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C–Closed Sets in L–Fuzzy Topological Spaces and Some of its Applications

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Abstract

We introduce and study the notion of C–closed sets in L–fuzzy topological spaces. Then, C–convergence theory for nets and ideals is established in terms of C–closedness. Finally, we give a new concept of C–continuity on L–fuzzy topological space by means of L–fuzzy C–closedness and investigate some of its properties and its relationships with other L–fuzzy mappings introduced previously. Then we systematically study the characterizations of this notion with the aid of the C–convergence of L–fuzzy nets and L–fuzzy ideals.

Keywords and phrases. L–fuzzy topology, Q_α–compactness, L–fuzzy C–closed set, L–fuzzy C–continuous mappings, L–fuzzy net, L–fuzzy ideal, C–convergence.

1. Introduction

Continuity and its weaker forms constitute an important and intensely investigated area in the field of general topological spaces. For example, the notions of almost continuous, N–continuous, H–continuous, C–continuous, weakly continuous and semi–continuous have been introduced by different authors, and their inter–relationships with other topological notions have been established. Most of these notions turn out to be local properties; hence the pointwise approach is generally preferred in their studies and definitions. The concept of C–continuity in general topology was introduced by Gentry and Hoyle [5] in 1970. The class of C–closed sets (compact and closed) was defined by Garg and Kumar [4] in 1989. Then several characterizations of C–continuous mappings in terms of C–closed sets are given. Recently, Dang, Behera and Nanda [3] extended the concept to fuzzy topology, and introduced the notion of fuzzy C–continuous function using the fuzzy compactness given by Mukherjee and Sinha [8]. However, the fuzzy compactness has some shortcomings, such as the Tychonoff product theorem does not

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hold, and it is contradicts some kinds of separation axioms. Hence, the notion of fuzzy C–continuous function in [3] is unsatisfactory. In this paper, we first define the concept of L–fuzzy C–closed sets by means of the concept of L–fuzzy Q_α–compactness in the sense of Wang [11]. Then by making use of L–fuzzy C–closed sets we introduce and study the C–convergence theory of L–fuzzy nets and L–fuzzy ideals. Finally, we give a new definition of fuzzy C–continuous which calls L–fuzzy C–continuity on the basis of the notion of L–fuzzy C–closedness in L–fuzzy topology, and systematically discuss its characterizations and properties by making use of C–converges theory of L–fuzzy nets and L–fuzzy ideals.

2. Preliminaries

Throughout this paper, L denotes a complete, completely distributive lattice; $M(L)$ denotes the set of all nonzero irreducible elements of L; and 0 and 1 denote the least and greatest element in L, respectively. L^X and L^Y denote the set of all L–fuzzy sets on crisp sets X and Y, respectively. Write $M(L^X) = \{x_\alpha \in L^X : x \in X, \alpha \in M(L)\}$, and call the elements in $M(L^X)$ molecules or L–fuzzy points on X. For $\varphi \subset L^X$, put $\varphi' = \{\mu' : \mu \in \varphi\}$.

Let (L^X, τ) be an L–fuzzy topological space, briefly L–fts. For each $\mu \in L^X$, $c\ell(\mu)$, $int(\mu)$ and μ' will denote the closure, interior and the complement of μ, respectively. 0_X and 1_X denote, respectively, the least and the greatest element of L^X. If $\mu \in L^X$ and $\mu = int(c\ell(\mu))$, then it is called regular open. The complement of regular open is called regular closed. The class of all L–fuzzy regular open (resp. regular closed) sets will be denoted by $RO(L^X, \tau)$ (resp. $RC(L^X, \tau)$). Let (X, T) be a crisp topological space and $\mu \in L^X$, if $\forall \alpha \in L$, $\{x \in X : \mu(x) \leq \alpha\} \in T'$, then we call μ a lower semi–continuous function. The set of all these functions is denoted by $\omega_L(T)$ and is an L–fuzzy topology on X generated by T.

Definition 2.1 [10]: Let (L^X, τ) be an L–fts and $x_\alpha \in M(L^X)$. $\lambda \in \tau'$ is called a remoted neighbourhood (R–nbd, for short) of x_α if $x_\alpha \not\leq \lambda$. The set of all R–nbds of x_α is denoted by R_{x_α}.

Definition 2.2 [1,10]: Let (L^X, τ) be an L–fts and $\mu \in L^X$. $\Psi \subset \tau'$ (resp. $\Psi \subseteq RC(L^X, \tau)$) is called an α–remoted (resp. α–regular closed remoted) neighbourhood family of μ, briefly α–RF (resp. α–rcRF) of μ, if for each L–fuzzy point $x_\alpha \leq \mu$, there is $\eta \in \Psi$ such that $\eta \in R_{x_\alpha}$.

Definition 2.3 [9,10]: Let (L^X, τ) be an L–fts. Then $\mu \in L^X$ is called:

(i) Q_α–compact (resp. nearly Q_α–compact) if for any $\alpha \in M(L)$ and every α–RF (resp. α–rcRF) Ψ of μ there exists a finite subfamily $\Psi_\circ$ of Ψ such that $\Psi_\circ$ is an α–RF of μ.

(ii) Strong Q–compact (resp. Strong nearly Q–compact) if it is Q_α–compact (resp. nearly Q–compact) for all $\alpha \in M(L)$.

If 1_X is Q_α–compact (resp. nearly Q_α–compact, strong Q–compact, strong nearly Q–compact), then we say that (L^X, τ) is a Q_α–compact (resp. nearly Q_α–compact, strong Q–compact, strong nearly Q–compact) space.

Definition 2.4 [7]: An (L^X, τ) is said to be :

(i) LFT_2–space (L–fuzzy Hausdorff space) iff $(\forall x_\alpha, y_\gamma \in M(L^X), x \neq y)$ $(\exists \eta \in R_{x_\alpha})(\exists \lambda \in R_{y_\gamma})(\eta \vee \lambda = 1_X)$.

(ii) LFR_2–space (L–fuzzy regular space) iff $(\forall x_\alpha \in M(L^X))(\forall \eta \in R_{x_\alpha})$ $(\exists \lambda \in R_{x_\alpha})(\exists \rho \in \tau')(\lambda \vee \rho = 1_X$ and $\eta \wedge \rho = 0_X)$.

(iii) Fully stratified if $\underline{\alpha} \in \tau$ for all $\alpha \in L$.

