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Traces of Maximal Ideals of Topological Algebras in Their Subalgebras

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Abstract

Let $\mathbb{K}$ be one of the fields $\mathbb{R}$ or $\mathbb{C}$, A a topological associative algebra over $\mathbb{K}$ with separately continuous multiplication, B a subalgebra of A and M a closed maximal regular left (right or two-sided) ideal of A such that the trace $M \cap B$ of M is a proper subset of B. In cases, when B is a Gelfand-Mazur subalgebra of A or a subalgebra of the centre $Z(A)$ of A, such classes of topological algebras in which $M \cap B$ (in the subset topology) is a closed maximal regular left (respectively, right or two-sided) ideal of B, are described.

Key words and phrases: Topological algebras, Gelfand-Mazur algebras, locally pseudoconvex algebras, galbed algebras, traces of ideals in subalgebras.

1. Introduction

Let $\mathbb{K}$ be one of the fields $\mathbb{R}$ of real numbers or $\mathbb{C}$ of complex numbers, A a topological associative algebra over $\mathbb{K}$ with separately continuous multiplication (in short, a topological algebra), B a subalgebra of A and M a closed maximal regular left (right or two-sided) ideal of A such that the trace $M \cap B$ of M in B is a proper subset of B. Then $M \cap B$ (in the subset topology) is a closed regular left (respectively, right or two-sided) ideal of B, but not necessarily a closed maximal ideal of B, as it is shown below. Therefore, it is interesting to know for which subalgebra B of a topological algebra A the trace $M \cap B$ of every closed maximal regular left (right or two-sided) ideal M of A is a maximal ideal

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in B This problem has arisen very often for several problems of topological algebras. An answer to this question, in case B is a Gelfand-Mazur subalgebra of A or a subalgebra of the centre $Z(A)$ of A, is given.

1. A topological algebra A is *locally pseudoconvex* if it has a base $\{U_\alpha : \alpha \in \mathcal{A}\}$ of neighbourhoods of zero which consists of balanced (that is, $\mu a \in U_\alpha$ if $a \in U_\alpha$ and $|\mu| \leqslant 1$) and pseudoconvex (that is, $U_\alpha + U_\alpha \subset 2^{\frac{1}{k_\alpha}} U_\alpha$ for some $k_\alpha \in (0,1]$) sets U_α. It is well known (see [24], p. 4, or [15], p. 189) that we can give the topology of A by a family $\{p_\alpha : \alpha \in \mathcal{A}\}$ of k_α-homogeneous seminorms. In particular, when A is a locally pseudoconvex algebra, in which for each $a \in A$ and $\alpha \in \mathcal{A}$ there are positive numbers $M(a,\alpha)$ and $N(a,\alpha)$ such that $p_\alpha(ab) \leqslant M(a,\alpha)p_\alpha(b)$ and $p_\alpha(ba) \leqslant N(a,\alpha)p_\alpha(b)$ for all $b \in A$, then A is a *locally absorbingly pseudoconvex* or *locally A-pseudoconvex* algebra. Furthermore, if all seminorms p_α satisfy the condition $p_\alpha(ab) \leqslant p_\alpha(a)p_\alpha(b)$ for all $a, b \in A$, then A is a *locally multiplicatively pseudoconvex* or *locally m-pseudoconvex* algebra. In the case when $k_\alpha = 1$ for each $\alpha \in \mathcal{A}$, then A is a *locally convex* (respectively, *locally A-convex* or *locally m-convex*) algebra, and when the topology of A is given by only one k-homogeneous seminorm for some $k \in (0,1]$, then A is a *locally bounded algebra*. Hence, the class of locally pseudoconvex algebras contains all locally convex algebras and all locally bounded algebras (thus, it contains all p-Banach (in particular, all Banach) algebras studied by W. Żelazko in [25] and in [26]).

2. Let l^0 denote the set of sequences (α_n) in $\mathbb{K}$ which have a finite number of nonzero elements α_n, l^1 the set of sequences (α_n) in $\mathbb{K}$ such that $\sum_{k=0}^{\infty} |\alpha_n|$ converges, and let $l = l^1 \setminus l^0$. A topological algebra A is a *galbed algebra* (see [5], [6] and [12]) if there is a sequence $(\alpha_n) \in l$ such that for each neighbourhood O of zero in A there is another neighbourhood U of zero such that

$$\Big\{ \sum_{k=0}^{n} \alpha_k a_k : a_0, \dots, a_n \in U \Big\} \subset O$$

for each $n \in \mathbb{N}$. In particular, when $\alpha_0 \neq 0$ and $\inf_{n>0} |\alpha_n|^{\frac{1}{n}} > 0$, then (see [5]) A is called a *strongly galbed algebra,* and when $\alpha_n = 2^{-n}$ for each $n \in \mathbb{N}$, then (see, for example, [3], [7], [8] and [23]) is called an *exponentially galbed algebra.* It is easy to see that every locally pseudoconvex algebra is an exponentially galbed algebra and every exponentially galbed algebra is a strongly galbed algebra, but there are topological algebras which are not galbed (see [6], Proposition 5).

