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Some Curvature Tensors on a Trans-Sasakian Manifold

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Abstract

The object of the present paper is to study the geometry of trans-Sasakian manifold when it is projectively semi-symmetric, Weyl semi-symmetric and concircularly semi-symmetric.

Key words and phrases: Trans-Sasakian, projectively flat, concircularly flat.

1. Introduction

In 1985, J.A. Oubina [9] introduced a new class of almost contact manifold namely trans-Sasakian manifold. Many geometers like [1, 2, 6], [5], [9], have studied this manifold and obtained many interesting results. The notion of semi-symmetric manifold is defined by $R(X,Y)\cdot R = 0$ and studied by many authors [10, 11, 12]. The conditions R(X,Y)·P=0, R(X,Y)·C=0 and R(X,Y)·$\overline{C}$ = 0 are called projectively semi-symmetric, Weyl semi-symmetric and concircularly semi-symmetric respectively, where R(X,Y) is considered as derivation of tensor algebra at each point of the manifold. In this paper we consider the trans-Sasakian manifold under the condition ϕ (grad α) = (2m-1) grad β satisfying the properties R(X,Y)·P = 0, R(X,Y)·C = 0 and R(X,Y)·$\overline{C}$ = 0 and show that such a manifold is either Einstein or η-Einstein.

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2. Preliminaries

Let M be a $(2m+1)$ dimensional almost contact metric manifold with an almost contact metric structure (ϕ,ξ,η,g), where ϕ is a (1,1) tensor field, ξ is a vector field, η is a 1-form and g is the associated Riemannian metric such that [3],

$$\begin{aligned}\phi^2=-I+\eta\otimes\xi,\ \ \eta(\xi) &= 1,\ \phi\xi=0,\ \ \eta o\phi=0,\\ g(\phi X,\phi Y) &= g(X,Y)-\eta(X)\eta(Y), \end{aligned} \tag{2.1}$$

$$g(X,\phi Y)=-g(\phi X,Y)\ and\ g(X,\xi)=\eta(X)\forall X,Y\in TM. \tag{2.2}$$

An almost Contact metric structure (ϕ,ξ,η,g) on M is called a trans-Sasakian structure [9],if $(M\times R,J,G)$ belongs to the class W_4 [7], where J is the almost complex structure on M ×R defined by J(X, f $\frac{d}{dt}$) = $(\phi X-f\xi,\eta(X)\frac{d}{dt})$ for all vector fields X on M and smooth functions f on M × R. This may be expressed by the condition [4],

$$(\nabla_X\phi)Y=\alpha(g(X,Y)\xi-\eta(Y)X)+\beta(g(\phi X,Y)\xi-\eta(Y)\phi X), \tag{2.3}$$

for some smooth functions α and β on M, and we say that the trans-Sasakian structure is of type (α,β).
From (1.3) it follows that

$$\nabla_X\xi=-\alpha\phi X+\beta(X-\eta(X)\xi),\quad(\nabla_X\eta)Y=-\alpha g(\phi X,Y)+\beta g(\phi X,\phi Y). \tag{2.4}$$

Trans-Sasakian manifolds have been studied by authors [5] and they obtained the following results:

$$\begin{aligned}R(X,Y)\xi&=(\alpha^2-\beta^2)(\eta(Y)X-\eta(X)Y)+2\alpha\beta(\eta(Y)\phi(X)-\eta(X)\phi(Y))\\ &\quad+(Y\alpha)\phi X-(X\alpha)\phi Y+(Y\beta)\phi^2X-(X\beta)\phi^2Y,\end{aligned} \tag{2.5}$$

$$R(\xi,X)\xi=(\alpha^2-\beta^2-\xi\beta)(\eta(X)\xi-X), \tag{2.6}$$

$$2\alpha\beta+\xi\alpha=0, \tag{2.7}$$

$$S(X,\xi)=(2m(\alpha^2-\beta^2)-\xi\beta)\eta(X)-(2m-1)X\beta-(\phi X)\alpha, \tag{2.8}$$

$$Q\xi=(2m(\alpha^2-\beta^2)-\xi\beta)\xi-(2m-1)grad\beta+\phi(grad\alpha). \tag{2.9}$$

