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Equi-Statistical Extension of the Korovkin Type Approximation Theorem

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Abstract

In this paper using equi-statistical convergence, which is stronger than the usual uniform convergence and statistical uniform convergence, we obtain a general Korovkin type theorem. Then, we construct examples such that our new approximation result works but its classical and statistical cases do not work.

Key Words: Equi-statistical convergence, positive linear operator, Korovkin type theorem.

1. Introduction

Throughout this paper $I := [0, \infty)$. $C(I)$ is the space of all real-valued continuous functions on I and $C_B(I) := \{f \in C(I) : f \text{ is bounded on } I\}$. The sup norm on $C_B(I)$ is given by

$$\|f\|_{C_B(I)} := \sup_{x \in I} |f(x)|, \qquad (f \in C_B(I)).$$

Also, let H_w be the space of all real valued functions f defined on I and satisfying

$$|f(x) - f(y)| \leq w\left(f; \left|\frac{x}{1+x} - \frac{y}{1+y}\right|\right), \tag{1.1}$$

where w is the modulus of continuity given by, for any $\delta > 0$,

$$w(f;\delta) := \sup_{\substack{x,y \in I \\ |x-y|<\delta}} |f(x) - f(y)|.$$

The idea of statistical convergence of a sequence of real numbers has been introduced in [14]. Recently, various kinds of statistical convergence for sequences of functions have been introduced in [1] (see also [7]). In [1] a kind of convergence (equi-statistical convergence for sequences of functions) lying between uniform and

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pointwise statistical convergence was presented. Using this concept, Korovkin type approximation theory was studied in [12]. First we recall the concept of equi-statistical convergence.

Let f and f_k belong to H_w. Then we use the following notations:

$$\begin{aligned} \Psi_n(x,\varepsilon) &:= \left|\{k \leq n : |f_k(x) - f(x)| \geq \varepsilon\}\right|, \quad x \in I \\ \Phi_n(\varepsilon) &:= \left|\left\{k \leq n : \|f_k - f\|_{C_B(I)} \geq \varepsilon\right\}\right| \end{aligned}$$

where $\varepsilon > 0$, $n \in \mathbb{N}$ and the symbol $|A|$ denotes the cardinality of the subset A.

Definition 1 *[12](f_n) is said to be statistically pointwise convergent to f on I if $st - \lim_{n\to\infty} f_n(x) = f(x)$ for each $x \in I$, i.e., for every $\varepsilon > 0$ and for each $x \in I$, $\lim_{n\to\infty} \frac{\Psi_n(x,\varepsilon)}{n} = 0$. Then, it is denoted by $f_n \to f$ $(stat)$ on I.*

Definition 2 *[12](f_n) is said to be equi-statistically convergent to f on I if for every $\varepsilon > 0$, $\lim_{n\to\infty} \frac{\Psi_n(x,\varepsilon)}{n} = 0$ uniformly with respect to $x \in I$, which means that $\lim_{n\to\infty} \frac{\|\Psi_n(.,\varepsilon)\|_{C_B(I)}}{n} = 0$ for every $\varepsilon > 0$. In this case, we denote this limit by $f_n \to f$ $(equi - stat)$ on I.*

Definition 3 *[12](f_n) is said to be statistically uniform convergent to f on I if st-$\lim_{n\to\infty} \|f_n - f\|_{C_B(I)} = 0$, or $\lim_{n\to\infty} \frac{\Phi_n(\varepsilon)}{n} = 0$. This limit is denoted by $f_n \rightrightarrows f$ $(stat)$ on I.*

Using the above definitions, we get the following result.

Lemma 1 *[12]$f_n \rightrightarrows f$ on I (in the ordinary sense) implies $f_n \rightrightarrows f$ $(stat)$ on I, which also implies $f_n \to f$ $(equi - stat)$ on I. Furthermore, $f_n \to f$ $(equi - stat)$ on I implies $f_n \to f$ $(stat)$ on I; and $f_n \to f$ on I (in the ordinary sense) implies $f_n \to f$ $(stat)$ on I.*

However, one can construct an example which guarantees that the converses of Lemma 1 are not always true. Such an example is in the following (see also [1]) example.

Example 1 *Define $g_n \in H_w, n \in \mathbb{N}$ by the formula*

$$g_n(x) := \begin{cases} 0, & x = \frac{1}{n} \\ 1, & x \neq \frac{1}{n} \end{cases}. \tag{1.2}$$

Then observe that $g_n \to g = 1 (equi - stat)$ on I, but (g_n) does not usual uniform convergent and statistically uniform convergent to the function $g = 1$ on I.

Now let $\{L_n\}$ be a sequence of positive linear operators acting from $C(X)$ into $C(X)$, which is the space of all continuous real valued functions on a compact subset X of the real numbers. In this case, Korovkin [13] first noticed the necessary and sufficient conditions for the uniform convergence of $L_n(f)$ to a function f by using the test function e_i defined by $e_i(x) = x^i$ $(i = 0, 1, 2)$. Many researchers have investigated these conditions for various operators defined on different spaces. In recent years, some matrix summability methods have been used in the approximation theory. Although some operators, such as interpolation operators of Hermite-Fejer [3], do not converge at points of simple discontinuity, the matrix summability method of Cesàro-type are strong enough to correct the lack of convergence [4]. Furthermore, uniform statistical convergence in Definition 3, which is a regular (non-matrix) summability transformation, has also been used in the Korovkin type approximation theory [6], [8], [9], [10], [11]. Recently, a Korovkin type approximation theorem has been studied in [12] via equi-statistical convergence which is stronger than the statistical uniform convergence. In this paper, using the concept of equi-statistical convergence we study a Korovkin type approximation theorem for positive linear operators which defined on $H_w(I^n)$. Also, we will construct sequences of positive linear operators such that while our new results work, their classical and statistical cases do not work.