3. Let A be a topological algebra, $m(A)$ the set of all closed regular two-sided ideals

of A, maximal as left or right ideals and let hom$A$ be the set of all nontrivial continuous linear multiplicative maps from A onto $\mathbb{K}$.

A topological algebra over $\mathbb{K}$ is called a *Gelfand-Mazur algebra* (see, for example, [3], [8], [10] and [13]) if A/M is topologically isomorphic to $\mathbb{K}$ for each $M \in m(A)$. One speaks about *real Gelfand-Mazur algebras* if $\mathbb{K} = \mathbb{R}$ and about *complex Gelfand-Mazur algebras* if $\mathbb{K} = \mathbb{C}$. It is easy to see that a Gelfand-Mazur algebra A is exactly a topological algebra for which there is a bijection between $m(A)$ and hom$A$. Therefore, only in case of Gelfand-Mazur algebras it is possible to use the Gelfand theory, well-known for commutative complex Banach algebras.

The class of Gelfand-Mazur algebras is quite large. In addition to complex normed, locally m-convex and locally bounded algebras, the class of Gelfand-Mazur algebras contains all complex locally pseudoconvex Fréchet algebras, all complex locally pseudoconvex Waelbroeck algebras[1], all complex locally A-pseudoconvex (in particular, locally m-pseudoconvex) algebras, all complex strongly galbed algebras with bounded elements[2] (see, for example, [3], [8] and [10]) and similar classes of commutative strongly real topological algebras (see [21]).

4. Let A be a topological algebra. A net $(a_\lambda)_{\lambda\in\Lambda}$ of elements of A is (a) *advertibly convergent* in A (see [9]) if there is an element $a \in A$ such that both[3] $(a \circ a_\lambda)_{\lambda\in\Lambda}$ and $(a_\lambda \circ a)_{\lambda\in\Lambda}$ converge to the zero element θ_A of A; (b) *Mackey convergent* (see [18], pp. 25–26) to an element $a_0 \in A$ if there exists a bounded set $B \subset A$ and for every $\varepsilon > 0$ an index $\lambda_0 \in \Lambda$ such that $a_\lambda - a_0 \in \varepsilon B$ whenever $\lambda > \lambda_0$; and (c) *Mackey advertibly convergent* (see [5]) if there is an element $a \in A$ such that both $(a \circ a_\lambda)_{\lambda\in\Lambda}$ and $(a_\lambda \circ a)_{\lambda\in\Lambda}$ are Mackey convergent to θ_A.

A topological algebra A is (see [20] and [5]) *advertibly complete* (*Mackey advertibly complete*) if every advertibly convergent (respectively, Mackey advertibly convergent) Cauchy net in A converges in A. In particular, when only every advertibly convergent (Mackey advertibly convergent) Cauchy sequence converges in A, then one speaks about *advertibly σ-complete* (respectively, *Mackey advertibly σ-complete*) topological algebras. It is easy to see that every complete topological algebra is advertibly complete and every advertibly complete topological algebra is Mackey advertibly complete. Moreover, the class of advertibly complete topological algebras contains all Q-algebras[4] (see [20], p.

[1]The definition of Waelbroeck algebra can be found in [20], p. 54.

[2]An element $a \in A$ is bounded if there is a number $\lambda \in \mathbb{K} \setminus \{0\}$ such that the set $\{(\frac{a}{\lambda})^n : n \in \mathbb{N}\}$ is bounded in A.

[3]Here and later on $a \circ b = a + b - ab$ for each $a, b \in A$.

[4]A topological algebra A is a Q-*algebra* if the set of all advertive (if A has the unit element, then

45).