When $\phi(grad\alpha)=(2m-1)grad\beta$, (1.8) and (1.9) reduce to

$$S(X,\xi)=2m(\alpha^2-\beta^2)\eta(X), \tag{2.10}$$

$$Q\xi=2m(\alpha^2-\beta^2)\xi. \tag{2.11}$$

3. Projectively flat Trans-Sasakian manifold

The Weyl-projective curvature tensor P is defined as

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{2m}\left[S(Y,Z)X - S(X,Z)Y\right], \tag{3.1}$$

where R is the curvature tensor and S is the Ricci tensor.
Suppose that P = 0. Then from (3.1), we have

$$R(X,Y)Z = \frac{1}{2m}\left[S(Y,Z)X - S(X,Z)Y\right]. \tag{3.2}$$

From (3.2), we have

$$'R(X,Y,Z,W) = \frac{1}{2m}\left[S(Y,Z)g(X,W) - S(X,Z)g(Y,W)\right], \tag{3.3}$$

where $'$R(X,Y,Z,W) = g(R(X,Y,Z),W).
Putting W = ξ in (3.3), we get

$$\eta(R(X,Y)Z) = \frac{1}{2m}\left[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)\right]. \tag{3.4}$$

Again taking X = ξ in (3.4), and using (2.1),(2.5) and (2.10), we get

$$S(Y,Z) = 2m(\alpha^2 - \beta^2)g(Y,Z). \tag{3.5}$$

Therefore, the manifold is Einstein. Hence we can state the following theorem

Theorem 3.1 *A Weyl projectively flat trans-Sasakian manifold is an Einstein manifold.*

4. Trans-Sasakian manifold satisfying R(X,Y)· P = 0

Using (2.2), (2.5) in (3.1), we get

$$\eta(P(X,Y)Z) = (\alpha^2 - \beta^2)\left[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\right] \\ -\frac{1}{2m}[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)]. \tag{4.1}$$

Putting Z = ξ in (4.1), we get

$$\eta(P(X,Y)\xi) = 0. \tag{4.2}$$

Again taking X = ξ in (4.1), we have

$$\eta(P(\xi,Y)Z) = (\alpha^2-\beta^2)g(Y,Z) - \frac{1}{2m}S(Y,Z), \tag{4.3}$$

where (2.1) and (2.10) are used.
Now,

$$(R(X,Y)P)(U,V)Z = R(X,Y)\cdot P(U,V)Z - P(R(X,Y)U,V)Z - P(U,R(X,Y)V)Z \\ -P(U,V)R(X,Y)Z.$$

As it has been considered R(X,Y)·P = 0,so we have

$$R(X,Y)\cdot P(U,V)Z - P(R(X,Y)U,V)Z - P(U,R(X,Y)V)Z \\ -P(U,V)R(X,Y)Z = 0. \tag{4.4}$$

Therefore,

$$g\left[R(\xi,Y)\cdot P(U,V)Z,\xi\right] - g\left[P(R(\xi,Y)U,V)Z,\xi\right] \\ -g[P(U,R(\xi,Y)V)Z,\xi] - g[P(U,V)R(\xi,Y)Z,\xi] = 0.$$

From this, it follows that,

$$-{}'P(U,V,Z,Y) + \eta(Y)\eta(P(U,V)Z) - \eta(U)\eta(P(Y,V)Z) + g(Y,U)\eta(P(\xi,V)Z) \\ -\eta(V)\eta(P(U,Y)Z) + g(Y,V)\eta(W(U,\xi)Z) - \eta(Z)\eta(W(U,V)Y) = 0, \tag{4.5}$$

where ${}'$P(U,V,Z,Y)=g(P(U,V)Z,Y).
Putting Y=U in (4.5), we get

$$-{}'P(U,V,Z,Y) + g(U,U)\eta(P(\xi,V)Z) - \eta(V)\eta(P(U,U)Z) \\ +g(U,V)\eta(P(U,\xi)Z) - \eta(Z)\eta(P(U,V)U) = 0. \tag{4.6}$$

Let $\{e_i\}$, i = 1,2, . . . ,(2m+1) be an orthonormal basis of the tangent space at any point. Then the sum for 1≤i ≤2m+1 of the relation (4.6) for U = e_i yields

$$\eta(P(\xi,V)Z) = \frac{1}{2m}\left[\frac{r}{2m} - (2m+1)(\alpha^2-\beta^2)\right]\eta(V)\eta(Z). \tag{4.7}$$

From (4.3) and (4.7), we have

$$S(V,Z) = \left[2m(\alpha^2-\beta^2)\right]g(V,Z) - \left[\frac{r}{2m} - (2m+1)(\alpha^2-\beta^2)\right]\eta(V)\eta(Z). \tag{4.8}$$

Taking Z = ξ in (4.8) and using (2.10) we obtain

$$r = 2m(2m+1)(\alpha^2 - \beta^2). \tag{4.9}$$

Now using (4.1),(4.2),(4.8) and (4.9) in (4.5) we get

$$-{}' P(U, V, Z, Y) = 0 \tag{4.10}$$

From (4.10) it follows that

$$P(U, V)Z = 0. \tag{4.11}$$

Therefore, the trans-Sasakian manifold under consideration is Weyl projectively flat.

