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Multipliers and the Relative Completion in $L^p_w(G)$

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Abstract

Quek and Yap defined a relative completion $\widetilde{A}$ for a linear subspace A of $L^p(G)$, $1 \leq p < \infty$; and proved that there is an isometric isomorphism, between $\mathrm{Hom}_{L^1(G)}(L^1(G), A)$ and $\widetilde{A}$, where $\mathrm{Hom}_{L^1(G)}(L^1(G), A)$ is the space of the module homomorphisms (or multipliers) from $L^1(G)$ to A. In the present, we defined a relative completion $\widetilde{A}$ for a linear subspace A of $L^p_w(G)$,where w is a Beurling's weighted function and $L^p_w(G)$ is the weighted $L^p(G)$ space, ([14]). Also, we proved that there is an algeabric isomorphism and homeomorphism, between $\mathrm{Hom}_{L^1_w(G)}(L^1_w(G), A)$ and $\widetilde{A}$. At the end of this work we gave some applications and examples.

Key words and phrases: Module homomorphism (or multiplier), relative completion, essential module, weighted $L^p(G)$ space. 1991 AMS subject classification codes 43.

1. Introduction and Preliminarles

Throughout this work, G will denote a non-compact and non-discrete locally compact Abelian group with Haar measure $d\mu$. The left and right translation operators are denoted by $L_y f(x) = f(x-y), R_y f(x) = f(x+y)$ for all $x, y \in G$. The Fourier transform for any $f \in L^1(G)$ is denoted by $\hat{f}$. A real-valued, measurable function w on G is said to be a weight function (Beurling's weight function) if $1 \leq w(x)$ and $w(x+y) \leq w(x)w(y)$ for $x, y \in G$. We say that $w_1 \prec w_2$ if and only if there exists $c > 0$ such that $w_1(x) \leq c w_2(x)$

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for all $x \in G$. Two weight functions are called equivalent and written $w_1 \approx w_2$, if $w_1 \prec w_2$ and $w_2 \prec w_1$. For $1 \leq p < \infty$, we set

$$L_w^p(G) = \{f : \ fw \in L^p(G)\},$$

where $L^p(G)$ denotes the usual Lebesgue space. It is a Banach space under the norm

$$\|f\|_{p,w} = \|fw\|_p = \left(\int\limits_G |f(x)|^p \, w^p(x) d\mu(x) \right)^{\frac{1}{p}}.$$

If $p = 1$, then $L_w^1(G)$ becomes an algebra under convolution, called Beurling algebra [14]. A weight function w is said to satisfy the Beurling Domar condition (shortly (BD)) if one has

$$\sum_{n \geq 1} \frac{\log w(nx)}{n^2} < \infty$$

for all $x \in G$ [4]. Let $(A, \|.\|_A)$ be a Banach algebra and let $(X, \|.\|_X)$ a left Banach A-module, i. e. X is module over A in algeabric sense and satisfies $\|a.b\|_X \leq \|a\|_A \|b\|_X$ for all $a \in A$ and $b \in X$. Then the closed linear subspace of X spanned by

$$AX = \{ax : \ a \in A, \ x \in X\}$$

is called the essential part of X and is denoted by X_e. If $X = X_e$, then X is said to be an essential left Banach A-module [5].

If V and W are left(right) Banach A-modules, then a multiplier (or module homomorphism) from V to W is a bounded linear operator T from V to W, which commutes with module multiplication i. e. $T(av) = aT(v)$ for $a \in A$ and $v \in V$. We denote by $\mathrm{Hom}_A(V, W)$ or $M(V, W)$ the space of multipliers from V to W.

2. The Main Results

Definition 2.1 *Let A be a linear subspace of $L_w^p(G)$, $1 \leq p < \infty$, with the following properties:*

(i) $(A, \|.\|_A)$ is a Banach $L_w^1(G)$-module under convolution with respect to norm $\|.\|_A$, satisfying $\|.\|_{p,w} \leq \|.\|_A$.

(ii) There is a bounded approximate identity $(e_\alpha)_{\alpha\in I}$ *of* $L^1_w(G)$ *such that* $\|f * e_\alpha - f\|_A \to 0$ *for all* $f \in A$, *and* $\|e_\alpha\|_{1,w} < M$ *for a number* $M > 0$ *and all* $\alpha \in I$.