5. Let $A = L^{\omega}$ be the set of all measurable functions (classes of equivalence) f on the unit interval $(0,1)$ such that

$$p_n(f) = \Big(\int_0^1 |f(t)|^n dt \Big)^{\frac{1}{n}} < \infty$$

for each $n \in \mathbb{N}$. Then (see [26], Example 10.5) A is a unital commutative locally convex Fréchet algebra with respect to the point-wise algebra operations and the topology defined on A by the countable family $\{p_n : n \in \mathbb{N}\}$ of seminorms. It is known (see [26], p. 125, or [17], Example 4.10.31) that A has closed ideals but no closed maximal ideals (which means that every maximal ideal of A is dense).

Let now B be a metrizable locally convex algebra, $\{q_n : n \in \mathbb{N}\}$ a countable family of seminorms on B which defines the topology of B and let $C = A \times B$. If we define all algebraic operations in C coordinate-wise and seminorms r_n on C by

$$r_n((a,b)) = \max\{p_n(a), q_n(b)\}$$

for each $(a,b) \in C$, then C is a metrizable locally convex algeba in the topology defined by the countable family $\{r_n : n \in \mathbb{N}\}$. Since $r_n((a, \theta_B)) = p_n(a)$ for each $a \in A$, then the map $\mu : A \to C$ defined by $\mu(a) = (a, \theta_B)$ for each $a \in A$, is a topological isomorphism. Hence, $\mu(A)$ is a unital commutative subalgebra of C for which the trace $M \cap \mu(A)$ of every closed maximal ideal M of C is a closed ideal in $\mu(A)$ but not a closed maximal ideal.

2. Maximality of traces of ideals in subalgebras

1. Let A be a topological algebra. The next result characterizes these subalgebras B of A in which all traces of closed regular ideals of A are closed maximal ideals.

Proposition 1 *Let A be a topological algebra, I a closed regular left* (*right or two-sided*) *ideal of A, u a right* (*respectively, left or two-sided*) *unit for I. If B is a subalgebra of A such that $u \in B$ and*

(*a*) *for each $b \in B$ there is a $\lambda \in \mathbb{K}$ such that $b - \lambda u \in I$,*

invertive) elements of A is open.

then the trace $I \cap B$ *of* I (*in the subspace topology*) *is a closed maximal regular left* (*respectively, right or two-sided*) *ideal of* B.

In particular, when B *is a commutative Gelfand-Mazur subalgebra of* A *and* $u \in B$, *then* $I \cap B \in m(B)$ *if and only if* B *satisfies the condition* (a).

Proof. Let I be a closed regular left ideal[5] of A and B a subalgebra of A. Then the trace $I \cap B$ of I in B is a closed regular left ideal. If $I \cap B$ is not maximal, then there is a left ideal J in B such that $I \cap B \subset J$ and there is an element $b_0 \in J \setminus (I \cap B)$. Because B and I satisfy the condition (a), then there is a number $\lambda_0 \in \mathbb{K}$ such that $b_0 - \lambda_0 u \in I$. Since $b_0 \notin I$, then $\lambda_0 \neq 0$. Therefore

$$u = \frac{1}{\lambda_0}[b_0 - (b_0 - \lambda_0 u)] \in J;$$

but this is impossible. Hence $I \cap B$ is a closed maximal regular left ideal of B.

Let now B be a commutative Gelfand-Mazur subalgebra of A and $u \in B$. If B satisfies the condition (a), then $I \cap B \in m(B)$; and if $I \cap B \in m(B)$, then there is a $\varphi \in \hom B$ such that $I \cap B = \ker\varphi$. Since $u \notin \ker\varphi$, then $\varphi(u) \neq 0$. Hence

$$b - \frac{\varphi(b)}{\varphi(u)} u \in I$$

for each $b \in B$. It means that B satisfies condition (a). □

Corollary 1 *Let* A *be a topological algebra with the unit element* e_A *and* I *a closed left* (*right or two-sided*) *ideal of* A. *If* B *is a subalgebra of* A *with the same unit element and*

(α) *for each* $b \in B$ *there is a* $\lambda \in \mathbb{K}$ *such that* $b - \lambda e_A \in I$,

then the trace $I \cap B$ *of* I *is a closed maximal left* (*respectively, right or two-sided*) *ideal of* B.

In particular, when B *is a commutative Gelfand-Mazur subalgebra of* A *such that* $e_A \in B$, *then* $I \cap B \in m(B)$ *if and only if* B *satisfies the condition* (α).

Corollary 2 *Let* A *be a topological algebra such that* $\hom A$ *is not empty and* B *a subalgebra of* A. *If* $\psi \in \hom A$ *and a unit* u *for* $\ker\psi$ *belongs to* B, *then* $\ker\psi \cap B \in m(B)$.

