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On Marcinkiewicz Integrals along flat surfaces

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Abstract

In this paper, we study Marcinkiewicz integral operators with rough kernels supported by surfaces given by flat curves. Under convexity assumptions on our surfaces, we establish an L^p boundedness result of such operators. Moreover, we obtain the L^p boundedness of the corresponding Marcinkiewicz integral operators that are related to area integral and Littlewood-Paley g_λ^* functions.

Key Words: Marcinkiewicz integral, rough kernels, flat curves, Fourier transform, area integral, Littlewood-Paley g_λ^* functions.

1. Introduction and Statement of Results

Let $\mathbf{R}^n$, $n \geq 2$ be the n-dimensional Euclidean space and $\mathbf{S}^{n-1}$ be the unit sphere in $\mathbf{R}^n$ equipped with the induced Lebesgue measure $d\sigma$. Let Ω be a homogeneous function of degree zero on $\mathbf{R}^n$ that is integrable on $\mathbf{S}^{n-1}$ and satisfies

$$\int_{\mathbf{S}^{n-1}} \Omega(y^{'}) d\sigma(y^{'}) = 0, \tag{1.1}$$

where $y' = \frac{y}{|y|}$ for $y \neq 0$.

For a smooth mapping $\Gamma : \mathbf{R}^n \to \mathbf{R}^d$, consider the Marcinkiewicz integral operator

$$\mu_{\Omega,\Gamma} f(x) = \left(\int_{-\infty}^{\infty} |F_{\Omega,\Gamma,t}(x)|^2 \, 2^{-2t} dt \right)^{\frac{1}{2}}, \tag{1.2}$$

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where

$$F_{\Omega,\Gamma,t}(x) = \int_{|y|\leq 2^t} f(x-\Gamma(y))\, |y|^{-n+1}\, \Omega(y)dy. \tag{1.3}$$

The problem regarding the operator $\mu_{\Omega,\Gamma}$ is that under what conditions on Γ and Ω, the operator $\mu_{\Omega,\Gamma}$ maps $L^p(\mathbf{R}^n)$ into $L^p(\mathbf{R}^d)$ for some $1 < p < \infty$. It is known that if $\Gamma(y) = y$ and $\Omega \in Lip_{\alpha}(\mathbf{S}^{n-1})$, $(0 < \alpha \leq 1)$, i.e., Ω in the Lipschitz function class of degree α on the unit sphere, E. M. Stein ([10]) proved that $\mu_{\Omega,\Gamma}$ is bounded on L^p for all $1 < p \leq 2$. Subsequently, A. Benedek, A. Calderón, and R. Panzone proved the L^p boundedness of $\mu_{\Omega,\Gamma}$, $\Gamma(y) = y$, for all $1 < p < \infty$ provided that $\Omega \in C^1\left(\mathbf{S}^{n-1}\right)$ ([3]).

In their study of singular integral operators, Grafakos and Stefanov ([9]) introduced the following condition:

$$\sup_{\xi\in\mathbf{S}^{n-1}} \int_{\mathbf{S}^{n-1}} \left|\Omega(y^{'})\right| \left(\log\left|\frac{1}{\xi\cdot y'}\right|\right)^{1+\alpha} d\sigma(y^{'}) < \infty. \tag{1.4}$$

For $\alpha > 0$, let $F_{\alpha}(\mathbf{S}^{n-1})$ be the space of all integrable functions on $\mathbf{S}^{n-1}$ which satisfy (1.4). Grafakos and Stefanov ([9]) showed that

$$\bigcap_{\alpha>0} F_{\alpha}(\mathbf{S}^{n-1}) \subsetneqq H^1(\mathbf{S}^{n-1}) \subsetneqq \bigcup_{\alpha>0} F_{\alpha}(\mathbf{S}^{n-1})$$

and

$$\bigcap_{\alpha>0} F_{\alpha}(\mathbf{S}^{n-1}) \subsetneqq L\log^+ L(\mathbf{S}^{n-1}),$$

