2,\ldots,N \tag{20d}$$

$$S_t \leq -20.10^6 \quad t=1,2,\ldots,N \tag{20e}$$

The critical period inflows used in this problem result in a T_d value and ending storages given in Tables 5 and 6, respectively. It is seen that the target value is much higher than the value obtained by CCLP where an annual operating rule was assumed.

Table 6. Ending Reservoir Storages Obtained From Critical Period With LP and DP Models ($10^6 m^3$/month)

Month / Model	O	N	D	J	F	M	A	M	J	J	A	S
CC LP	20	24.5	31	36.4	40.5	40	38.9	36.2	32.6	27.8	23.2	21
LP	20	30.6	90.8	104.6	114	110	104.6	96.2	86	74.6	63.6	56.4
	52	53.8	57.6	60	61	59.5	56.5	51.8	45.2	36.3	28.9	24.3
DP	20	30.8	90.6	109.5	112.2	109.3	104.7	98.1	90.2	81.5	71.9	64.9
	60.3	62	65.3	67.8	68.6	67.8	64.9	60.3	53.6	44.5	36.2	24.3

5. Solution With Dynamic Programming (DP)

In order to compare the results of the LP formulation, the optimization of the system was carried out with a DP model. For this purpose, the OPTIMA program (Duranyıldız, 1988) for optimizing the operation of a river basin was used. Since the maximization of the firm water is under investigation, the objectiv function can be written

$$maxT = max \left[\begin{matrix} min(V_f) \\ t \end{matrix} \right], t = 1, 2, \ldots, N (21)$$

and the transition equation for any stage

$$f_t^\star(S_t) = max[minV_f(S_t + 1), f_{t-1}\star(S_t)] \quad (22)$$
$$t = 1, 2, \ldots, N$$

where V_f is the return function. The constraints of this model are the same as for the LP model (18 b, c). The DP model has been applied to the 6 flow series determined earlier with the same initial storages as the LP models, and the results are summarized in Tables 3 and 5. The ending storages corresponding to these results are in Tables 4 and 6. The results of the DP and LP models are very close.

6. Conclusions

In recent times water supply has become an important problem, especially for large cities. The rapid increase in water need and the limited amount of available water necessitate the effective use of water resources. The basis of the solution of this problem is the determination of the optimal operation of the water resource systems, and it is of great importance, in this context, to take into account the random nature of the inflows. In this study a chance constrained (CC) LP model able to take into account this random nature of the flows was developed. Optimal monthly operating policies for a one-year period were determined for different inflows with different exceedance probability levels for the water supply system to which the model was applied. Here the deficit percentage (f_t) of the target water was taken to be zero and the firm yield maximization was investigated. Similarly, different results may be obtained with different deficit percentages and the risk can be calculated. No difference was found between the results obtained from LP and DP models. Especially for short term operation optimization, the LP model proposed above, which takes into account the stochastic nature of the natural flows seems more suitable because of the ready-to-use solution packages for obtaining rapid results. It is seen also that the model yields much smaller results when a yearly operation rule is introducted into the model.

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