The relative completion $\widetilde{A}$ *of* A *is defined in the following way:*

$$\widetilde{A} = \left\{ f \in L^p_w(G) : f * e_\alpha \in A \text{ for all } \alpha \in I, \sup_{\alpha\in I} \|f * e_\alpha\|_A < \infty \right\}.$$

It is easy to see that $\widetilde{A}$ *is a normed space with the norm* $\|.\|_{\widetilde{A}}$ *, defined by*

$$\|f\|_{\widetilde{A}} = \sup_{\alpha\in I} \|f * e_\alpha\|_A .$$

Proposition 2.2 *Let* A *be as in Definition 2.1. The following properties are satisfied:*

(i) If $f \in \widetilde{A}$ *and* $g \in L^1_w(G)$, *then* $f * g \in \widetilde{A}$ *and*

$$\|f * g\|_{\widetilde{A}} \le \|f\|_{\widetilde{A}} \|g\|_{1,w} .$$

(ii) If $f \in A$, *then*

$$\|f\|_{\widetilde{A}} \le M \|f\|_A \quad \text{and} \quad \|f\|_A \le \|f\|_{\widetilde{A}} ,$$

i.e. the norms $\|.\|_A$ *and* $\|.\|_{\widetilde{A}}$ *are equivalent on* A.

(iii) A *is a closed subspace of* $\widetilde{A}$.

Proof. (i) Let $f \in \widetilde{A}$ and $g \in L^1_w(G)$. Then $f * e_\alpha \in A$ for all $\alpha \in I$ and $\sup_{\alpha\in I} \|f * e_\alpha\|_A < \infty$. Also, we can write $(f * g) * e_\alpha = (f * e_\alpha) * g \in A$, because A is a Banach $L^1_w(G)$-module. Hence

$$\begin{aligned} \|f * g\|_{\widetilde{A}} &= \sup_{\alpha\in I} \|(f * g) * e_\alpha\|_A \le \sup_{\alpha\in I} (\|f * e_\alpha\|_A \|g\|_{1,w}) \\ &= \|g\|_{1,w} \sup_{\alpha\in I} \|f * e_\alpha\|_A = \|f\|_{\widetilde{A}} \|g\|_{1,w} . \end{aligned}$$

(ii) Let $f \in A$. We have $f * e_\alpha \in A$ for all $\alpha \in I$, because A is a Banach $L^1_w(G)$-module. By Definition 2.1, we have

$$\|f\|_{\widetilde{A}} = \sup_{\alpha\in I} \|f * e_\alpha\|_A \le \sup_{\alpha\in I} (\|f\|_A \|e_\alpha\|_{1,w}) \le M \|f\|_A .$$

Now let any $\varepsilon > 0$ be given. Then we can choose $\alpha_0 \in I$ such that

$$\|f - f * e_\alpha\|_A < \varepsilon$$

for all $\alpha \geq \alpha_0$. Hence, we obtain

$$\|f\|_A \leq \|f - f * e_{\alpha_0}\|_A + \|f * e_{\alpha_0}\|_A < \varepsilon + \sup_{\alpha \in I} \|f * e_\alpha\|_A = \varepsilon + \|f\|_{\widetilde{A}}\,.$$

This gives the inequality $\|f\|_A \leq \|f\|_{\widetilde{A}}$.

(iii) It is easy to see that $A \subset \widetilde{A}$. We will show that A is a complete space with respect to the norm $\|.\|_{\widetilde{A}}$. Let (f_n) be a Cauchy sequence in $(A, \|.\|_{\widetilde{A}})$. Then it is a Cauchy sequence in $(A, \|.\|_A)$ by (ii). Since $(A, \|.\|_A)$ is a Banach space, there is a function $f \in A$ such that (f_n) converges to f. Also, since the norms $\|.\|_A$ and $\|.\|_{\widetilde{A}}$ are equivalent on A, this implies that (f_n) converges to f with respect to the norm $\|.\|_{\widetilde{A}}$. Hence, $(A, \|.\|_{\widetilde{A}})$ is a complete space. This shows that A is closed in $\widetilde{A}$. □

Proposition 2.3 *The definition of the space $\widetilde{A}$ does not depend on the approximate identity chosen.*