where $H^1(\mathbf{S}^{n-1})$ is the Hardy space on $\mathbf{S}^{n-1}$ (in the sense of Coifman and Weiss [5]) and $L\log^+ L(\mathbf{S}^{n-1})$ is the space of all functions Ω with $|\Omega|\log^+(|\Omega|)$ is integrable on $\mathbf{S}^{n-1}$. Recently, when $\Omega \in F_{\alpha}(\mathbf{S}^{n-1})$ for some $\alpha > 0$ that satisfies (1.1) and $\frac{\partial^{\gamma}\Gamma}{\partial y^{\gamma}}(0) \neq 0$ for some multi-index $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_n)$, $\gamma_1, \gamma_2, \ldots, \gamma_n$ are non negative integers, there has been a notable progress in obtaining L^p boundedness results of the operator $\mu_{\Omega,\Gamma}$ (see [1], [4], among others). In particular, Al-Qassem and Al-Salman ([1]) proved that $\mu_{\Omega,\Gamma}$ is bounded on L^p, $\frac{2\alpha+2}{2\alpha+1} < p < 2+2\alpha$, provided that Ω is in a subspace of $F_{\alpha}(\mathbf{S}^{n-1})$ for some $\alpha > 0$ and $\Gamma : \mathbf{R}^n \to \mathbf{R}^d$, $d \geq 1$ is a polynomial mapping. Our main focus in this paper is investigating the L^p boundedness of $\mu_{\Omega,\Gamma}$ if $\Omega \in F_{\alpha}(\mathbf{S}^{n-1})$ for some $\alpha > 0$ and

$\frac{\partial^\gamma \Gamma}{\partial y^\gamma}(0) = 0$ for all multi-indices γ. More specifically, we let $\Gamma(y) = \varphi(|y|)y'$, where φ is a real valued function defined on $\mathbf{R}^+$. Here we allow φ to be flat at the origin. In what follows we shall simply denote $\mu_{\Omega,\Gamma}$ by μ_φ. Also, in this paper, we shall establish an L^p boundedness result of the corresponding Marcinkiewicz integral operators that are related to area integral and Littlewood-Paley g^*_λ functions. More specifically, let $F_{\Omega,\varphi,t} = F_{\Omega,\Gamma,t}$ be given by (1.3) with $\Gamma(y) = \varphi(|y|)y'$ and define the operators $\tilde{\mu}_\varphi$ and $\mu^*_{\varphi,\lambda}$ for $\lambda > 1$ by

$$\tilde{\mu}_\varphi f(x) = \left(\int_{\Upsilon(x)} |F_{\Omega,\varphi,t}(z)|^2 \, 2^{-(2+n)t} dz dt \right)^{\frac{1}{2}}, \tag{1.5}$$

$$\mu^*_{\varphi,\lambda} f(x) = \left(\int \int_{\mathbf{R}^{n+1}} \left(\frac{2^t}{2^t + |x-z|} \right)^{n\lambda} |F_{\Omega,\varphi,t}(z)|^2 \, 2^{-(2+n)t} dz dt \right)^{\frac{1}{2}}, \tag{1.6}$$

where $\Upsilon(x) = \{(z,t) \in \mathbf{R}^{n+1} : |x-z| < 2^t\}$.

Throughout the rest of this paper, the functions φ in the statements of the results are assumed to be second continuously differentiable, i.e., $\varphi \in \mathcal{C}^{\,2}$. Our main results in this paper are the following:

Theorem 1.1. *Suppose that $\varphi : \mathbf{R}^+ \to \mathbf{R}$ is an increasing convex function with $\varphi(0) = 0$. If* $\Omega \in F_\alpha(\mathbf{S}^{n-1})$ for some $\alpha > 0$ and satisfies (1.1), *then μ_φ is bounded on $L^p(\mathbf{R}^n)$ for $p \in (\frac{2\alpha+2}{2\alpha+1}, 2+2\alpha)$.*

Theorem 1.2. *Suppose that $\varphi : \mathbf{R}^+ \to \mathbf{R}$ is an increasing convex function with $\varphi(0) = 0$,* $\Omega \in F_\alpha(\mathbf{S}^{n-1})$ for some $\alpha > 0$ and satisfies (1.1). *Then the operators $\tilde{\mu}_\varphi$* and $\mu^*_{\varphi,\lambda}$ are *bounded on $L^p(\mathbf{R}^n)$ for $2 \le p < 2+2\alpha$.*

Throughout this paper, the letter C is a positive constant that may vary at each occurrence but it is independent of the essential variables.