(iv) Weakly induced if each nonempty element of τ is a lower semi–continuous mapping from $(X, [\tau])$ to L.

(v) Induced if it is both fully stratified and weakly induced.

The family of all crisp open (resp. closed) sets in τ is denoted by $[\tau]$ (resp. $[\tau']$). Obviously, $(X, [\tau])$ is a crisp topological space.

Theorem 2.5 [7]: A topological space (X, T) is a T_2-space iff an L–fts $(L^X, \omega_L(T))$ is a LFT_2-space.

Theorem 2.6 [6]: For fully stratified L–fts (L^X, τ) and $\mu \in L^X$, if for each $\alpha \in M(L)$, $\mu_{w\alpha} \in [\tau']$, then $\mu \in \tau'$, where $\mu_{w\alpha} = \{x \in X : \mu(x) \geq \alpha$ and $\alpha \in M(L)\}$.

Theorem 2.7 [9]: Each strong Q–compact L–fuzzy set in a fully stratified LFT_2–space is L–fuzzy closed.

Theorem 2.8 [9]: Every L–fuzzy closed set of a Q_α–compact (resp., strong Q–compact) L–fts is Q_α–compact (resp., strong Q–compact).

Theorem 2.9 [9]: Let (X,T) be a topological space. Then L–fuzzy set $\mu \in L^X$ is Q_α–compact in $(L^X, \omega_L(T))$ iff $\mu_{w\alpha}$ is compact in (X,T), for all $\alpha \in M(L)$.

Theorem 2.10 [9]: Let (L^Y, Δ) be an LFT_2–space and $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L–fuzzy continuous mapping [12] and $\mu \in L^X$ be a strong Q–compact in (L^X, τ), then $f(\mu)$ is a strong Q–compact L–fuzzy set in (L^Y, Δ).

Theorem 2.11 [9]: Let (L^X, τ) be an LFR_2–space. Then every strong nearly Q–compact set is strong Q–compact.

Theorem 2.12 [9]: Let (L^X, τ) be an induced L–fts. Then the concepts of N–compactness and strong Q–compactness are equivalent.

Definition 2.13 [12,15]: Let (L^X, τ) be an L–fts. An L–fuzzy net in (L^X, τ) is a mapping $S : D \to M(L^X)$ denoted by $S = \{S(n), n \in D\}$, where D is a directed set. S is said to be in $\mu \in L^X$ if $\forall n \in D$, $S(n) \leq \mu$.

Definition 2.14 [12,13]: The non empty family $\mathcal{L} \subset L^X$ is called an L–fuzzy ideal if, for each μ_1, $\mu_2 \in L^X$ the following satisfies:

(i) If $\mu_1 \leq \mu_2$ and $\mu_2 \in \mathcal{L}$, then $\mu_1 \in \mathcal{L}$.

(ii) If μ_1, $\mu_2 \in \mathcal{L}$, then $\mu_1 \vee \mu_2 \in \mathcal{L}$.

(iii) $1_X \notin \mathcal{L}$.

Theorem 2.15 [12,13]: Let (L^X, τ) be an L–fts, $\mu \in L^X$ and $x_\alpha \in M(L^X)$. Then $x_\alpha \leq c\ell(\mu)$ iff there exists an L–fuzzy net in μ (resp., an L–fuzzy ideal $\mathcal{L}$ not containing μ) which converges to x_α (see Definitions 3.9 and 3.11).

Other unexplained notations and definitions in this paper can be found in [1,2,9,12,13].

3. L–fuzzy C–closure and C–interior operators.

In this section, we introduce and study the concepts of C–closure operator and C–interior operator by having the aid of the notion of Q_α–compactness and discuss their properties. Then we present the concepts of C–limit and C–cluster points of L–fuzzy nets and L–fuzzy ideals.

Definition 3.1: Let (L^X, τ) be an L–fts and $\mu \in L^X$. An L–fuzzy point $x_\alpha \in M(L^X)$ is called an C–adherent (resp. N^*–adherent) point of μ, written as $x_\alpha \leq C.c\ell(\mu)$ (resp. $x_\alpha \leq N^*.c\ell(\mu)$) iff $\mu \nleq \lambda$ for each $\lambda \in CR_{x_\alpha}$ (resp. $\lambda \in N^*R_{x_\alpha}$), where CR_{x_α} (resp. $N^*R_{x_\alpha}$) is the family of all strong Q–compact (resp. strong nearly Q–compact) R–nbds of x_α. $C.c\ell(\mu)$ (resp. $N^*.c\ell(\mu)$) is said to be C–closure (resp. N^*–closure) of μ. If $C.c\ell(\mu) \leq \mu$ (resp. $N^*.c\ell(\mu) \leq \mu$), then μ is called L–fuzzy C–closed (resp. N^*–closed). The complement of an L–fuzzy C–closed (resp. N^*–closed) set is called L–fuzzy C–open (resp. N^*–open) set.

In [1], Chen and Wang have introduced the concept of L–fuzzy N–closed sets by using N–compactness due to Zhao [14]. It is easy to see that every L–fuzzy N^*–closed set is N–closed. So the properties and characterizations of N^*–closed set and its related notions are similar to those of N–closed set.

Theorem 3.2: Let (L^X, τ) be an L–fts and $\mu, \eta \in L^X$. Then the following statements hold:

(i) $\mu \leq c\ell(\mu) \leq N^*.c\ell(\mu) \leq C.c\ell(\mu)$.

(ii) If $\mu \leq \eta$, then $C.c\ell(\mu) \leq C.c\ell(\eta)$.

(iii) $C.c\ell(C.c\ell(\mu)) = C.c\ell(\mu)$.

(iv) $C.c\ell(\mu) = \wedge\{\rho \in L^X : \rho$ is a C–closed set containing $\mu\}$.

Proof: It is similar to that of Theorem 3.1 in [2].

Theorem 3.3: Let (L^X, τ) be an L–fts. The following statements hold:

(i) 1_X and 0_X are both C–closed.

(ii) Every strong Q–compact closed set is C–closed.

(iii) The union of finite C–closed sets is C–closed.

(iv) The intersection of arbitrary C–closed sets is C–closed.

(v) $\mu \in L^X$ is C–closed iff there exists $\eta \in CR_{x_\alpha}$ such that $\mu \leq \eta$ for each $x_\alpha \in M(L^X)$ with $x_\alpha \nleq \mu$.