Proof. Let $(e_\alpha)_{\alpha \in I}$ be as in Definition 2.1. Suppose that $(v_\beta)_{\beta \in J}$ is an other bounded approximate identity in $L^1_w(G)$ such that $\|f * v_\beta - f\|_A \to 0$ for all $f \in A$ and $\|v_\beta\|_{1,w} < M_1$for some $M_1 > 0$. Now, we define

$$X = \left\{ f \in L^p_w(G) : f * v_\beta \in A \ \ for\ all\ \beta \in J,\ \sup_{\beta \in J} \|f * v_\beta\|_A < \infty \right\}$$

with the norm $\|.\|_X$, defined by

$$\|f\|_X = \sup_{\beta \in J} \|f * v_\beta\|_A\,.$$

Assume that $f \in \widetilde{A}$. Then we write $\|f\|_{\widetilde{A}} < \infty$ and $f * e_\alpha \in A$ for all $\alpha \in I$. Since A is a Banach $L^1_w(G)$-module, we have $f * e_\alpha * v_{\beta_0} = f * v_{\beta_0} * e_\alpha \in A$ for a fix $\beta_0 \in J$. So, we write

$$\|f * e_\alpha * v_{\beta_0} - f * v_{\beta_0}\|_{\widetilde{A}} \leq \|f\|_{\widetilde{A}} \|e_\alpha * v_{\beta_0} - v_{\beta_0}\|_{1,w}$$

by Proposition 2.2. Also, since $(e_\alpha)_{\alpha\in I}$ is an approximate identity in $L^1_w(G)$, the net $(f * e_\alpha * v_{\beta_0})_{\alpha\in I}$ converges to $f * v_{\beta_0}$. This shows that $f * v_{\beta_0} \in A$, because A is a closed subspace of $\widetilde{A}$ by Proposition 2.2. Therefore, we have $f * v_\beta \in A$ for all $\beta \in J$. By (i) and (ii) in Proposition 2.2, we have

$$\begin{aligned}
\|f * v_\beta\|_A &\leq \|f * v_\beta\|_{\widetilde{A}} \leq \|f * v_\beta - f * v_\beta * e_\alpha\|_{\widetilde{A}} + \|f * v_\beta * e_\alpha\|_{\widetilde{A}} \\
&\leq \|f\|_{\widetilde{A}} \|v_\beta - v_\beta * e_\alpha\|_{1,w} + \|f * e_\alpha\|_{\widetilde{A}} \|v_\beta\|_{1,w} \\
&\leq \|f\|_{\widetilde{A}} \|v_\beta - v_\beta * e_\alpha\|_{1,w} + MM_1 \|f * e_\alpha\|_A \\
&\leq \|f\|_{\widetilde{A}} \|v_\beta - v_\beta * e_\alpha\|_{1,w} + MM_1 \|f\|_{\widetilde{A}} .
\end{aligned}$$

Hence

$$\begin{aligned}
\|f\|_X &= \sup_{\beta\in J} \|f * v_\beta\|_A \leq \|f\|_{\widetilde{A}} \sup_{\beta\in J} \|v_\beta - v_\beta * e_\alpha\|_{1,w} + MM_1 \|f\|_{\widetilde{A}} \\
&\leq \|f\|_{\widetilde{A}} \sup_{\beta\in J} (\|v_\beta\|_{1,w} + \|v_\beta\|_{1,w} \|e_\alpha\|_{1,w}) + MM_1 \|f\|_{\widetilde{A}} \\
&\leq \|f\|_{\widetilde{A}} (M_1 + MM_1) + MM_1 \|f\|_{\widetilde{A}} = \|f\|_{\widetilde{A}} (M_1 + 2MM_1).
\end{aligned}$$

Thus, since $\|f\|_{\widetilde{A}} < \infty$, we have $\|f\|_X < \infty$. That means $f \in X$. This shows that $\widetilde{A} \subset X$.

Similary, we can show that $X \subset \widetilde{A}$. This gives $X = \widetilde{A}$. □

It is easy to prove the following Lemma 2.4.

Lemma 2.4 *$L^p_w(G)$ is an essential Banach module over $L^1_w(G)$.*

Now, we offer the statement of Corollary 8.10 in [13], used in the proof of the following lemma:

If A is a Banach algebra with approximate identity, and if W is an essential A-module which is reflexive as a Banach space, then

$$Hom_A(A, W) \cong W.$$

Lemma 2.5 *If $p > 1$, then there exists an isometric module isomorphism, between the spaces $Hom_{L^1_w(G)}(L^1_w(G), L^p_w(G))$ and $L^p_w(G)$, i. e.*

$$Hom_{L^1_w(G)}(L^1_w(G), L^p_w(G)) \cong L^p_w(G).$$

Proof. It is known that $L_w^1(G)$ is a Banach algebra with approximate identity. By Lemma 2.4, $L_w^p(G)$ is an essential Banach $L_w^1(G)$-module. If $p > 1$, it is also known that $L_w^p(G)$ is reflexive as a Banach space [10]. Then, by Corollary 8.10 in [13], we have

$$\mathrm{Hom}_{L_w^1(G)}(L_w^1(G), L_w^p(G)) \cong L_w^p(G).$$

$\square$

Theorem 2.6 *Let A be as in Definition 2.1. Then the spaces $Hom_{L_w^1(G)}(L_w^1\ (G), A)$ and $\widetilde{A}$ are algebrically isomorphic and homeomorphic (i.e. $Hom_{L_w^1(G)}(L_w^1\ (G), A) \cong \widetilde{A}$), if $p > 1$, or if $p = 1$ and $Hom_{L_w^1(G)}(L_w^1\ (G), A) \subset L_w^1\ (G)$.*

Proof. Assume that $f \in \widetilde{A}$. We now define a mapping T_f such that $T_f(g) = f * g$ for all $g \in L_w^1(G)$. By Definition of $\widetilde{A}$, we have $f * e_i - f * e_j \in A$. Hence, by (i) and (ii) of Proposition 2.2,

$$\begin{aligned} \|f * g * e_i - f * g * e_j\|_A &\leq \|f * g * e_i - f * g * e_j\|_{\widetilde{A}} \\ &\leq \|f\|_{\widetilde{A}} \|g * e_i - g + g - g * e_j\|_{1,w} \\ &\leq \|f\|_{\widetilde{A}} (\|g * e_i - g\|_{1,w} + \|g - g * e_j\|_{1,w}). \end{aligned}$$

Then, since $(e_\alpha)_{\alpha \in I}$ is an approximate identity in $L_w^1(G)$, $(f * g * e_i)_{i \in I}$ is a Cauchy net in the space A. (It is known that, for a normed space, the definitions of completeness in the mean of topological vector space and in the mean of metric space are equivalent. Hence one can use net instead of sequence in the normed space). Since A is a Banach space, there is a function $h \in A$ such that $f * g * e_i \to h$ in A. In addition, by (i) of Proposition 2.2, $f * g$ is an element of $\widetilde{A}$, where $f \in \widetilde{A}$ and $g \in L_w^1(G)$. Hence

$$\begin{aligned} \|f * g - h\|_{\widetilde{A}} &\leq \|f * g - f * g * e_\alpha\|_{\widetilde{A}} + \|f * g * e_\alpha - h\|_{\widetilde{A}} \\ &\leq \|f\|_{\widetilde{A}} \|g - g * e_\alpha\|_{1,w} + M \|f * g * e_\alpha - h\|_A . \end{aligned}$$

Since the net $(f * g * e_\alpha)_{\alpha \in I}$ converges to $h \in A$ and $(e_\alpha)_{\alpha \in I}$ is an approximate identity in $L_w^1(G)$, this shows that $T_f(g) = f * g = h \in A$. Again, by Lemma 2.2, we obtain

$$\begin{aligned} \|T_f\| &= \sup_{g \neq 0} \frac{\|T_f(g)\|_A}{\|g\|_{1,w}} = \sup_{g \neq 0} \frac{\|f * g\|_A}{\|g\|_{1,w}} \\ &\leq \sup_{g \neq 0} \frac{\|f * g\|_{\widetilde{A}}}{\|g\|_{1,w}} \leq \sup_{g \neq 0} \frac{\|f\|_{\widetilde{A}} \|g\|_{1,w}}{\|g\|_{1,w}} = \|f\|_{\widetilde{A}} \end{aligned}$$

for each $g \in L^1_w(G)$. This shows that T_f is a continuous mapping from $L^1_w(G)$ to A. Then, it is easy to verify that T_f is a continuous module homomorphism from $L^1_w(G)$ to A. By Definition 2.1, we have

$$\begin{aligned} \|T_f\| &= \sup_{g\neq 0}\frac{\|T_f(g)\|_A}{\|g\|_{1,w}} = \sup_{g\neq 0}\frac{\|f*g\|_A}{\|g\|_{1,w}} \geq \sup_{\alpha\in I}\frac{\|f*e_\alpha\|_A}{\|e_\alpha\|_{1,w}} \\ &\geq \sup_{\alpha\in I}\frac{\|f*e_\alpha\|_A}{M} = \frac{1}{M}\sup_{\alpha\in I}\|f*e_\alpha\|_A = \frac{1}{M}\|f\|_{\widetilde{A}}. \end{aligned}$$