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2. Preliminary Estimates

Throughout the rest of this paper, we shall need the following simple observation:

Proposition 2.1. Suppose that $\varphi : \mathbf{R}^+ \to \mathbf{R}$ is second continuously differentiable increasing convex function with $\varphi(0) = 0$. Then
(i) $\varphi(2r) \geq 2\varphi(r)$ for every $r > 0$.
(ii) $r\varphi'(r) \geq \varphi(r)$ for every $r > 0$.

A proof of Proposition 2.1 is straightforward. In fact, the inequality (**ii**) is an easy consequence of the fact that the function $g(r) = r\varphi'(r) - \varphi(r)$ is an increasing function (since φ is convex, i.e., $\varphi''(r) \geq 0$) and the fact that $g(0) = 0$. The inequality (**i**) follows by use of the Mean value theorem, the fact that φ' is increasing (since φ is convex), and (**ii**).

It is interesting to notice that the inequalities (**i**) and (**ii**) in Proposition 2.1 may not hold if the convexity of the function φ is dropped. For example, the nonconvex function $\varphi(r) = \sqrt{r}$ does not satisfy (**i**) and (**ii**).

For a smooth mapping $\varphi : \mathbf{R}^+ \to \mathbf{R}$, a homogeneous function Ω of degree zero on $\mathbf{R}^n$ that is integrable on $\mathbf{S}^{n-1}$ and satisfies (1.1), a $\xi \in \mathbf{R}^n$, and a nonnegative real number u, let

$$G(\varphi, \Omega, \xi, u) = \int_{\frac{u}{2} \leq |y| < u} e^{-i\varphi(|y|)\xi \cdot y'} |y|^{-n+1} \Omega(y) dy. \tag{2.1}$$

Then we have the following lemma.
Lemma 2.2. *If* $\Omega \in F_\alpha(\mathbf{S}^{n-1})$ for some $\alpha > 0$ and φ *is an increasing convex function with* $\varphi(0) = 0$, then there exist a constant $C > 0$ independent of u and ξ such that

$$|G(\varphi, \Omega, \xi, u)| \leq uC \min\{|\varphi(u)\xi|, (\log\left|\varphi(\frac{u}{2})\xi\right|)^{-1-\alpha}\}. \tag{2.2}$$

Proof. First, by the cancelation property (1.1), we have

$$|G(\varphi, \Omega, \xi, u)| = |G(\varphi, \Omega, \xi, u) - G(\varphi, \Omega, 0, u)| \leq uC\,|\varphi(u)\xi|\,. \tag{2.3}$$

Secondly, using polar coordinates, it is easy to see that

$$|G(\varphi, \Omega, \xi, u)| \leq u \int_{\mathbf{S}^{n-1}} \left|\Omega(y')\right| J(\varphi, \xi, u) d\sigma(y'), \tag{2.4}$$

where

$$J(\varphi, \xi, u) = \left| \int_1^2 e^{-i\varphi(\frac{u}{2}r)\xi \cdot y'} dr \right|. \tag{2.5}$$

Now by integration by parts and Proposition 2.1(ii), it follows that

$$J(\varphi, \xi, u) \leq \left|\varphi(\frac{u}{2})\xi \cdot y'\right|^{-1}. \tag{2.6}$$