Proof: It is similar to that of Theorem 3.2 in [2].

Theorem 3.4: Let (L^X, τ) be an L–fts and $\mu \in L^X$. Then the families

$$\tau_C = \{\mu \in L^X : \mu' = C.c\ell(\mu')\} \text{ and } \tau_{N^*} = \{\mu \in L^X : \mu' = N^*.c\ell(\mu')\}$$

of all L–fuzzy C–open and N^*–open sets in X are L–fuzzy topologies on X associated with τ. We call (L^X, τ_C) and (L^X, τ_{N^*}) L–fuzzy C–space and L–fuzzy N^*–space, respectively, induced by (L^X, τ).

Proof: It is an immediate consequnce of Definition 3.1 and Theorems 3.2 and 3.3.

Theorem 3.5: Let (L^X, τ) be an L–fts. Then

(i) $\tau_C \leq \tau_{N^*} \leq \tau$.

(ii) If (L^X, τ) is strong Q–compact (resp. strong nearly Q–compact), then $\tau = \tau_C$ (resp. $\tau = \tau_{N^*}$).

(iii) If (L^X, τ) is LFR_2–space, then $\tau_C = \tau_{N^*}$.

(iv) If (L^X, τ) is induced L–fts, then $\tau_{N^*} = \tau_N$[1].

Proof: Follows from Theorems 2.11, 2.12 and 3.4.

Definition 3.6: Let (L^X, τ) be an L–fts, $\mu \in L^X$ and $C.int(\mu) = \vee\{\rho \in L^X : \rho$ is an L–fuzzy C–open set contained in $\mu\}$. We say that $C.int(\mu)$ is the C–interior of μ.

The following theorem shows the relationships between C–closure operator and C–interior operator.

Theorem 3.7: Let (L^X, τ) be an L–fts and $\mu \in L^X$. Then the following are true.

(i) μ is C–open iff $\mu = C.int(\mu)$.

(ii) $C.int(\mu) \leq int(\mu) \leq \mu$.

(iii) $C.int(\mu) = (C.c\ell(\mu'))'$.

(iii) If $\eta \in L^X$ and $\mu \leq \eta$, then $C.int(\mu) \leq C.int(\eta)$.

(iv) $C.int(C.int(\mu)) = C.int(\mu)$.

Dually, we have the following results.

Theorem 3.8: Let (L^X, τ) be an L–fts. The following statements hold:

(i) 1_X and 0_X are both C–open.

(ii) The intersection of finite C–open sets is C–open.

(iii) The union of arbitrary C–open sets is C–open.

Definition 3.9: Let S be an L–fuzzy net in an L–fts (L^X, τ) and $x_\alpha \in M(L^X)$. Then x_α is said to be a:

(i) limit point of S [12] or S converges to x_α, in symbol $\mathcal{S} \rightarrow x_\alpha$, if
$(\forall \lambda \in R_{x_\alpha})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda)$.

(ii) C–limit point of S or S C–converges to x_α, in symbol $\mathcal{S} \xrightarrow{C} x_\alpha$, if
$(\forall \lambda \in CR_{x_\alpha})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda)$.

The union of all limit (resp. C-limit) points of S is denoted by $lim(S)$ (resp. $C.lim(S)$).

Proposition 3.10: Suppose that S is an L–fuzzy net in (L^X, τ), $\mu \in L^X$ and $x_\alpha \in M(L^X)$. Then the following results are true:

(i) $x_\alpha \leq C.lim(S)$ iff $\mathcal{S} \xrightarrow{C} x_\alpha$.

(ii) $lim(S) \leq C.lim(S)$.

(iii) $x_\alpha \leq C.c\ell(\mu)$ iff there is an L–fuzzy net in μ which C-converges to x_α.

(iv) $C.lim(S)$ is an L–fuzzy C–closed set in L^X.

Proof: (i) Let $\mathcal{S} \xrightarrow{C} x_\alpha$, so by definition $x_\alpha \leq C.lim(S)$. Conversely, let $x_\alpha \leq C.lim(S)$ and $\lambda \in CR_{x_\alpha}$. Since $x_\alpha \not\leq \lambda$, so we have $C.lim(S) \geq \alpha > \lambda(x)$. Thus $C.lim(S) \not\leq \lambda$. Therefore there exists $y_\beta \in M(L^X)$ such that $\mathcal{S} \xrightarrow{C} y_\beta$, but $y_\beta \not\leq \lambda$ and so $\lambda \in CR_{y_\beta}$. Hence $(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda)$. Thus $\mathcal{S} \xrightarrow{C} x_\alpha$.

(ii) Let $x_\alpha \leq lim(S)$ and $\eta \in CR_{x_\alpha}$. Since $CR_{x_\alpha} \leq R_{x_\alpha}$, then $\eta \in R_{x_\alpha}$. And since $x_\alpha \leq lim(S)$, then $(\forall \lambda \in R_{x_\alpha})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda)$ and so $S(m) \not\leq \eta$. Hence $x_\alpha \leq C.lim(S)$. So $lim(S) \leq C.lim(S)$.

(iii) Let $x_\alpha \leq C.c\ell(\mu)$. Then $(\forall\lambda \in CR_{x_\alpha})(\mu \not\leq \lambda)$ and so there exists $\alpha(\mu,\lambda) \in L\backslash\{0\}$ such that $x_{\alpha_{(\mu,\lambda)}} \leq \mu$ and $x_{\alpha_{(\mu,\lambda)}} \not\leq \lambda$. Since the pair $(CR_{x_\alpha}, \geq)$ is a directed set, we can define an L–fuzzy net $S : CR_{x_\alpha} \to M(L^X)$ given by $S(\lambda) = x_{\alpha_{(\mu,\lambda)}}, \forall\lambda \in CR_{x_\alpha}$. Then S is an L–fuzzy net in μ. Now let $\rho \in CR_{x_\alpha}$ such that $\rho \geq \lambda$, so we have the situation in which there exists $S(\rho) = x_{\alpha_{(\mu,\rho)}} > \rho \geq \lambda$. Then $x_{\alpha_{(\mu,\rho)}} \not\leq \lambda$. So $\mathcal{S}\overset{C}{\to}x_\alpha$. Conversely, let S be an L–fuzzy net in μ with $\mathcal{S}\overset{C}{\to}x_\alpha$. Then $(\forall\lambda \in CR_{x_\alpha})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda)$. Since S is an L–fuzzy net in μ, then $\mu \geq S(m) > \lambda$. Hence $(\mu \not\leq \lambda)(\forall\lambda \in CR_{x_\alpha})$. So $x_\alpha \leq C.c\ell(\mu)$.