Conversely, assume that $T \in \mathrm{Hom}_{L^1_w(G)}(L^1_w(G), A)$. By Definition 2.1, since $A \subset L^p_w(G)$ and $\|.\|_{p,w} \leq \|.\|_A$, we obtain

$$\|T(g)\|_{p,w} \leq \|T(g)\|_A \leq \|T\| \, \|g\|_{1,w},$$

for all $g \in L^1_w(G)$. Then $T \in \mathrm{Hom}_{L^1_w(G)}(L^1_w(G), L^p_w(G))$. Hence, by Lemma 2.5 for the case $p > 1$ and by our hypothesis for the case $p = 1$, there is a function $f \in L^p_w(G)$ such that $T(g) = f * g$ for all $g \in L^1_w(G)$. Also, by Definition 2.1, we have

$$\begin{aligned} \|f\|_{\widetilde{A}} &= \sup_{\alpha\in I}\|f*e_\alpha\|_A = M\sup_{\alpha\in I}\frac{\|f*e_\alpha\|_A}{M} \leq M\sup_{\alpha\in I}\frac{\|f*e_\alpha\|_A}{\|e_\alpha\|_{1,w}} \\ &\leq M\sup_{g\neq 0}\frac{\|f*g\|_A}{\|g\|_{1,w}} = M\sup_{g\neq 0}\frac{\|T(g)\|_A}{\|g\|_{1,w}} = M\|T\| < \infty. \end{aligned}$$

This gives $f \in \widetilde{A}$. Thus, every $T \in \mathrm{Hom}_{L^1_w(G)}(L^1_w(G), A)$ is of the form T_f for some $f \in \widetilde{A}$ with $\|T_f\| \leq \|f\|_{\widetilde{A}}$ and $\|f\|_{\widetilde{A}} \leq M\|T_f\|$. Hence, the mapping $f \to T_f$ is an algebraic isomorphism and homeomorphism. □

Applications

Let w be a weight function on a locally compact abelian group G and $S_w(G)$ be the subalgebra of the Beurling algebra $L^1_w(G)$ satisfying the following conditions:

1) $S_w(G)$ is dense in $L^1_w(G)$.

2) $S_w(G)$ is a Banach algebra under some norm $\|.\|_{S_w}$ and invariant under translations.

3) For each $f \in S_w(G)$, $\|L_y f\|_{S_w} \leq w(y) \|f\|_{S_w}$ for all $y \in G$.

4) Given any $f \in S_w(G)$ and $\varepsilon > 0$ there exists a neighbourhood U of the unit element $e \in G$ such that $\|L_y f - f\|_{S_w} < \varepsilon$ for all $y \in U$.

5) For all $f \in S_w(G)$, $\|f\|_{1,w} \leq \|f\|_{S_w}$.

Some properties and multipliers of the space $S_w(G)$ have been discussed in [1], [2], [3].

Proposition 2.7 $S_w(G)$ *is an essential Banach convolution ideal in* $L^1_w(G)$.

Proof. By definition of $S_w(G)$, we have $\|f\|_{1,w} \leq \|f\|_{S_w}$ for all $f \in S_w(G)$. Hence, for any compact subset $K \subset G$, we write the inequality

$$\|f.\chi_K\|_1 \leq \int_K |f(x)|\, w(x)dx \leq \|f\|_{1,w} \leq \|f\|_{S_w} . \tag{3.1}$$

Hence $S_w(G)$ is a Banach function space(shortly BF-Space). Also since $S_w(G)$ is translation invariant and the translation operator is continuous from G into $S_w(G)$, then $S_w(G)$ is an essential Banach convolution ideal in a Beurling algebra $L^1_{w_0}(G)$, where $w_0(x) = \max\{1, \|L_x\|\}$ [7]. A simple calculation shows that $\|L_x\|_{1,w} \leq w(x)$. Thus we have $w_0(x) \leq w(x)$ for all $x \in G$ and $L^1_w(G) \subset L^1_{w_0}(G)$. Now let $f \in S_w(G)$ and $g \in L^1_w(G)$. Then we write

$$\|f * g\|_{S_w} \leq \|f\|_{S_w} \|g\|_{1,w_0} \leq \|f\|_{S_w} \|g\|_{1,w} . \tag{3.2}$$