Therefore, by (2.6) and the trivial estimate $J(\varphi, \xi, u) \leq 1$, it is easy to see that

$$J(\varphi, \xi, u) \leq C\{\log |\xi' \cdot y'|^{-1}\}^{1+\alpha}\{\log \left|\varphi(\frac{u}{2})\xi\right|\}^{-1-\alpha}, \tag{2.7}$$

where C is a constant independent of ξ, y', and u. Thus by (2.4) and (2.7), we have

$$\begin{aligned} |G(\varphi, \Omega, \xi, u)| &\leq uC\{\log \left|\varphi(\frac{u}{2})\xi\right|\}^{-1-\alpha} \int_{\mathbf{S}^{n-1}} \left|\Omega(y^{'})\right| \{\log |\xi' \cdot y'|^{-1}\}^{1+\alpha} d\sigma(y') \\ &\leq uC\{\log \left|\varphi(\frac{u}{2})\xi\right|\}^{-1-\alpha} \sup_{\eta \in \mathbf{S}^{n-1}} \int_{\mathbf{S}^{n-1}} \left|\Omega(y^{'})\right| \{\log |\eta' \cdot y'|^{-1}\}^{1+\alpha} d\sigma(y'); \end{aligned}$$

which when combined with the condition (1.4), implies that

$$|G(\varphi, \Omega, \xi, u)| \leq uC(\log \left|\varphi(\frac{u}{2})\xi\right|)^{-1-\alpha}.$$

This concludes the proof of lemma. □

Now, by following a similar argument as in the proof of Lemma 4.1 in ([7]), we get the following relation between the operators μ_{φ} and $\mu^{*}_{\varphi,\lambda}$.

Lemma 2.3. Let $\lambda > 1$. Then for any nonnegative function g, we have

$$\int_{\mathbf{R}^n} (\mu^{*}_{\varphi,\lambda} f(x))^2 g(x)dx \leq C_\lambda \int_{\mathbf{R}^n} (\mu_{\varphi} f(x))^2 (Hg)(x)dx, \tag{2.8}$$

where H is the classical Hardy-Littlewood maximal operator on $\mathbf{R}^n$.

Proof. The inequality (2.8) is an immediate consequence of the definition of $\mu^{*}_{\varphi,\lambda} f$ in (1.6) and the following simple inequality:

$$\sup_{t \in \mathbf{R}} 2^{-nt} \int_{\mathbf{R}^n} \left(\frac{2^t}{2^t + |x-y|}\right)^{n\lambda} g(x)dx \leq C_\lambda (Hg)(y).$$

□

3. Proof of Main Result

Proof of Theorem 1.1. Suppose that $\Omega \in F_{\alpha}(\mathbf{S}^{n-1})$ for some $\alpha > 0$ and satisfies (1.1). Let $\Gamma(y) = \varphi(|y|)y'$, where $\varphi : \mathbf{R}^{+} \to \mathbf{R}$ is an increasing convex function with $\varphi(0) = 0$. We start by writing our operator μ_{φ} as

$$\mu_{\varphi}(f)(x) = \sum\nolimits_{j=0}^{\infty} 2^{-j} \mu_{\varphi,j}(f)(x), \tag{3.1}$$

where $\mu_{\varphi,j}$ is given by

$$\mu_{\varphi,j}(f)(x) = (\int_{-\infty}^{\infty} |\Delta_{j,t}(f)(x)|^2 \, dt)^{\frac{1}{2}} \tag{3.2}$$

and

$$\Delta_{j,t}(f)(x) = 2^{-(t-j)} \int_{2^{t-j-1} < |y| \leq 2^{t-j}} f(x - \Gamma(y)) \, |y|^{-n+1} \, \Omega(y) dy. \tag{3.3}$$

To prove that $\left\| \mu_{\varphi}(f) \right\|_p \leq \|f\|_p$ for all $p \in (\frac{2\alpha+2}{2\alpha+1}, 2 + 2\alpha)$, it suffices to show that

$$\left\| \mu_{\varphi,j}(f) \right\|_p \leq C \, \|f\|_p \tag{3.4}$$

for all $p \in (\frac{2\alpha+2}{2\alpha+1}, 2+2\alpha)$ and $j \geq 0$ with constant C independent of j. To establish (3.4), we argue as in ([2]).