[5] For right and two-sided ideals the proof is similar.

Proof. Since $\ker\psi$ is a closed maximal regular two-sided ideal of A, $u \notin \ker\psi$ and

$$b - \frac{\psi(b)}{\psi(u)} u \in \ker\psi$$

for each $b \in B$, then $\ker\psi \cap B \in m(B)$ by Proposition 1. □

Corollary 3 *Let A be a topological algebra with the unit element e_A such that* hom$A$ *is not empty and B a subalgebra of A with the same unit element. Then* $\ker\psi \cap B \in m(B)$ *for each $\psi \in$* hom$A$.

2. Let A be a topological algebra with the unit element e_A, M a closed maximal left (right or two-sided) ideal of A, $A - M$ the quotient space of A defined by M, π_M the canonical map from A onto $A-M$ and $\mathcal{L}(A-M)$ the algebra of all linear maps from $A-M$ into $A - M$. Then $A - M$ is a left (right) A-module if we define the left (respectively, right) multiplication on $A - M$ by $a \cdot \pi_M(b) = \pi_M(ab)$ (respectively, $\pi_M(b) \cdot a = \pi_M(ba)$ for each $a, b \in A$. Moreover, $\mathcal{L}(A - M)$ is an algebra over $\mathbb{K}$ if we define the addition and the multiplication over $\mathbb{K}$ in $\mathcal{L}(A - M)$ pointwise and the multiplication of elements in $\mathcal{L}(A - M)$ by the composition.

For each element $a \in A$ let L_a^M be a map from $A - M$ into $A - M$ defined by

$L_a^M(x) = a \cdot x$ ($L_a^M(x) = x \cdot a$) for each $x \in A-M$ if M is a maximal left (respectively, right) ideal in A and $\mathcal{L}(A - M)$ defined by $L_M(a) = L_a^M$ for each $a \in A$. Then L_M is a representation of A on $A - M$ with

$$\ker L_M = \{a \in A : aA \subset M\}$$

if M is a maximal left ideal and

$$\ker L_M = \{a \in A : Aa \subset M\}$$

if M is a maximal right ideal of A. In both cases $\ker L_M \subset M$ is called a *primitive ideal* of A. Let

$$\mathcal{D}_A^M = \{T \in \mathcal{L}(A - M) \colon a \cdot (T(x)) = T(a \cdot x) \text{ for each } a \in A \text{ and } x \in A - M\}$$

and

$$\mathcal{B}_M^A = \{a \in A : (ba - ab)A \subset M \text{ for each } b \in A\}$$

for a closed maximal left ideal M of A. Then $\mathcal{D}_A^M$ is a division subalgebra of $\mathcal{L}(A-M)$ (see [16], p.127), $\mathcal{B}_M^A$ is a subalgebra of A, $e_A \in \mathcal{B}_M^A$ and

$$L_M(\mathcal{B}_M^A) = L_M(A) \cap \mathcal{D}_A^M,$$

because $(ab)\cdot x = L_M(ab)(x) = L_M(a)[L_M(b)(x)] = a\cdot(b\cdot x)$ for each $a, b \in A$ and and for each $x \in A - M$. It is known (see [11], Theorem 1) that $\mathcal{D}_A^M$ is topologically isomorphic to $\mathbb{C}$ (we denote this isomorphism by μ) for any closed maximal left (right) ideal M of A if A is a unital locally m-pseudoconvex Hausdorff algebra over $\mathbb{C}$ or a unital locally pseudoconvex Fréchet algebra over $\mathbb{C}$.

Proposition 2 *Let A be a locally m-pseudoconvex Hausdorff algebra over $\mathbb{C}$ or a locally pseudoconvex Fréchet algebra over $\mathbb{C}$, e_A the unit element of A and M a closed maximal left* (*right*) *ideal of A. Then for every $a \in \mathcal{B}_M^A$ there is a number $\lambda \in \mathbb{C}$ such that $a - \lambda e_A \in M$.*

Proof. Let $a \in \mathcal{B}_M^A$. Then $L_M(a) \in \mathcal{D}_A^M$. Therefore there is a $\lambda \in \mathbb{C}$ such that $\mu(L_M(a)) = \lambda = \mu(L_M(\lambda e_A))$. Hence $a - \lambda e_A \in \ker L_M \subset M$. □

By Corollary 1 and Proposition 2 we have the following theorem.

