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On Some Properties of Szasz-Mirakyan Operators in Hölder Spaces

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Abstract

We study some properties of modified Szasz-Mirakyan operators in Hölder exponential weighted spaces. We give theorems on the degree of approximation of functions by these operators.

Key Words: Szasz-Mirakyan operator, Hölder space, modulus of smoothness, degree of approximation.

1. Introduction

1.1. Paper [1] examined approximation properties of Szasz-Mirakyan operators

$$S_n(f;x) := \sum_{k=0}^{\infty} p_k(nx) f\left(\frac{k}{n}\right), \qquad (1)$$

$x \in R_0 = [0, +\infty)$, $n \in N := \{1, 2, \cdots\}$, where

$$p_k(t) := e^{-t}\frac{t^k}{k!} \qquad \text{for} \quad t \in R_0,\ k \in N_0 := N \cup \{0\}, \qquad (2)$$

in exponential weighted spaces C_q. The space C_q, with a fixed $q > 0$, is related with the weighted function $v_q(x) := e^{-qx}$, $x \in R_0$, and C_q is the set of all real-valued functions f

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continuous on R_0 for which $v_q f$ is uniformly continuous and bounded on R_0. The norm in C_q is defined by

$$\|f\|_q \equiv \|f(\cdot)\|_q := \sup_{x \in R_0} v_q(x)|f(x)|. \tag{3}$$

It is obvious that $C_q \subset C_p$ if $0 < q < p < +\infty$.

In [1] was proved that S_n defined by (1) is a positive linear operator from the space C_q into C_p provided that $p > q > 0$ and $n > q/\ln(p/q)$.

Recently in many papers were introduced various modifications of operators S_n (see e.g. [3, 4, 5, 8, 10]).

In the paper [8] were introduced for $f \in C_q$, $q > 0$, the following modified Szasz-Mirakyan operators

$$S_{n,q}(f;x) := \sum_{k=0}^{\infty} p_k(nx) f\left(\frac{k}{n+q}\right), \qquad x \in R_0,\ n \in N, \tag{4}$$

where $p_k(\cdot)$ is defined by (2). Also in [8] was proved that, for every $n \in N$ and $q > 0$, $S_{n,q}$ is positive linear operator from the space C_q into C_q and

$$\|S_{n,q}(f;\cdot)\|_q \leq \|f\|_q, \qquad n \in N, \tag{5}$$

for every $f \in C_q$. Moreover, in [8] were given approximation theorems for $f \in C_q$ and $S_{n,q}(f)$.

1.2. The purpose of this paper is the examination of approximation properties of operators $S_{n,q}$ in Hölder spaces related with exponential weighted space C_q, $q > 0$.

Approximation of 2π-periodic functions in the Hölder spaces first was considered by S. Prössdorf and J. Prestin in the papers [6] and [7].

The results given in [6] and [7] were extend by many authors.

Since the Szasz-Mirakyan operators are important in approximation theory and $S_{n,q}$ are operators from the space C_q into C_q, we shall consider the properties of these operators in the Hölder spaces.

In this paper, as had been similarly done in [6] and [7], we shall apply the modulus of smoothness of the order $r \in N$ ([2], [9]) of function $f \in C_q$:

$$\omega_r(f;q;t) := \sup_{0 \leq h \leq t} \|\Delta_h^r f(\cdot)\|_q\,, \qquad t \geq 0, \tag{6}$$

where

$$\Delta_h^1 f(x) := f(x+h) - f(x), \quad \Delta_h^r f(x) := \Delta_h^1 \left(\Delta_h^{r-1} f(x)\right) \quad \text{if} \quad r \geq 2, \tag{7}$$

for $x, h \in R_0$. It is known ([9]) that

$$\Delta_h^r f(x) = \sum_{j=0}^{r} \binom{r}{j} (-1)^{r-j} f(x+jh) \quad \text{for } x, h \in R_0 \text{ and } r \in N. \tag{8}$$

Moreover from (6) – (8) we deduce that if $f \in C_q$, $q > 0$, and $r \in N$, then $\omega_r(f, q; \cdot)$ is non-negative and non-decreasing function and $lim_{t \to 0+} \omega_r(f; q; t) = 0$.