Hence we can state the next theorem

Theorem 4.1 *If in a trans-Sasakian manifold M of dimension 2m+1, m >0, the relation $R(X,Y) \cdot P$ holds, then the manifold is Weyl-projectively flat.*

But from theorem 3.1, a Weyl-projectively flat trans-Sasakian manifold is an Einstein manifold. Hence we can state the following theorem.

Theorem 4.2 *A trans-Sasakian manifold M of dimension 2m+1, m>0, satisfying $R(X,Y) \cdot P=0$ is an Einstein manifold and also it is a manifold of constant curvature 2m(2m+1)$(\alpha^2 - \beta^2)$.*

5. Conformally flat Trans-Sasakian manifold

The Weyl-conformal curvature tensor C is defined by

$$\begin{aligned} C(X,Y)Z &= R(X,Y)Z - \frac{1}{2m-1}\left[g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y\right] \\ &\quad + \frac{r}{2m(2m-1)}\left[g(Y,Z)X - g(X,Z)Y\right]. \end{aligned} \tag{5.1}$$

Suppose that C=0. Then form (5.1), we get

$$\begin{aligned} R(X,Y)Z = \frac{1}{2m-1}&\left[g(y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y\right] \\ &+ \frac{r}{2m(2m-1)}\left[g(Y,Z)X - g(X,Z)Y\right]. \end{aligned} \tag{5.2}$$

From (5.2) we get

$$'R(X,Y,Z,W) = \frac{1}{2m-1}[g(Y,Z)g(QX,W) - g(X,Z)g(QY,W) + S(Y,Z)g(X,W) - S(X,Z)g(Y,W)] + \frac{r}{2m(2m-1)}\left[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)\right]. \tag{5.3}$$

where $'$R(X,Y,Z,W)= g(R(X,Y,Z),W).
Putting W=ξ in (5.3), we get

$$\eta(R(X,Y)Z) = \frac{1}{2m-1}[g(Y,Z)g(QX,\xi) - g(X,Z)g(QY,\xi) + S(Y,Z)\eta(X) - S(X,Z)\eta(Y)] + \frac{r}{2m(2m-1)}\left[g(Y,Z)\eta(X) - g(X,Z)\eta(Y))\right]. \tag{5.4}$$

Again taking X = ξ in (5.4), and using (2.1),(2.2),(2.5) and (2.10) we get

$$S(Y,Z) = -\left[\frac{r}{2m} + (\alpha^2 - \beta^2)\right] g(Y,Z) + \left[\frac{r}{2m} + (2m+1)(\alpha^2 - \beta^2)\right] \eta(Y)\eta(Z). \tag{5.5}$$

Therefore the manifold is η-Einstein. Hence we can state this theorem:

Theorem 5.1 *A conformally flat trans-Sasakian manifold is* $\eta - Einstein$.

6. Trans-Sasakian manifold satisfying R(X,Y)·C=0

From (5.1), (2.2) and (2.5) we have

$$\eta(C(X,Y)Z) = (\alpha^2 - \beta^2)\left[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\right] - \frac{1}{(2m-1)}\left[g(Y,Z)\eta(QX) - g(X,Z)\eta(QY) + S(Y,Z)\eta(X) - S(X,Z)\eta(Y)\right] + \frac{r}{2m(2m-1)}\left[g(Y,Z)\eta(X) - g(X,Z)\eta(Y))\right]. \tag{6.1}$$

Putting $X = \xi$ in (6.1) we get

$$\eta(C(X,Y)\xi) = 0. \tag{6.2}$$

Again taking X=ξ in (6.1), we have

$$\eta(C(\xi,Y)Z) = \frac{1}{(2m-1)}\left[\frac{r}{2m} - (\alpha^2-\beta^2)\right][g(Y,Z) - \eta(Y)\eta(Z)] - \frac{1}{2m-1}\left[S(Y,Z) - 2m(\alpha^2-\beta^2)\eta(Y)\eta(Z)\right], \tag{6.3}$$

where (2.1),(2.2) and (2.10) are used.
Now,
(R(X,Y)C)(U,V)Z = R(X,Y)·C(U,V)Z - C(R(X,Y)U,V)Z - C(U,R(X,Y)V)Z -C(U,V)R(X,Y)Z.
Let R(X,Y)·C = 0, then we have

$$R(X,Y)\cdot C(U,V)Z - C(R(X,Y)U,V)Z - C(U,R(X,Y)V)Z - C(U,V)R(X,Y)Z = 0. \tag{6.4}$$