By an elementary procedure, choose a collection of $\mathcal{C}^{\infty}$ functions $\{\omega_k\}_{k \in \mathbf{Z}}$ on $(0, \infty)$ with the following properties:

$$\begin{aligned} supp(\omega_k) &\subseteq [\frac{1}{\varphi(2^{k+1})}, \frac{1}{\varphi(2^{k-1})}]; \ 0 \leq \omega_k \leq 1; \\ \left| \frac{d^s \omega_k}{du^s}(u) \right| &\leq \frac{C_s}{u^s}; \ \sum_{k \in \mathbf{Z}} \omega_k(u) = 1. \end{aligned} \tag{3.5}$$

For $k \in \mathbf{Z}$, let ψ_k be the function defined on $\mathbf{R}^n$ by $\hat{\psi}_k(y) = \omega_k(|y|)$. Then, it is easy to see that

$$\mu_{\varphi,j}(f)(x) \leq \sum_{k \in \mathbf{Z}} \mu_{\varphi,j,k}(f)(x), \tag{3.6}$$

where

$$\mu_{\varphi,j,k}(f)(x) = (\int_{-\infty}^{\infty} \left|\Delta_{j,t}(f * \psi_{\lfloor t-j \rfloor + k}(x)\right|^2 dt)^{\frac{1}{2}}. \tag{3.7}$$

Here, $\lfloor x \rfloor$ is the greatest integer function less than or equal to x.

First, we claim that

$$\left\|\mu_{\varphi,j,k}(f)\right\|_p \leq C \left\|f\right\|_p \tag{3.8}$$

for all $p \in (1, \infty)$ with constant C independent of j and k. To see this, define the operators $M_{j,k}$ and $S_{j,k}$ by

$$M_j(f)(x) = \sup_{t \in \mathbf{R}} |\Delta_{j,t}(f)(x)| \tag{3.9}$$

$$S_{j,k}f(x) = (\int_{-\infty}^{\infty} \left|f * \psi_{\lfloor t-j \rfloor + k}(x)\right|^2 dt)^{\frac{1}{2}}. \tag{3.10}$$

Then, by a similar justification as in ([2] , see also [12], P. 46 and[11], P. 245-246), it follows that

$$\left\|S_{j,k}(f)\right\|_p \leq C \left\|f\right\|_p \tag{3.11}$$

for all $p \in (1, \infty)$ with constant C depends only on p and the dimension of the underlying space $\mathbf{R}^n$.

Now, using polar coordinates, we have

$$M_j(f)(x) \leq \int_{\mathbf{S}^{n-1}} \left|\Omega(y^{'})\right| \{\sup_{r>0} \frac{1}{r} \int_{\frac{r}{2}}^{r} |f(x - \varphi(v)y')| \, dv\} d\sigma(y'). \tag{3.12}$$

By convexity of φ and Proposition on page 477 in ([12]), we have

$$\left\|\sup_{r>0} \frac{1}{r} \int_{\frac{r}{2}}^{r} |f(x - \varphi(v)y')| \, dv\right\|_p \leq C \left\|f\right\|_p \tag{3.13}$$

for all $p \in (1, \infty)$ with constant C independent of $y' \in \mathbf{S}^{n-1}$. Therefore, by (3.12), (3.13), and Minkowski inequality, we obtain

$$\left\|M_j(f)\right\|_p \leq C \left\|f\right\|_p \tag{3.14}$$

for all $p \in (1, \infty)$ with constant C independent of j. Thus, by (3.11), (3.14), the fact that $2^{-(t-j)} G(\varphi, |\Omega|, 0, 2^{t-j}) \le C$ where $G(\varphi, |\Omega|, 0, 2^{t-j})$ is given by (2.1), and Theorem 3.1 in ([1]), we obtain (3.8).

Secondly, we claim that for $p \in (\frac{2\alpha+2}{2\alpha+1}, 2+2\alpha)$, there exist constants $\theta(p) > 0$ and $\beta(p) > 1$ such that

$$\left\| \mu_{\varphi,j,k}(f) \right\|_p \le A(k) \left\| f \right\|_p , \tag{3.15}$$

where

$$A(k) = \begin{cases} 2^{-\theta(p)k} & , \text{ if } k \ge -1 \\ |k|^{-\beta(p)} & , \text{ if } k < -1. \end{cases} \tag{3.16}$$

To prove (3.15), we proceed as follows.