1.3. Let $r \in N$ and let Ω_r be the set of functions of $\omega_r(f; q; \cdot)$ type, i.e. Ω_r is the set of all functions ω satisfying the following conditions:

(i) ω is defined, non-negative and increasing on R_0;

(ii) $\omega(t) \to 0$ as $t \to 0+$;

(iii) $\omega(t)t^{-r}$ is decreasing for $t > 0$.

Similarly as in [6] and [7], for given $r \in N$, $\omega \in \Omega_r$ and $q > 0$, we define the generalized Hölder spaces $H_q^{r,\omega}$ and $\widetilde{H}_q^{r,\omega}$. The space $H_q^{r,\omega}$ is the set of all functions $f \in C_q$ for which

$$\|f\|_q^{*\,r,\omega} := \sup_{h>0} \frac{\|\Delta_h^r f(\cdot)\|_q}{\omega(h)} < +\infty \tag{9}$$

and the norm is defined by the formula

$$\|f\|_{H_q^{r,\omega}} := \|f\|_q + \|f\|_q^{*\,r,\omega}. \tag{10}$$

The space $\widetilde{H}_q^{r,\omega}$ is the set of all functions $f \in H_q^{r,\omega}$ for which

$$\lim_{t \to 0+} \frac{\omega_r(f; q; t)}{\omega(t)} = 0 \tag{11}$$

and the norm is defined by (10).

From definition of $H_q^{r,\omega}$ we deduce that $f \in H_q^{r,\omega}$ if and only if there exists a positive constant $M_1(f)$ depending only on f and such that

$$\omega_r(f,q;t) \leq M_1(f)\omega(t) \qquad \text{for} \quad t \geq 0. \tag{12}$$

The spaces $H_q^{r,\omega}$ and $\widetilde{H}_q^{r,\omega}$, with fixed $q > 0$, $r \in N$ and $\omega(t) = t^\alpha$, $0 < \alpha \leq r$, are classical Hölder - Lipschitz - Zygmund spaces. Moreover, we observe that if $\omega, \mu \in \Omega_r$, $r \in N$, and

$$\lambda(t) := \frac{\omega(t)}{\mu(t)}, \qquad t > 0, \tag{13}$$

is increasing function, then for every $q > 0$ we have

$$H_q^{r,\omega} \subset H_q^{r,\mu}, \qquad \widetilde{H}_q^{r,\omega} \subset \widetilde{H}_q^{r,\mu} \tag{14}$$

for every $q > 0$.

2. Main Results

2.1. First we shall give certain inequalities for operators $S_{n,q}$ and $f \in C_q$.

Denote by C_q^p, $p \in N$, $q > 0$, the class of all functions $f \in C_q$ which have derivatives $f^{(k)}$, $k = 1, \ldots, p$, on R_0 and these $f^{(k)}$ belong also to C_q.

Theorem 1 *Let $n \in N$ and $q > 0$ be fixed numbers. Then $S_{n,q}$ defined by* (4) *is an operator from the space C_q into C_q^∞. Moreover, for every $p \in N$ and $f \in C_q$, we have*

$$\| [S_{n,q}(f)]^{(p)} \|_q \leq (1+e)^p n^p \|f\|_q. \tag{15}$$

If $f \in C_q^p$, $p \in N$, $q > 0$, then

$$\| [S_{n,q}(f)]^{(p)} \|_q \leq e^p \|f^{(p)}\|_q, \qquad n \in N. \tag{16}$$

Proof. From (4), (2) and (7) we derive the formula

$$[S_{n,q}(f;x)]^{(p)} = n^p S_{n,q}\left(\Delta^p_{1/(n+q)} f(t); x\right),$$

for every $f \in C_q$ and $n, p \in N$. By (3) and (5) we have

$$\| [S_{n,q}(f)]^{(p)} \|_q \leq n^p \|\Delta^p_{1/(n+q)} f(\cdot)\|_q, \qquad n, p \in N. \tag{17}$$

Applying (8) and (3), we get

$$\|\Delta^p_{1/(n+q)} f(\cdot)\|_q \le \|f\|_q \sum_{k=0}^{p} \binom{p}{k} e^{kq/(n+q)} \le$$

$$\le \|f\|_q (1+e)^p, \qquad n, p \in N.$$

From the above follows (15).