Therefore,

$$g\left[R(\xi,Y)\cdot C(U,V)Z,\xi\right] - g\left[C(R(\xi,Y)U,V)Z,\xi\right] - g\left[C(R(\xi,Y)V)Z,\xi\right] - g\left[C(U,V)R(\xi,Y)Z,\xi\right] = 0.$$

From this it follows that

$$-{}'C(U,V,Z,Y) + \eta(Y)\eta(C(U,V)Z) - \eta(U)\eta(C(Y,V)Z) + g(Y,U)\eta(C(\xi,V)Z) - \eta(V)\eta(C(U,Y)Z) + g(Y,V)\eta(C(U,\xi)Z) - \eta(Z)\eta(C(U,V)Y) = 0, \tag{6.5}$$

where ${}'$C(U,V,Z,Y) = g(C(U,V)Z,Y).
Putting Y = U in (6.5), we get

$$-{}'C(U,V,Z,Y) + g(U,U)\eta(C(\xi,V)Z) - \eta(V)\eta(C(U,U)Z) + g(U,V)\eta(C(U,\xi)Z) - \eta(Z)\eta(C(U,V)U) = 0. \tag{6.6}$$

Let $\{e_i\}$, i=1,2,...,(2m+1) be an orthonormal basis of the tangent space at any point. Then the sum for $1 \leq i \leq 2m+1$ of the relation (6.6) for U = e_i, yields

$$\eta(C(\xi,V)Z) = \frac{1}{2m(2m-1)}\left[(\alpha^2-\beta^2) - \frac{r}{2m}\right]\eta(V)\eta(Z). \tag{6.7}$$

From (6.3) and (6.7) we have

$$S(V,Z) = \left[\frac{r}{2m} - (\alpha^2 - \beta^2)\right] g(V,Z)$$
$$+ \left[\left((2m+1) - \frac{1}{2m}\right)(\alpha^2-\beta^2) + \frac{r}{2m}\left(\frac{1}{2m} - 1\right)\right]\eta(V)\eta(Z). \tag{6.8}$$

Taking $Z = \xi$ in (6.8) and using (2.10) we obtain

$$r = 2m(\alpha^2 - \beta^2). \tag{6.9}$$

Now using (6.1),(6.2),(6.8) and (6.9) in (6.5), we get

$$-^{'}C(U,V,Z,Y) = 0. \tag{6.10}$$

From (6.10) it follows that

$$C(U,V)Z = 0. \tag{6.11}$$

Therefore the trans-Sasakian manifold is conformally flat. Hence, we can state the following theorem

Theorem 6.1 *If in a trans-Sasakian manifold M of dimension 2m+1, $m > 0$, the relation $R(X,Y) \cdot C = 0$ holds, then the manifold is conformally flat.*

Theorem 5.1 says that a conformally flat trans-Sasakian manifold is an η-Einstein manifold. Therefore, we can state this theorem:

Theorem 6.2 *A trans-Sasakian manifold M of dimension 2m+1, $m > 0$, satisfying $R(X,Y) \cdot C = 0$ is an η-Einstein manifold and also a manifold of constant curvature $2m(\alpha^2 - \beta^2)$.*

7. Trans-Sasakian manifold satisfying R(X,Y)$\cdot\overline{C}$=0

The concircular curvature tensor $\overline{C}$ is defined as

$$\overline{C}(X,Y)Z = R(X,Y)Z - \frac{r}{2m(2m+1)}\left[g(Y,Z)X - g(X,Z)Y\right], \tag{7.1}$$

where R is the curvature tensor and r is the scalar curvature.
Hence, in view of (2.2) and (2.5), we get

$$\eta\left(\overline{C}(X,Y)Z\right) = \left[(\alpha^2-\beta^2) - \frac{r}{2m(2m+1)}\right]\left[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\right]. \tag{7.2}$$