(iv) Let $x_\alpha \leq C.c\ell(C.lim(S))$ and $\lambda \in CR_{x_\alpha}$. Then $C.lim(S) \not\leq \lambda$. So there exists $y_\beta \in M(L^X)$ such that $y_\beta \leq C.lim(S)$ and $y_\beta \not\leq \lambda$. Then $(\forall\rho \in CR_{y_\beta})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \rho)$ and so $S(m) \not\leq \lambda$. Hence $x_\alpha \leq C.lim(S)$. Thus $C.c\ell(C.lim(S)) \leq C.lim(S)$ and so $C.lim(S)$ is a L–fuzzy C–closed set.

Definition 3.11: Let $\mathcal{L}$ be an L–fuzzy ideal in an L–fts (L^X, τ) and $x_\alpha \in M(L^X)$. Then x_α is said to be:

(i) a limit point of $\mathcal{L}$ [13] or $\mathcal{L}$ converges to x_α, in symbol $\mathcal{L} \to x_\alpha$, if $R_{x_\alpha} \subseteq \mathcal{L}$.

(ii) C–limit point of $\mathcal{L}$ or $\mathcal{L}$ C–converges to x_α, in symbol $\mathcal{L}\overset{C}{\to}x_\alpha$, if $CR_{x_\alpha} \subseteq \mathcal{L}$.

The union of all limit points (resp., C–limit points) of $\mathcal{L}$ is denoted by $lim(\mathcal{L})$ (resp. $C.lim(\mathcal{L})$).

Proposition 3.12: Suppose that $\mathcal{L}$ is an L–fuzzy ideal in (L^X, τ), $\mu \in L^X$ and $x_\alpha \in M(L^X)$. Then the following results are true:

(i) $x_\alpha \leq C.lim(\mathcal{L})$ iff $\mathcal{L}\overset{C}{\to}x_\alpha$.

(ii) $lim(\mathcal{L}) \leq C.lim(\mathcal{L})$.

(iii) $x_\alpha \leq C.c\ell(\mu)$ iff there is an L–fuzzy ideal $\mathcal{L}$ which C-converges to x_α and $\mu \not\leq \mathcal{L}$.

(iv) $C.lim(\mathcal{L})$ is an L–fuzzy C–closed set in L^X.

Proof: The proof of the statements (i), (ii) and (iv) are similar to the correspondence statements of Proposition 3.10.

(iii) Let $x_\alpha \leq C.c\ell(\mu)$. Let $\mathcal{L}(CR_{x_\alpha}) = \{\rho \in L^X \colon \exists\lambda \in CR_{x_\alpha} \ni \rho \leq \lambda\}$. It easy to show that $\mathcal{L}(CR_{x_\alpha})$ is an L–fuzzy ideal. Now we show that $\mu \notin \mathcal{L}(CR_{x_\alpha})$. Since $x_\alpha \leq C.c\ell(\mu)$, then for each $\lambda \in CR_{x_\alpha}$, $\mu \not\leq \lambda$. So by definition of $\mathcal{L}(CR_{x_\alpha})$ we have $\mu \notin \mathcal{L}(CR_{x_\alpha})$. Finally, we show that $\mathcal{L}\overset{C}{\rightarrow}x_\alpha$. Let $\lambda \in CR_{x_\alpha}$ and since $\lambda \leq \lambda$, then $\lambda \in \mathcal{L}(CR_{x_\alpha})$. So $CR_{x_\alpha} \subseteq \mathcal{L}(CR_{x_\alpha})$. Thus $\mathcal{L}\overset{C}{\rightarrow}x_\alpha$. Conversely, let $\mathcal{L}$ be an L–fuzzy ideal, $\mu \notin \mathcal{L}$ and $\mathcal{L}\overset{C}{\rightarrow}x_\alpha$. Then for each $\lambda \in CR_{x_\alpha}$, $\lambda \in \mathcal{L}$. Since $\lambda \in \mathcal{L}$, $\mu \notin \mathcal{L}$, then $\mu \not\leq \lambda$ and so $x_\alpha \leq C.c\ell(\mu)$.

4. L–fuzzy C–continuous mappings.

Definition 4.1: An L–fuzzy mapping $f : (L^X, \tau) \to (L^Y, \Delta)$ is said to be :

(i) An L–fuzzy C–continuous if $f^{-1}(\eta) \in \tau'$ for each strong Q–compact L–fuzzy closed set η in L^Y.

(ii) An L–fuzzy C–continuous at L–fuzzy point $x_\alpha \in M(L^X)$ if $f^{-1}(\lambda) \in R_{x_\alpha}$ for each $\lambda \in CR_{f(x_\alpha)}$.

Theorem 4.2: A mapping $f : (X, T_1) \to (Y, T_2)$ is C-continuous iff an L–fuzzy mapping $f : (L^X, \omega_L(T_1)) \to (L^Y, \omega_L(T_2))$ is L–fuzzy C-continuous.

Proof: Let $f : (L^X, T_1) \to (L^Y, T_2)$ be C-continuous and let $\mu \in L^Y$ be strong Q–compact L–fuzzy closed. Then by Theorem 3.2 in [6] and Theorem 2.9, we have $\mu_{w_\alpha} \subseteq Y$ is compact and closed in (Y, T_2), $\forall \alpha \in M(L)$. Since $f^{-1}(\mu_{w_\alpha}) = (f^{-1}(\mu))_{w_\alpha}$, then $f^{-1}(\mu_{w_\alpha}) \in T_1'$ for each $\alpha \in M(L)$ and so $f^{-1}(\mu) \in \omega_L(T_1') = (\omega_L(T_1))'$. Thus $f : (L^X, \omega_L(T_1)) \to (L^Y, \omega_L(T_2))$ is L–fuzzy C-continuous. Conversely; let $f : (L^X, \omega_L(T_1)) \to (L^Y, \omega_L(T_2))$ be L–fuzzy C-continuous and let $A \subseteq Y$ be compact and closed. Then, by Theorem 2.9, $1_A \in L^Y$ is Q_α–compact and L–fuzzy closed in $(L^Y, \omega_L(T_2))$. Since $1_{f^{-1}(A)} = f^{-1}(1_A) \in \omega_L(T_1')$ so $f^{-1}(A) \in T_1'$. Hence $f : (X, T_1) \to (Y, T_2)$ is C-continuous.