It is known ([2], [9]) that if $f \in C^p_q$ with fixed $p \in N$ and $q > 0$, then

$$\Delta^p_h f(x) = \int_0^h \cdots \int_0^h f^{(p)}(x + u_1 + \cdots + u_p) du_1 \cdot \cdots \cdot du_p, \qquad h > 0,$$

which by (3) implies

$$\|\Delta^p_h f(\cdot)\|_q \le \|f^{(p)}\|_q e^{hpq} h^p, \qquad h > 0.$$

From this and by (17) we get

$$\| \left[S_{n,q}(f)\right]^{(p)} \|_q \le \left(\frac{n}{n+q}\right)^p e^{pq/(n+q)} \|f^{(p)}\|_q \le e^p \|f^{(p)}\|_q, \quad n, p \in N.$$

Thus the proof is completed. □

2.2. Let $f \in C_q$, $q > 0$, and let $r \in N$. It is known ([2], Section 6.1) that the modulus of smoothness $\omega_r(f; q; \cdot)$ defined by (6) is equivalent to the weighted K-functional

$$K_r\left(f; q; t^r\right) := \inf_{\varphi \in C^r_q} \left\{ \|f - \varphi\|_q + t^r \|\varphi^{(r)}\|_q \right\}, \tag{18}$$

i.e. there exists a positive constant M independent on f and t such that

$$M^{-1} \omega_r(f; q; t) \le K_r\left(f; q; t^r\right) \le M \omega_r(f; q; t) \qquad \text{for} \quad t > 0. \tag{19}$$

Using this equivalece, we shall prove theorem on modulus of smoothness of $S_{n,q}(f)$.

Theorem 2 *Let $q > 0$ and $r \in N$ be fixed numbers. Then there exists the suitable positive constant $M_2(r)$ depending only on r and such that for every $f \in C_q$ and $n \in N$ we have*

$$\omega_r\left(S_{n,q}(f);q;t\right) \leq M_2(r)\omega_r(f;q;t), \qquad t \geq 0. \tag{20}$$

Proof. By Theorem 1 we have $S_{n,q}(f) \in C_q^\infty$ if $f \in C_q$. Hence for $S_{n,q}(f)$ we can apply K- functional (18) and the inequality (19). For given $f \in C_q$ and $n, r \in N$ we have

$$\omega_r\left(S_{n,q}(f);q;t\right) \leq MK_r\left(S_{n,q}(f);q;t^r\right) =$$

$$= M \inf_{\varphi \in C_q^r} \left\{ \|S_{n,q}(f) - \varphi\|_q + t^r \|\varphi^{(r)}\|_q \right\} \leq$$

$$\leq M \left\{ \|S_{n,q}(f) - S_{n,q}(\Psi)\|_q + t^r \| \left[S_{n,q}(\Psi)\right]^{(r)} \|_q \right\}$$

for $t \geq 0$, where Ψ is arbitrary fixed function in C_q^r.

By (4) and (5) we have

$$\|S_{n,q}(f) - S_{n,q}(\Psi)\|_q = \|S_{n,q}(f - \Psi; \cdot)\|_q \leq \|f - \Psi\|_q$$

for all $n \in N$. Moreover by Theorem 1, we have

$$\| \left[S_{n,q}(\Psi)\right]^{(r)} \|_q \leq e^r \|\Psi^{(r)}\|_q, \qquad n \in N.$$

From the above we get

$$\omega_r\left(S_{n,q}(f);q;t\right) \leq Me^r \left\{ \|f - \Psi\|_q + t^r \|\Psi^{(r)}\|_q \right\}, \qquad t \geq 0,$$

for every fixed $f \in C_q$, $n, r \in N$ and for every $\Psi \in C_q^r$, which by (18) and (19) yields

$$\omega_r\left(S_{n,q}(f);q;t\right) \leq Me^r \inf_{\Psi \in C_q^r} \left\{ \|f - \Psi\|_q + t^r \|\Psi^{(r)}\|_q \right\} \leq$$

$$\leq Me^r K_r\left(f;q;t^r\right) \leq M^2 e^r \omega_r(f;q;t)$$

for $t \geq 0$, $f \in C_q$ and $n, r \in N$. Thus the inequality (20) was proved. □