Theorem 1 *Let A be a unital locally m-pseudoconvex Hausdorff algebra over $\mathbb{C}$ or a unital locally pseudoconvex Fréchet algebra over $\mathbb{C}$, M a closed maximal left* (*right or two-sided*) *ideal of A and B a subalgebra of A with the same unit as A. If $B \subset \mathcal{B}_M^A$, then $M \cap B$ is a closed maximal left* (*respectively, right or two-sided*) *ideal in B.*

3. Maximality of traces of maximal ideals in the center

Now we consider the case when a subalgbera B of A belongs to the center $Z(A)$ of A. It is well-known (see [14], Theorem 2.3) that if A is a complex Banach algebra with the unit element e_A and M is a maximal left (right or two-sided) ideal of A, then for each $z \in Z(A)$, there is some $\lambda \in \mathbb{C}$ such that $z - \lambda e_A \in M$. Next we prove the following generalization of this result.

Proposition 3 *Let A be one of the following unital complex topological algebras:*

a) *a locally A-pseudoconvex algebra;*
b) *a locally pseudoconvex Fréchet algebra;*

c) *a strongly galbed Fréchet algebra with bounded elements;*

or

d) *a Mackey advertibly σ-complete topologically primitive*[6] *strongly galbed Hausdorff algebra with bounded elements.*

Then for each closed maximal left (right or two-sided) ideal M of A and each $z \in Z(A)$ there is some $\lambda \in \mathbb{C}$ such that $z - \lambda e_A \in M$.

Proof. Cases (a) and (b) have been proved in [1], Theorem 3 (see also [2], Corollary 1, or [3], Corollary 3.2). To prove the cases (c) and (d), let M be a closed maximal left[7] ideal of A, P_M the primitive ideal of A defined[8] by M and let $\pi_M : A \to A/P_M$ be the canonical homomorphism. Then P_M is a closed primitive ideal in A. Hence, in case (c), $Z(A/P_M)$ is a strongly galbed (by Proposition 2.1 in [4]) Fréchet algebra (by Theorem 2 in [19], p. 138) with bounded elements (see the proof of Theorem 2.1 in [3]). Moreover, A/P_M is topologically primitive by Proposition 9, p. 136, from [16]. Hence, in case (c), $Z(A/P_M)$ is topologically isomorphic (this isomorphism we denote by μ) to $\mathbb{C}$ by Theorem 3.1 from [4] (or by Corollary 7 from [5]). Since $\pi_M(Z(A)) \subset Z(A/P_M)$, then for each $z \in Z(A)$ we can find a number $\lambda \in \mathbb{C}$ such that $\mu(\pi_M(b)) = \lambda = \mu(\pi_M(\lambda e_A))$. Therefore from $\pi_M(z) = \pi_M(\lambda e_A)$ follows that $z - \lambda e_A \in P_M \subset M$.

In case (d) $Z(A) = \mathbb{C}e_A$ by Corollary 9 from [5]. Consequently, for each $z \in Z(A)$ there is some $\lambda \in \mathbb{C}$ such that $z - \lambda e_A = \theta_A \in M$. □

Theorem 2 *Let A be one of the following unital topological algebras:*

a) *a (real commutative) locally A-pseudoconvex algebra;*

b) *a (real commutative) locally pseudoconvex Fréchet algebra;*

c) *a complex strongly galbed Fréchet algebra with bounded elements*

or

d) *a complex Mackey advertibly σ-complete topologically primitive strongly galbed Hausdorff algebra with bounded elements;*

Moreover, let M be a closed maximal left (right or two-sided) ideal[9] *of A and B a subalgebra of $Z(A)$ with the same unit as A. Then the trace $M \cap B$ of M is a closed maximal ideal in B.*

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[6] A topological algebra A is a *topologically primitive algebra* if there is a closed maximal regular left (right) ideal M of A such that $\{a \in A : aA \subset M\} = \{\theta_A\}$ (respectively, $\{a \in A : Aa \subset M\} = \{\theta_A\}$).

[7] The proof for closed maximal right ideal is similar.

[8] If M is a closed maximal two-sided ideal in A, then $P_M = M$ (see [22], Theorem 2.2.9(ii)).

[9] In real case we assume that M satisfies the condition: if $a^2 + b^2 \in M$, then $a \in M$ and $b \in M$.

Proof. Since every $b \in B$ defines some $\lambda \in \mathbb{K}$ such that $z - \lambda e_A \in M$ by Proposition 3 (in complex case) from the present paper and Corollary 4 from [21] (in real case), then Theorem 2 holds by Corollary 1. □

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