Theorem 4.3: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L–fuzzy mapping. Then the following are equivalent:

(i) f is L–fuzzy C–continuous;

(ii) f is L–fuzzy C–continuous at x_α for each L–fuzzy point $x_\alpha \in M(L^X)$;

(iii) For each $\eta \in \Delta$ with η' is strong Q–compact, then $f^{-1}(\eta) \in \tau$.

These statements are implied by

(iv) If $\eta \in L^Y$ is strong Q–compact, then $f^{-1}(\eta) \in \tau'$.

Moreover, if (L^Y, Δ) is fully stratified LFT_2–space, all the statements are equivalent.

Proof : $(i) \implies (ii)$ Suppose that f is L–fuzzy C–continuous, $x_\alpha \in M(L^X)$ and $\lambda \in CR_{f(x_\alpha)}$, then $f^{-1}(\lambda) \in \tau'$. Since $f(x_\alpha) \not\leq \lambda$ is equivalent to $x_\alpha \not\leq f^{-1}(\lambda)$, so $f^{-1}(\lambda) \in R_{x_\alpha}$, and hence f is L–fuzzy C–continuous at x_α.

$(ii) \implies (i)$ Let f be an L–fuzzy C–continuous at x_α for each $x_\alpha \in M(L^X)$. If f is not L–fuzzy C–continuous, then there is C-closed L–fuzzy set $\eta \in L^Y$ with $c\ell(f^{-1}(\eta)) \not\leq f^{-1}(\eta)$. Then there exists $x_\alpha \in M(L^X)$ such that $x_\alpha \leq c\ell(f^{-1}(\eta))$ and $x_\alpha \not\leq f^{-1}(\eta)$. Since $x_\alpha \not\leq f^{-1}(\eta)$ implies that $f(x_\alpha) \not\leq \eta$, so $\eta \in CR_{f(x_\alpha)}$. But $f^{-1}(\eta) \notin R_{x_\alpha}$, a contradiction. Therefore, f must be L–fuzzy C–continuous.

$(i) \Leftrightarrow (iii)$ Follows straightforward from Definition 4.1.

$(iv) \implies (iii)$ Let $\eta \in \Delta$ with η' is strong Q–compact. By (iv), we have $f^{-1}(\eta') \in \tau'$. Thus, $f^{-1}(\eta) = (f^{-1}(\eta'))' \in \tau$.

Now suppose that (L^Y, Δ) is fully stratified LFT_2–space.

$(iii) \implies (iv)$ Let $\eta \in L^Y$ be strong Q–compact set. Since (L^Y, Δ) is fully stratified LFT_2–space, then $\eta \in \Delta'$ and so $\eta' \in \Delta$. By (iii), $f^{-1}(\eta') \in \tau$. Thus $f^{-1}(\eta) = (f^{-1}(\eta'))' \in \tau'$.

By view of Theorems 4.2 and 4.3 the following example shows that LFT_2 is necessary when showing (i) implies (iii) in the above Theorem.

Example 4.4: Let $X = \{1, 2, 3\}$, $Y = R$, $\tau = \omega_L(S)$, where $S = \{X, \emptyset, \{3\}, \{2, 3\}\}$ and $\Delta = \omega_L(T)$, where T be a topology on Y generated by $\{(-\infty, -r) \cup (r, \infty) : r \in Y\}$. Then the mapping $f : (X, S) \to (Y, T)$ defined by $f(x) = x$ for each $x \in X$ is C–continuous (See, Example 1 in [5]). Hence by Theorem 4.2, the mapping $f : (L^X, \omega_L(S)) \to (L^Y, \omega(_L(T))$ is L–fuzzy C–continuous but does not satisfy statement (iii) in Theorem 4.3.

Theorem 4.5: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an surjective L–fuzzy mapping. Then the following conditions are equivalent:

(i) f is L–fuzzy C–continuous;

(ii) For each $\mu \in L^X$, $f(c\ell(\mu)) \leq C.c\ell(f(\mu))$.

(iii) For each $\eta \in L^Y$, $c\ell(f^{-1}(\eta)) \leq f^{-1}(C.c\ell(\eta))$.

(iv) For each $\eta \in L^Y$, $f^{-1}(C.int(\eta)) \leq int(f^{-1}(\eta))$.

(v) $f^{-1}(\rho)$ is L–fuzzy open in L^X, for each L–fuzzy C-open set ρ in L^Y.

(vi) $f^{-1}(\lambda)$ is L–fuzzy closed in L^X, for each L–fuzzy C-closed set λ in L^Y.

Proof: $(i) \implies (ii)$ Let $\mu \in L^X$ and $x_\alpha \in M(L^X)$ with $x_\alpha \leq c\ell(\mu)$. Then $f(x_\alpha) \leq f(c\ell(\mu))$. Let $\lambda \in CR_{f(x_\alpha)}$. So by (i), $f^{-1}(\lambda) \in R_{x_\alpha}$. Since $x_\alpha \leq c\ell(\mu)$ and $f^{-1}(\lambda) \in R_{x_\alpha}$, then $\mu \not\leq f^{-1}(\lambda)$. Thus $f(\mu) \not\leq \lambda$ and $\lambda \in CR_{f(x_\alpha)}$ and so $f(x_\alpha) \leq C.c\ell(f(\mu))$. Hence $f(c\ell(\mu)) \leq C.c\ell(f(\mu))$.

$(ii) \implies (iii)$ Let $\eta \in L^Y$. Then $f^{-1}(\eta) \in L^X$. By (ii), we have $f(c\ell(f^{-1}(\eta)) \leq C.c\ell(ff^{-1}(\eta)) \leq C.c\ell(\eta)$ and so $f(c\ell(f^{-1}(\eta)) \leq C.c\ell(\eta)$ Thus $f^{-1}f(c\ell(f^{-1}(\eta))) \leq f^{-1}(C.c\ell(\eta))$. Since $c\ell(f^{-1}(\eta)) \leq f^{-1}f(c\ell(f^{-1}(\eta)))$, then $c\ell(f^{-1}(\eta)) \leq f^{-1}(C.c\ell(\eta))$.