2.3. Now we shall give the main property of $S_{n,q}$ in Hölder spaces.

Theorem 3 *We assume that $q > 0$ and $r \in N$ are fixed numbers and $\omega \in \Omega_r$ is a given function. Then $S_{n,q}$, $n \in N$, defined by (4) is a positive linear operator from the space $H_q^{r,\omega}$ $(\widetilde{H}_q^{r,\omega})$ into $H_q^{r,\omega}$ $(\widetilde{H}_q^{r,\omega})$.*

Proof. Let $f \in H_q^{r,\omega}$ and $n \in N$. By Theorem 1, (20) and (12) we have $S_{n,q}(f) \in C_q^{\infty}$ and

$$\omega_r\left(S_{n,q}(f); q; t\right) \leq M_3(f,r)\omega(t), \qquad t \geq 0, \tag{21}$$

where $M_3(f,r) = \text{const.} > 0$ depending only on f and r. The inequality (21) and Section 1.3 show that if $f \in H_q^{r,\omega}$ then also $S_{n,q}(f) \in H_q^{r,\omega}$.

Now, assume that $f \in \widetilde{H}_q^{r,\omega}$. The condition (11) and the inequality (20) imply that

$$\lim_{t \to 0+} \frac{\omega_r\left(S_{n,q}(f); q; t\right)}{\omega(t)} = 0, \qquad n \in N,$$

which proves that $S_{n,q}(f) \in \widetilde{H}_q^{r,\omega}$, $n \in N$. □

2.4. In [8] (p.127) was proved the following theorem.

Theorem 4 *If $f \in C_q^2$, $q > 0$, then*

$$v_q(x)|S_{n,q}(f;x) - f(x)| \leq \|f'\|_q \frac{qx}{n+q} + \tag{22}$$

$$+\|f''\|_q \left(\frac{(4e^2+1)q^2x^2}{(n+q)^2} + \frac{4x}{n+q} \right)$$

for $x \in R_0$ and $n \in N$.

Arguing as in [1] and [8], we shall prove the following theorem.

Theorem 5 *Suppose that $a > 0$ and $q > 0$ are fixed numbers. Then there exists a positive constant $M_4(a,q)$ (depending only on a and q) such that*

$$\sup_{0 \leq x \leq a} v_q(x)|S_{n,q}(f;x) - f(x)| \leq M_4(a,q)\omega_2\left(f; q; 1/\sqrt{n+q}\right) \tag{23}$$

for every $f \in C_q$ and $n \in N$.

Proof. Analogously as [1] and [8] we use the Steklov function f_h of $f \in C_q$:

$$f_h(x) := \frac{4}{h^2} \int_0^{\frac{h}{2}} \int_0^{\frac{h}{2}} [2f(x+s+t) - f(x+2(s+t))]dsdt$$

for $x \in R_0$ and $h > 0$. It is known ([1], [8]) that $f_h \in C_q^2$ if $f \in C_q$ and

$$\|f_h - f\|_q \leq \omega_2 (f, q; h); \tag{24}$$

$$\|f'_h\|_q \leq 5e^{qh} h^{-1} \omega_1 (f; q; h); \tag{25}$$

$$\|f''_h\|_q \leq 9h^{-2} \omega_2 (f; q; h); \tag{26}$$

for $h > 0$. Applying (5), (22), and (24)–(26), we get

$$v_q(x) |S_{n,q}(f; x) - f(x)| \leq v_q(x) \{|S_{n,q} (f - f_h; x)| +$$

$$+ |S_{n,q} (f_h; x) - f_h(x)| + |f_h(x) - f(x)|\} \leq 2 \|f_h - f\|_q +$$

$$+ \|f'_h\|_q \frac{qx}{n+q} + \|f''_h\|_q \left(\frac{(4e^2+1)q^2x^2}{(n+q)^2} + \frac{4x}{n+q} \right) \leq$$

$$\leq \frac{5qe^{qh}x}{n+q} \omega_1 (f; q; h) + \left\{ 2 + 9h^{-2} \left(\frac{(4e^2+1)q^2x^2}{(n+q)^2} + \frac{4x}{n+q} \right) \right\} \omega_2 (f; q; h),$$