Putting Z $=\xi$ in (7.2) we get

$$\eta\left(\overline{C}(X,Y)\xi\right) = 0. \tag{7.3}$$

Again taking X $= \xi$ in (7.2), we have

$$\eta\left(\overline{C}(\xi,Y)Z\right) = \left[(\alpha^2-\beta^2) - \frac{r}{2m(2m+1)}\right]\left[g(Y,Z) - \eta(Y)\eta(Z)\right], \tag{7.4}$$

where (2.2) and (2.10) are used.
Now,
(R(X,Y)$\overline{C}$)(U,V)Z = R(X,Y)$\overline{C}$(U,V)Z -$\overline{C}$(R(X,Y)U,V)Z
-$\overline{C}$(U,R(X,Y)V)Z -$\overline{C}$(U,V)R(X,Y)Z.
As it has been considered $R(X,Y)\cdot\overline{C}$=0, we have

$$\begin{aligned} R(X,Y)\cdot\overline{C}(U,V)Z - \overline{C}(R(X,Y)U,V)Z - \overline{C}(U,R(X,Y)V)Z \\ -\overline{C}(U,V)R(X,Y)Z = 0. \end{aligned} \tag{7.5}$$

Therefore,

$$\begin{aligned} g\left[R(\xi,Y)\cdot\overline{C}(U,V)Z,\xi\right] - g\left[\overline{C}\left(R(\xi,Y)U,V\right)Z,\xi\right] \\ -g\left[\overline{C}\left(U,R(\xi,Y)V\right)Z,\xi\right] - g\left[\overline{C}(U,V)R(\xi,Y)Z,\xi\right] = 0. \end{aligned}$$

From this it follows that

$$\begin{aligned} -{}'\overline{C}(U,V,Z,Y) + \eta(Y)\eta(\overline{C}(U,V)Z) - \eta(U)\eta(\overline{C}(Y,V)Z) \\ +g(Y,U)\eta(\overline{C}(\xi,V)Z) - \eta(V)\eta(\overline{C}(U,Y)Z) \\ +g(Y,V)\eta(\overline{C}(U,\xi)Z) - \eta(Z)\eta(\overline{C}(U,V)Y) = 0, \end{aligned} \tag{7.6}$$

where $'\overline{C}$(U,V,Z,Y)=g($\overline{C}$(U,V)Z,Y).
Putting Y = U in (7.6), we get

$$\begin{aligned} -{}'\overline{C}(U,V,Z,Y) + g(U,U)\eta(\overline{C}(\xi,V)Z) - \eta(V)\eta(\overline{C}(U,U)Z) \\ +g(U,V)\eta(\overline{C}(U,\xi)Z) - \eta(Z)\eta(\overline{C}(U,V)U) = 0. \end{aligned} \tag{7.7}$$

Let $\{e_i\}, i = 1, 2, ..., 2m+1$ be an orthonormal basis of the tangent space at any point of the manifold. Then the sum for $1 \leq i \leq 2m+1$ of the relation (7.7) for $U = e_i$, yields

$$\eta(\overline{C}(\xi, V)Z) = \frac{1}{2m}S(V, Z) - \frac{r}{2m(2m+1)}g(V, Z) + \frac{1}{2m}\left[\frac{r}{2m} - (2m+1)(\alpha^2 - \beta^2)\right]\eta(V)\eta(Z). \tag{7.8}$$

From(7.4) and (7.8), we have

$$S(V, Z) = 2m(\alpha^2 - \beta^2)g(V, Z) - \left[\frac{r}{2m(2m+1)} - (\alpha^2 - \beta^2)\right]\eta(V)\eta(Z). \tag{7.9}$$

Taking $Z = \xi$ in (7.9) and using (2.10), we have

$$r = 2m(2m+1)(\alpha^2 - \beta^2). \tag{7.10}$$

Now using (7.2),(7.4),(7.9) and (7.10) in (7.6), we get

$$-{}'\overline{C}(U, V, Z, Y) = 0. \tag{7.11}$$

From (7.11) it follows that

$$\overline{C}(U, V)Z = 0. \tag{7.12}$$

Therefore, the trans-Sasakian manifold is concircularly flat. Hence we can state the next theorem.

Theorem 7.1 *If in a trans-Sasakian manifold M of dimension 2m+1, $m > 0$, the relation $R(X, Y) \cdot \overline{C} = 0$ holds then the manifold is concircularly flat.*

As we know, in general, a concircularly flat Riemannian manifold is Einstein and so, in particular, a concircularly flat trans-Sasakian manifold is Einstein. Hence we can state

Theorem 7.2 *A trans-Sasakian manifold M of dimension 2m+1, $m > 0$, satisfying $R(X, Y)\cdot\overline{C}=0$ is an Einstein manifold and a manifold of constant curvature 2m (2m+1)$(\alpha^2 - \beta^2)$.*

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