$(iii) \implies (iv)$ Let $\eta \in L^Y$. By (iii), $c\ell(f^{-1}(\eta')) \leq f^{-1}(C.c\ell(\eta'))$. Since $c\ell(f^{-1}(\eta')) = c\ell(f^{-1}(\eta)') = (int(f^{-1}(\eta)))'$ and $f^{-1}(C.c\ell(\eta')) = (f^{-1}(C.int(\eta)))'$. So, $(int(f^{-1}(\eta)))' \leq (f^{-1}(C.int(\eta)))'$ and by the complement, $int(f^{-1}(\eta)) \geq f^{-1}(C.int(\eta))$.

$(iv) \implies (v)$ Let ρ be C-open in L^Y. Then $f^{-1}(\rho) = f^{-1}(C.int(\rho))$ and by (iv), $f^{-1}(C.int(\rho)) \leq int(f^{-1}(\rho))$, so $f^{-1}(\rho) \leq int(f^{-1}(\rho))$. Thus $f^{-1}(\rho) \in \tau$.

$(v) \implies (vi)$ Let λ be C-closed in L^Y. By (v), $f^{-1}(\lambda') \in \tau$. Then $(f^{-1}(\lambda))' = f^{-1}(\lambda') \in \tau$. So $f^{-1}(\lambda) \in \tau'$.

$(vi) \implies (i)$ Let η be strongly Q–compact and closed set in L^Y. Then by Theorem 3.3 (ii), we have η is C–closed set in L^Y. Hence by (vi), $f^{-1}(\eta) \in \tau'$. Hence f is L–fuzzy C-continuous.

Theorem 4.6: Every L–fuzzy continuous mapping in the sense of Wang [12] is L–fuzzy C–continuous.

Proof : Straightforward.

By view of Theorems 4.2 and 4.6 the following example shows that not every L–fuzzy C–continuous mapping is L–fuzzy continuous.

Example 4.7: Let R be the set of reals with the usual topology T_U and define f :

$(R, T_U) \rightarrow (R, T_U)$ by

$$f(x) = \begin{cases} \frac{1}{x} & : \quad x \neq 0 \\ \frac{1}{2} & : \quad x = 0. \end{cases}$$

Then f is C–continuous but not continuous (See, Example 2 in [5]). Hence by Theorem 4.2, $f : (L^R, \omega_L(T_U)) \rightarrow (L^R, \omega_L(T_U))$ is L–fuzzy C–continuous but not L–fuzzy continuous.

In the following two Theorems we discuss the conditions which the L–fuzzy C–continuity is equivalent to the L–fuzzy continuity.

Theorem 4.8: A mapping $f : (L^X, \tau) \rightarrow (L^Y, \Delta_C)$ is L–fuzzy continuous iff it is L–fuzzy C–continuous.

Proof: Since $\Delta'_C \leq \Delta'$, then necessity is evident. Now, suppose that f is L–fuzzy C–continuous and $\eta \in \Delta'_C$. Then by Theorem 4.5 (iii) we have $f^{-1}(\eta) = f^{-1}(C.c\ell(\eta)) \geq c\ell(f^{-1}(\eta))$ and so $f^{-1}(\eta) \in \tau'$.

Theorem 4.9: Let $f : (L^X, \tau) \rightarrow (L^Y, \Delta)$ be an L–fuzzy mapping and (L^Y, Δ) be strong Q–compact space. Then f is L–fuzzy continuous iff f is L–fuzzy C–continuous.

Proof: By Theorem 4.6 we need only to investigate the sufficiency. Let $\eta \in \Delta'$. Since (L^Y, Δ) is strong Q–compact then, by Theorem 2.8, η is strong Q–compact and so η is L–fuzzy C–closed set. By L–fuzzy C–continuity of f, we have $f^{-1}(\eta) \in \tau'$. Hence f is L–fuzzy continuous.

In [1] Chen and Wang have introduced and studied the concept of L–fuzzy N–continuous mapping by using nearly N–compactness due to Chen and Wang [1]. Here we redefine this concept by using strong nearly Q–compactness due to Nouh [9]. However, its detailed treatment is beyond the scope of this paper and will be dealt elsewhere.

Definition 4.10: An L–fuzzy mapping $f : (L^X, \tau) \rightarrow (L^Y, \Delta)$ is said to be :

(i) An L–fuzzy N^*–continuous if $f^{-1}(\eta) \in \tau'$ for each strong nearly Q–compact L–fuzzy closed set η in L^Y.

(ii) An L–fuzzy N^*–continuous at L–fuzzy point $x_\alpha \in M(L^X)$ if $f^{-1}(\lambda) \in R_{x_\alpha}$ for each $\lambda \in N^* R_{f(x_\alpha)}$.

Theorem 4.11: Every L–fuzzy N^*–continuous mapping is L–fuzzy N–continuous in the sense of Chen and Wang [1].

Proof: Follows from the fact that every N–compact L–fuzzy set is strong Q–compact.

The converse of Theorem 4.11 is not true in general as can be seen from Example 1 in [15]. However, we have the following result.

Theorem 4.12: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L–fuzzy mapping and (L^Y, Δ) be induced L–fts. Then f is L–fuzzy N^*–continuous iff f is L–fuzzy N–continuous.

Proof: Follows from Theorems 2.12 and 4.11.

Theorem 4.13: Every L–fuzzy N^*–continuous mapping is L–fuzzy C–continuous.

Proof: Follows from the fact that every strong Q–compact set is strong nearly Q–compact.

Theorem 4.14: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L–fuzzy mapping and (L^Y, Δ) be LFR_2–space. Then f is L–fuzzy N^*–continuous iff f is L–fuzzy C–continuous.

Proof: Follows from Theorems 2.11 and 4.13.

Remark 4.15: For an L–fuzzy mapping $f : (L^X, \tau) \to (L^Y, \Delta)$, we obtain the following implications:

L–fuzzy continuity ⇒ L–fuzzy N^*–continuity ⇒ L–fuzzy C–continuity.

The following counterexample shows that none of these implications are reversible.