for $x \in R_0$, $n \in N$ and $h > 0$. Hence, for fixed $a > 0$, $n \in N$ and $h = 1/\sqrt{n+q}$, we obtain

$$\sup_{0 \leq x \leq a} v_q(x) |S_{n,q}(f; x) - f(x)| \leq M_5(a, q) \left\{ \frac{1}{\sqrt{n+q}} \omega_1 \left(f; q; \frac{1}{\sqrt{n+q}} \right) + \right.$$

$$\left. + \omega_2 \left(f; q; \frac{1}{\sqrt{n+q}} \right) \right\},$$

which implies the desired estimation (23). □

2.5. Applying Theorem 5 we shall give two approximation theorems for function f belonging to Hölder spaces $H_q^{2,\omega}$ and $\widetilde{H}_q^{2,\omega}$. For $f \in H_q^{2,\omega}$ with fixed $q > 0$, $\omega \in \Omega_2$ and for a fixed $a > 0$ we write

$$\|f\|_{q,a} := \sup_{0 \leq x \leq a} v_q(x) |f(x)|, \tag{27}$$

$$\|f\|_{q,a}^{*\,2,\omega} := \sup_{0 < h \leq 1} \frac{\left\| \Delta_h^2 f(\cdot) \right\|_{q,a}}{\omega(h)}, \tag{28}$$

$$\|f\|_{q,a}^{2,\omega} := \|f\|_{q,a} + \|f\|_{q,a}^{*\,2,\omega}. \tag{29}$$

Theorem 6 *Suppose that $a, q > 0$ are fixed numbers, $\omega, \mu \in \Omega_2$ are given functions such that $\lambda(\cdot)$ defined by (13) is increasing and $f \in H_q^{2,\omega}$. Then there exists a positive constant $M_6 \equiv M_6(a, q, f, \mu(1))$ (depending only on indicated parameters) such that*

$$\|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a}^{2,\mu} \le M_6 \lambda(1/\sqrt{n+q}), \qquad n \in N. \tag{30}$$

Proof. By our assumptions follows (14) and by Theorem 3 and (29) we have

$$\|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a}^{2,\mu} = \|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a} + \|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a}^{*\,2,\mu}$$

for $n \in N$. But by (27) and (23) and (12) we get

$$\|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a} \le M_4(a,q)\omega_2\left(f; q; 1/\sqrt{n+q}\right) \le$$

$$\le M_7(a,q,f)\omega\left(1/\sqrt{n+q}\right) \le M_7(a,q,f)\mu(1)\lambda(1/\sqrt{n+q}), \qquad n \in N.$$

Using definition (28) and denoting by

$$A = \{h : 0 < h \le 1/\sqrt{n+q}\}, \qquad B = \{h : 1/\sqrt{n+q} < h \le 1\},$$

we can write

$$\|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a}^{*\,2,\mu} \le (\sup_{h\in A} + \sup_{h\in B}) \frac{\left\|\Delta_h^2 \left[S_{n,q}(f;\cdot) - f(\cdot)\right]\right\|_{q,a}}{\mu(h)} := W_1 + W_2.$$

By (27), (3), (7), (6) and Theorem 2 we deduce that

$$\left\|\Delta_h^2 \left[S_{n,q}(f;\cdot) - f(\cdot)\right]\right\|_{q,a} \le \left\|\Delta_h^2 \left[S_{n,q}(f;\cdot) - f(\cdot)\right]\right\|_q \le$$

$$\le \left\|\Delta_h^2 S_{n,q}(f;\cdot)\right\|_q + \left\|\Delta_h^2 f(\cdot)\right\|_q \le$$

$$\le \omega_2\left(S_{n,q}(f); q; h\right) + \omega_2\left(f; q; h\right) \le M_8 \omega_2\left(f; q; h\right)$$

for $h > 0$. From this and by (12) we get

$$W_1 \le M_9(f) \sup_{h\in A} \frac{\omega(h)}{\mu(h)} \le M_9(f)\lambda(1/\sqrt{n+q}), \qquad n \in N.$$