Notice that

$$(\Delta_{j,t}(f * \psi_{\lfloor t-j \rfloor + k}))\hat{}(\xi) = 2^{-(t-j)} G(\varphi, \Omega, \xi, 2^{t-j}) \omega_{\lfloor t-j \rfloor + k}(|\xi|) \hat{f}(\xi), \tag{3.17}$$

where $G(\varphi, \Omega, \xi, 2^{t-j})$ is given by (2.1). Therefore, by Lemma 2.2, we obtain

$$\left| (\Delta_{j,t}(f * \psi_{\lfloor t-j \rfloor + k}))\hat{}(\xi) \right| \le C \left| \varphi(2^{t-j})\xi \right| \left| \omega_{\lfloor t-j \rfloor + k}(|\xi|) \right| \left| \hat{f}(\xi) \right| \tag{3.18}$$

$$\left| (\Delta_{j,t}(f * \psi_{\lfloor t-j \rfloor + k}))\hat{}(\xi) \right| \le C (\log \left| \varphi(2^{t-j-1})\xi \right|)^{-1-\alpha} \left| \omega_{\lfloor t-j \rfloor + k}(|\xi|) \right| \left| \hat{f}(\xi) \right| \tag{3.19}$$

with constant C independent of j, k, t, and ξ. For $k \in \mathbf{Z}$, $\xi \in \mathbf{R}^n$ and $j > 0$, let $b(k,j,\xi) = \log_2(2^{j-k-1} \varphi^{-1}(|\xi|^{-1}))$ and $d(k,j,\xi) = \log_2(2^{j-k+2} \varphi^{-1}(|\xi|^{-1}))$. Thus, if $k > 1$, by Plancherel's theorem, (3.5), and (3.18), we have

$$\begin{aligned} \left\| \mu_{\varphi,j,k}(f) \right\|_2^2 &= C \int_{\mathbf{R}^n} \left| \hat{f}(\xi) \right|^2 \int_{b(k,j,\xi)}^{d(k,j,\xi)} \left| \varphi(2^{t-j})\xi \right|^2 dt d\xi \\ &\le (\log 2) C 2^{-k} \int_{\mathbf{R}^n} \left| \hat{f}(\xi) \right|^2 \int_{b(k,j,\xi)}^{d(k,j,\xi)} dt d\xi \\ &\le (\log 2) C 2^{-k} \left\| f \right\|_2^2 \end{aligned} \tag{3.20}$$

Here, in the second inequality in (3.20), we used Proposition 2.1 (i). Thus

$$\left\|\mu_{\varphi,j,k}(f)\right\|_2 \leq C2^{-k} \|f\|_2 . \tag{3.21}$$

We notice here that by the fact $2^{-(t-j)}G(\varphi, |\Omega|, 0, 2^{t-j}) \leq C$, the inequality (3.21) also holds for $k = -1, 0, 1$. Therefore, by interpolating between (3.8) and (3.21) along with the remark just mentioned for every $1 < p < \infty$, we get (3.15) for $k \geq -1$.

On the other hand, if $k < -1$, then by Plancherel's theorem, (3.5), (3.19), Proposition 2.2 (i), and similar argument as for the case $k > 1$, we obtain

$$\left\|\mu_{\varphi,j,k}(f)\right\|_2 \leq C |k|^{-1-\alpha} \|f\|_2 . \tag{3.22}$$

Therefore, if $p \in (\frac{2\alpha+2}{2\alpha+1}, 2+2\alpha)$, then by interpolating between (3.22) and (3.8) for any $1 < p < \infty$, there exists a constant $\beta(p) > 1$ such that

$$\left\|\mu_{\varphi,j,k}(f)\right\|_p \leq C |k|^{-\beta} \|f\|_p . \tag{3.23}$$

This concludes the proof of (3.15).

Hence, (3.4) follows by (3.6), (3.15), and Minkowski inequality. This completes the proof of Theorem 1.1. □

Finally, Theorem 1.2 follows by Lemma 2.3, Theorem 1.1, and a similar argument as in ([7]). We omit the details.

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