Counterexample 4.16: Let (L^X, τ) and (L^X, Δ) be two L–fts's, where (L^X, τ) is fully stratified LFT_2 and (L^X, Δ) is not LFR_2. Let $f : (L^X, \tau) \to (L^X, \Delta)$ be the identity mapping. Then :

(i) If Δ is strictly finer than τ and (L^X, τ) is LFR_2, then f is L–fuzzy N^*–continuous but not L–fuzzy continuous.

(ii) If $\tau \neq \Delta$ and (L^X, τ) is not LFR_2, then f is L–fuzzy C–continuous but not L–fuzzy N^*–continuous.

However, if (L^Y, Δ) is strong Q–compact (resp. LFR_2) space, then Theorem 4.9 (resp. Theorem 4.14) implies that the concepts of L–fuzzy continuity (resp. N^*–continuity) and L–fuzzy C–continuity are equivalent.

Definition 4.17 [1]: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L–fuzzy mapping and $A \subseteq X$. Define an L–fuzzy mapping $f|_A : L^A \to L^Y$ as follows:

$(f|_A)(\mu) = f(\mu) \wedge 1_A = f(\mu^*)$, for each $\mu \in L^A$

And call $f|_A$ the restriction of f on A. Where μ^* denote the extension of μ in L^X, that is for each $x \in X$,

$$\mu^*(x) = \begin{cases} \mu(x) & : \quad x \in A \\ 0 & : \quad x \notin A \end{cases}$$

Theorem 4.18: If $f : (L^X, \tau) \to (L^Y, \Delta)$ is an L–fuzzy C–continuous and $A \subseteq X$, then $f|_A : (L^A, \tau_A) \to (L^Y, \Delta)$ is an L–fuzzy C–continuous mapping.

Proof: Let $\mu \in L^Y$ be C–closed. Since f is L–fuzzy C–continuous, so $f^{-1}(\mu) \in \tau'$ and $(f|_A)^{-1}(\mu) = f^{-1}(\mu) \wedge 1_A \in \tau'_A$. Hence $f|_A : (L^A, \tau_A) \to (L^Y, \Delta)$ is L–fuzzy C–continuous.

The composition of two L–fuzzy C–continuous mappings need not be L–fuzzy C–continuous (See, Example 3.14 in [3]). However, we have the following result.

Theorem 4.19: If $f : (L^X, \tau_1) \to (L^Y, \tau_2)$ is L–fuzzy continuous mapping and $g : (L^Y, \tau_2) \to (L^Z, \tau_3)$ is L–fuzzy C–continuous mapping, then $g \circ f : (L^X, \tau_1) \to (L^Z, \tau_3)$ is L–fuzzy C–continuous.

Proof: Obvious.

Theorem 4.20: If $(L^X, \tau), (L^Y, \Delta)$ are L–fts's and $1_X = 1_A \vee 1_B$, where 1_A and 1_B are closed of L^X and $f : (L^X, \tau) \to (L^Y, \Delta)$ is L–fuzzy mapping such that $f|_A$ and $f|_B$ are L–fuzzy C–continuous, then f is L–fuzzy C–continuous.

Proof: Let $1_A, 1_B \in \tau'$. Let $\mu \in L^Y$ be C–closed. Then $(f|_A)^{-1}(\mu) \vee (f|_B)^{-1}(\mu) = (f^{-1}(\mu) \wedge 1_A) \vee (f^{-1}(\mu) \wedge 1_B) = f^{-1}(\mu) \wedge (1_A \vee 1_B) = f^{-1}(\mu) \wedge 1_X = f^{-1}(\mu)$. Hence $f^{-1}(\mu) = (f|_A)^{-1}(\mu) \vee (f|_B)^{-1}(\mu) \in \tau'$. So $f : (L^X, \tau) \to (L^Y, \Delta)$ is L–fuzzy C–continuous.

5. More Characterizations of L–fuzzy C–continuous mappings

Theorem 5.1: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be L–fuzzy C–continuous and (L^Y, Δ) be a fully stratified LFT_2–space. If $f(1_X)$ is contained in some strong Q–compact set of L^Y, then f is L–fuzzy continuous.

Proof: Let $\mu \in L^Y$ be a strong Q–compact containing $f(1_X)$ and let $\rho \in \Delta'$. Since μ is strong Q–compact in (L^Y, Δ) which is fully stratified LFT_2-space, so $\mu \in \Delta'$. Thus $\mu \wedge \rho \in \Delta'$. Hence by Theorem 2.8, $\mu \wedge \rho \in L^Y$ is strong Q–compact. Thus

$\mu \wedge \rho \in L^Y$ is C–closed. Since f is L–fuzzy C–continuous, then $f^{-1}(\mu \wedge \rho) \in \tau'$. But, $f^{-1}(\mu \wedge \rho) = f^{-1}(\mu) \wedge f^{-1}(\rho) = f^{-1}(\rho) \wedge 1_X = f^{-1}(\rho)$. So $f^{-1}(\rho) \in \tau'$. Hence f is L–fuzzy continuous.

Theorem 5.2: Let (L^X, τ) be an L–fts and (L^Y, Δ) be fully stratified LFT_2–space. If $f : (L^X, \tau) \to (L^Y, \Delta)$ is a bijective and L–fuzzy continuous, then $f^{-1} : (L^Y, \Delta) \to (L^X, \tau)$ is L–fuzzy C–continuous.

Proof: Let $\eta \in L^X$ be strong Q–compact. Since f is L–fuzzy continuous, then by Theorem 2.10, $f(\eta)$ is strong Q–compact. Since (L^Y, Δ) is fully stratified LFT_2-space, then $f(\eta) \in \Delta'$. Hence by Theorem 4.3, f^{-1} is L–fuzzy C–continuous.

Corollary 5.3: Let (L^X, τ) be a strong Q–compact space and (L^Y, Δ) be fully stratified LFT_2-space. If $f : (L^X, \tau) \to (L^Y, \Delta)$ is a bijective and L–fuzzy continuous, then f is an L–fuzzy homeomorphism.

Proof: Follows from Theorems 5.1 and 5.2.

Theorem 5.4: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an surjective L–fuzzy mapping. Then the following conditions are equivalent:

(i) f is L–fuzzy C–continuous;

(ii) For each $x_\alpha \in M(L^X)$ and each L–fuzzy net S in L^X, $f(\mathcal{S}) \overset{C}{\to} f(x_\alpha)$ if $S \to x_\alpha$.

(iii) $f(lim(S)) \leq C.lim(f(S))$, for each L–fuzzy net S in L^X.