Let $(e_\alpha)_{\alpha \in I}$ be a bounded approximate identity in $L^1_w(G)$. Since $(e_\alpha)_{\alpha \in I}$ is also a bounded approximate identity in $L^1_{w_0}(G)$, using (3.2) we have

$$\lim_\alpha \|f * e_\alpha - f\|_{S_w} = 0$$

for all $f \in S_w(G)$. Thus $S_w(G)$ is an essential Banach convolution ideal in $L^1_w(G)$ by Corollary 15.3 in [5]. Since the conditions of Definition 2.1 are satisfied, we have

$$\mathrm{Hom}_{L^1_w(G)}(L^1_w(G), S_w(G)) \cong \widetilde{S_w}(G)$$

by Theorem 2.7. $\square$

Examples 1) Let $1 \leq p, q < \infty$ and let w, ω be weight functions on a locally compact abelian group G and its dual group $\hat{G}$ respectively. We set

$$A^{p,q}_{w,\omega}(G) = \left\{ f : \ f \in L^p_w(G), \ \hat{f} \in L^q_w(G) \right\}$$

and equip this space with the norm

$$\|f\|^{p,q}_{w,\omega} = \|f\|_{p,w} + \left\| \hat{f} \right\|_{q,\omega},$$

where $(\wedge)$ is the generalized Fourier transform. It has been proved in [8] that $A^{p,q}_{w,\omega}(G)$ is a Banach space. Also if w satisfies Beurling-Domar condition (BD), then $A^{p,q}_{w,\omega}(G)$ admits an approximate identity bounded in $L^1_w(G)$. Furhermore, $A^{p,q}_{w,\omega}(G)$ is an essential Banach convolution module over $L^1_w(G)$ by Proposition 1.13 (a) in [8]. Hence

$$\mathrm{Hom}_{L^1_w(G)}(L^1_w(G), A^{p,q}_{w,\omega}(G)) \cong \tilde{A}^{p,q}_{w,\omega}(G)$$

by Theorem. 2.7.

2) Let G be a locally compact Abelian group. Suppose that $A_w(G) = L^1_w(G) \cap L^p_w(G)$, $1 \leq p < \infty$. In [11], Oztop and Gürkanlı proved that $A_w(G)$ is a Banach convolution algebra with the norm $\|f\|_{A_w} = \|f\|_{1,w} + \|f\|_{p,w}$, and investigated some properties of this space. It is easy to see that $A_w(G) \subset L^p_w(G)$ and $\|.\|_{p,w} \leq \|.\|_{A_w}$. Also, $A_w(G)$ is a Banach $L^1_w(G)$-module [11]. It is known that $L^1_w(G)$ has a bounded approximate identity [14]. It is also an approximate identity for $A_w(G)$ [10]. Hence the condition of Definiton 2.1 are satisfied. Thus we have

$$\mathrm{Hom}_{L^1_w(G)}(L^1_w(G), A_w(G)) \cong \tilde{A}_w(G).$$

3) Let G be a locally compact Abelian group. In [2], Feichtinger and Gürkanlı defined the linear space

$$A^p_w(G) = \left\{ f : \ f \in L^1_w(G), \ \hat{f} \in L^p_w(G) \right\}, \ 1 \leq p < \infty.$$

by Theorem 2.7. In the mentioned paper, it is shown that $A^p_w(G)$ is a Banach convolution algebra with the norm $\|.\|^p_w$, defined by $\|f\|^p_w = \|f\|_{1,w} + \left\| \hat{f} \right\|_{p,w}$. Also, they have investigated some properties of this space. It is easy to see that $A_w^p(G) \subset L_w^1(G)$ and $\|.\|_{1,w} \leq \|.\|_w^p$. Also, $(A_w^p(G), \|.\|_w^p)$ is a Banach ideal over $L_w^1(G)$. Since w satisfies (BD), then $L_w^1(G)$ has a bounded approximate identity $(e_\alpha)_{\alpha\in I}$, whose Fourier transforms have compact support and $\|f * e_\alpha - f\|_w^p \to 0$ for all $f \in A_w^p(G)$ by Theorem 4.2 in [2]. Hence

$$\mathrm{Hom}_{L_w^1(G)}(L_w^1(G), A_w^p(G)) \cong \widetilde{A_w^p}(G)$$

by Theorem 2.7.

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