Applying (8) and (27) we get $\left\|\Delta_h^2 f(\cdot)\right\|_{q,a} \leq M_{10}(a,q)\|f\|_{q,a}$ and

$$W_2 \leq M_{10}(a,q) \sup_{h\in B} \frac{\|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a}}{\mu(h)},$$

which by Theorem 5 and (12) implies

$$W_2 \leq M_{11}(a,q)\omega_2\left(f;q;1/\sqrt{n+q}\right)/\mu(1/\sqrt{n+q}) \leq$$
$$\leq M_{12}(a,q,f)\lambda(1/\sqrt{n+q}), \qquad n \in N.$$

Combining these, we obtain estimation (30). □

Analogously we can prove the following theorem

Theorem 7 *Suppose that a, q, ω, μ and λ satisfy the assumptions of Theorem 6. If $f \in \widetilde{H}_q^{2,\omega}$, then*

$$\|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a}^{2,\mu} = o\left(\lambda(1/\sqrt{n+q})\right) \qquad as \quad n \to \infty.$$

From Theorem 6 and Theorem 7 we derive the following corollary.

Corollary. Let $a, q > 0$ be fixed numbers and let $\omega(t) = t^\alpha$, $\mu(t) = t^\beta$ for $t \geq 0$ and for fixed $0 < \beta < \alpha \leq 2$.

1. If $f \in H_q^{2,\omega}$, then

$$\|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a}^{2,\mu} = O\left((n+q)^{(\beta-\alpha)/2}\right) \quad as\ n \to \infty.$$

2. If $f \in \widetilde{H}_q^{2,\omega}$, then

$$\|S_{n,q}(f;\cdot) - f(\cdot)\|_{q,a}^{2,\mu} = o\left((n+q)^{(\beta-\alpha)/2}\right) \quad as\ n \to \infty.$$

Remark. Analogous theorems we can obtain for operators of Kantorovitch type:

$$T_{n,q}(f;x) := \sum_{k=0}^{\infty} p_k(nx)(n+q)\int_{k/(n+q)}^{(k+1)/(n+q)} f(t)dt, \qquad x \in R_0,\ n \in N,$$

which are positive linear operators from the space C_q into C_q and $\|T_{n,q}\|_q \leq e\|f\|_q$ for every $f \in C_q$ and $n \in N$.

Similar theorems can also be obtained for Szasz-Mirakyan operators (1) in the Hölder polynomial weighted spaces related with the polynomial weighted spaces C_p, $p \in N_0$, and the weighted function $w_p(x) := 1/(1 + x^p)$ if $p \geq 1$, and $w_0(x) := 1$ for $x \in R_0$.

Cleary, approximation properties of other operators (e.g. operators considered in [3, 4, 10]) can be examined in suitable Hölder spaces.

References

[1] M. Becker, D. Kucharski, R.J. Nessel, Global approximation theorems for the Szasz-Mirakjan operators in exponential weight spaces, In: Linear Spaces and Approximation (Proc. Conf. Oberwolfach, 1977), Birkhuser Verlag, Basel ISNM, 40(1978), 319 - 333.

[2] Z. Ditzian, V. Totik, Moduli of Smoothness, Springer Series in Computational Mathematics, Vol. 9, Springer-Verlag, New York Inc. 1987.

[3] V. Gupta, R.P. Pant, Rate of convergence of the modified Szasz-Mirakyan operators on functions of bounded variation, J. Math. Anal. Appl., 233(199), 476-483.

[4] V. Gupta, G.S. Srivastava, Approximation by Durrmeyer type operators, Annal. Polonici Math., LXIV(1996), 153-159.

[5] B. Firlej, L. Rempulska, Approximation of functions of several variables by some operators of Szasz-Mirakyan type, Fasciculi Mate., 27(1997), 15-28.

[6] J. Prestin, S. Prössdorf, Error estimates in generalized trigonometric Hölder norms, Z. Anal. Anwendungen, 9(1990), 343-349.

[7] S. Prössdorf, Zur konvergenz der Fourierreihen hölderstetiger funktionen,

Math. Nachr., 69(1975), 7-14.

[8] L. Rempulska, Z. Walczak, Approximation properties of certain modified

Szasz-Mirakyan operators, Le Matematiche 55(2001)1, 121-132.

[9] A.F. Timann, Theory of Approximation of Functions of a Real Variable, New York 1963.

[10] [10] Z. Walczak, Approximation of functions of two variables by modified Szasz-Mirakyan operators, Fasciculi Math., 31(2001), 117-128.

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