Proof: $(i) \implies (ii)$ Let $x_\alpha \in M(L^X)$ and $S = \{x^n_{\alpha_n} : n \in D\}$ be an L–fuzzy net in L^X which converges to x_α. Let $\eta \in CR_{f(x_\alpha)}$, by (i) $f^{-1}(\eta) \in R_{x_\alpha}$. Since $S \to x_\alpha$, then $(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq f^{-1}(\eta))$. Then $f(S(m)) \not\leq ff^{-1}(\eta) = \eta$. Thus $f(S(m)) \not\leq \eta$. Hence $f(\mathcal{S}) \overset{C}{\to} f(x_\alpha)$.

$(ii) \implies (iii)$ Let $x_\alpha \leq lim(S)$, then $f(x_\alpha) \leq f(lim(S))$, by (ii) $f(x_\alpha) \leq C.lim(f(S))$. Thus $f(lim(S)) \leq C.lim(f(S))$.

$(iii) \implies (i)$ Let $\eta \in L^Y$ be L–fuzzy C–closed and $x_\alpha \in M(L^X)$ with $x_\alpha \leq c\ell(f^{-1}(\eta))$, by Theorem 2.15, there exists an L–fuzzy net S in $f^{-1}(\eta)$ which converges to x_α. Thus $x_\alpha \leq lim(S)$ and so $f(x_\alpha) \leq f(lim(S))$. By (iii), $f(x_\alpha) \leq f(lim(S)) \leq C.limf(S)$ and so, $f(\mathcal{S}) \overset{C}{\to} f(x_\alpha)$. Since S is L–fuzzy net in $f^{-1}(\eta)$, then for each $n \in D$, $S(n) \leq f^{-1}(\eta)$

and so $f(S(n)) \leq ff^{-1}(\eta) \leq \eta$. Hence $f(S(n)) \leq \eta$ for each $n \in D$. Thus $f(S)$ is L–fuzzy net in η. So we have $f(\mathcal{S}) \overset{C}{\to} f(x_\alpha)$ and $f(\mathcal{S})$ is L–fuzzy net in η so by Proposition 3.10 (iii), $f(x_\alpha) \leq C.c\ell(\eta)$. But since η is C–closed, so $\eta = C.c\ell(\eta)$. Thus $f(x_\alpha) \leq \eta$. Hence $x_\alpha \leq f^{-1}(\eta)$. So $c\ell(f^{-1}(\eta)) \leq f^{-1}(\eta)$. Hence $f^{-1}(\eta) \in \tau'$. Then f is L–fuzzy C–continuous.

Theorem 5.5: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L–fuzzy mapping. Then the following conditions are equivalent:

(i) f is L–fuzzy C–continuous;

(ii) For each $x_\alpha \in M(L^X)$ and each L–fuzzy ideal $\mathcal{L}$ in L^X which converges to x_α in L^X, $f^*(\mathcal{L})$ C-converges to $f(x_\alpha)$, where $f^*(\mathcal{L}) = \{\eta \in L^Y : \exists \mu \in \mathcal{L}$ such that for any $x_\alpha \in M(L^X), f(x_\alpha) \nleq \eta$ if $x_\alpha \nleq \mu\}$ is an L–fuzzy ideal in L^Y.

(iii) $f(lim(\mathcal{L})) \leq C.lim(f^*(\mathcal{L}))$, for each L–fuzzy ideal $\mathcal{L}$ in L^X.

Proof: The proof is similar to that of Theorem 5.4.

Similarly, we have the following result.

Theorem 5.6: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L–fuzzy mapping. Then the following conditions are equivalent:

(i) f is L–fuzzy C–continuous;

(ii) For each $x_\alpha \in M(L^X)$ and each L–fuzzy ideal $\mathcal{L}$ in L^X which converges to x_α in L^X, then $(f(\mathcal{L}'))'$ C-converges to $f(x_\alpha)$.

(iii) $f(lim(\mathcal{L})) \leq C.lim((f(\mathcal{L}'))')$, for each L–fuzzy ideal $\mathcal{L}$ in L^X.

References

[1] Chen, S.L., Wang, X.G.: L–fuzzy N–continuous mappings. *J. Fuzzy Math.*, 4 (3). 621–629 (1996).

[2] Chen, S.L., Chen, S.T.: A new extension of fuzzy convergence. *Fuzzy Sets and Systems*, 109, 199–204 (2000).

[3] Dang, S., Behera, A., Nanda, S.: Fuzzy C–continuous functions. *J. Fuzzy Math.*, 2 (1), 187–196 (1994).

[4] Garg, G. L., Kumar, B.: C–closed functions. *Math. Vesnik*, 41, 233–238 (1989).

[5] Gentry, K. R., Hoyle, H. B.: III, C-continuous functions. *Yokohama Math. J.*, *18 (2)*, 71–76 (1970).

[6] Liu, Y. M., Luo, M. K.: Fuzzy Stone–Cech–type compactifications. *Fuzzy Sets and Systems*, 33, 355–372 (1989).

[7] Liu, Y. M., Luo, M. K.: Separation in lattice–valued induced spaces. *Fuzzy Sets and Systems*, 36, 55–66 (1990).

[8] Mukherjee, M. N., Sinha, S. P.: Almost compact fuzzy sets in fuzzy topological spaces. *Fuzzy Sets and Systems*, 38, 386–396 (1990).

[9] Nouh, A. A.: On Q_α–compactness in L–fuzzy topological spaces. Submitted

[10] Wang, G. J.: A new fuzzy compactness defined by fuzzy nets. *J. Math. Anal. Appl.*, 94, 59–67 (1983).

[11] Wang, G. J.: Generalized topological molecular lattices. *Scientia Sinica*, (Ser. A) 27 (8), 785–793 (1984).

[12] Wang, G. J.: Theory of L–fuzzy topological spaces. Shaanxi Normal University Press. Xi'an 1988.

[13] Yang, Z. Q.: Ideal in topological molecular lattices. Acta *Mathematica Sinica*, 29 (2), 276–279 (1986).

[14] Zhao, D. S.: The N-compactness in L–fuzzy topological space. *J. Math. Anal. Appl.*, 128, 64–79 (1987).

[15] Zhongfu, L.: Compactness in fuzzy topological spaces. *Kexue Tongbao*, 29(5), 582–585 (1984) .

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