2. Equi-Statistical Convergence of Positive Linear Operators

Using usual uniform convergence, Çakar and Gadjiev [5] obtained Korovkin type approximation theorem on the space H_w:

Theorem 1 *[5]Let $\{L_n\}$ be a sequence of positive linear operators from H_w into $C_B(I)$. Then, for any $f \in H_w$,*

$$L_n f \rightrightarrows f \quad (\textit{in the ordinary sense})$$

is satisfied if the following holds:

$$L_n f_i \rightrightarrows f_i \quad (\textit{in the ordinary sense}), \qquad (i = 0, 1, 2),$$

where

$$f_0(u) = 1, \;\; f_1(u) = \frac{u}{1+u}, \quad f_2(u) = \left(\frac{u}{1+u}\right)^2.$$

Now we have the following result.

Theorem 2 *Let $\{L_n\}$ be a sequence of positive linear operators from H_w into $C_B(I)$. Then, for any $f \in H_w$,*

$$L_n f \to f \;\; (equi-stat) \tag{2.1}$$

is satisfied if the following holds:

$$L_n f_i \to f_i \;\; (equi-stat), \qquad (i = 0, 1, 2), \tag{2.2}$$

where

$$f_0(u) = 1, \;\; f_1(u) = \frac{u}{1+u}, \quad f_2(u) = \left(\frac{u}{1+u}\right)^2$$

Proof. Let $f \in H_w$ and $x \in I$ be fixed. Then, we immediately see from [5], [8] that, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that

$$|L_n(f;x) - f(x)| \leq \varepsilon + K\{|L_n(f_0;x) - f_0(x)| + |L_n(f_1;x) - f_1(x)| + |L_n(f_2;x) - f_2(x)|\}, \tag{2.3}$$

where $K := \varepsilon + \|f\|_{C_B(I)} + \frac{4\|f\|_{C_B(I)}}{\delta^2}$. For a given $r > 0$, choose $\varepsilon > 0$ such that $\varepsilon < r$. Then, for each $i = 0, 1, 2$, setting

$$\Psi_n(x, r) := |\{k \leq n : |L_k(f; x) - f(x)| \geq r\}|$$

and

$$\Psi_{i,n}(x, r) := \left|\left\{k \leq n : |L_k(f_i; x) - f_i(x)| \geq \frac{r - \varepsilon}{3K}\right\}\right| \qquad (i = 0, 1, 2),$$

it follows from (2.3) that

$$\Psi_n(x, r) \leq \sum_{i=0}^{2} \Psi_{i,n}(x, r),$$

which gives

$$\frac{\|\Psi_n(., r)\|_{C_B(I)}}{n} \leq \sum_{i=0}^{2} \frac{\|\Psi_{i,n}(., r)\|_{C_B(I)}}{n}. \tag{2.4}$$

Then using the hypothesis (2.2) and considering Definition 2, the right-hand side of (2.4) tends to zero as $n \to \infty$. Therefore, we have

$$\lim_{n\to\infty} \frac{\|\Psi_n(., r)\|_{C_B(I)}}{n} = 0 \quad \text{for every } r > 0,$$

whence the result. □

Now we give an example such that Theorem 2 works but the cases of classical and statistical do not work.

Remark 1 Suppose that $I = [0, \infty)$. We consider the following positive linear operators defined on H_w:

$$T_n(f; x) = \frac{g_n(x)}{(1+x)^n} \sum_{k=0}^{n} f\left(\frac{k}{n-k+1}\right) \binom{n}{k} x^k,$$

where $f \in H_w$, $x \in I$, $n \in \mathbb{N}$ and $g_n(x)$ is given by (1.2). If $g_n(x) = 1$ then T_n turn out to be the operators of Bleimann, Butzer and Hahn [2]. If we use the definition of T_n and the fact that

$$\binom{n}{k+1} = \frac{n}{k+1}\binom{n-1}{k} \quad , \quad \binom{n}{k+2} = \frac{n(n-1)}{(k+1)(k+2)}\binom{n-2}{k},$$

we can see that

$$\begin{aligned}
T_n(f_0; x) &= g_n(x), \\
T_n(f_1; x) &= \frac{n}{n+1} g_n(x) \left(\frac{x}{1+x}\right), \\
T_n(f_2; x) &= g_n(x) \frac{x^2}{(1+x)^2} \left(\frac{n(n-1)}{(n+1)^2}\right) + g_n(x) \frac{x}{1+x} \frac{n}{(n+1)^2}.
\end{aligned}$$

We show that conditions (2.2) in the Theorem 2 hold.

1. Since $g_n \to 1(equi - stat)$ on I, it is clear that $T_n f_0 \to f_0(equi - stat)$ on I.
2. Since $|T_n(f_1;x) - f_1(x)| = f_1(x)\left|\frac{n}{n+1}g_n(x) - 1\right|$, we can write

$$|T_n(f_1;x) - f_1(x)| < \left|\frac{n}{n+1}g_n(x) - 1\right|.$$

Also, we know that $\lim_{n\to\infty}\frac{n}{n+1} = 1$ and $g_n \to 1(equi - stat)$ on I. Then we have $\frac{n}{n+1}g_n(x) \to 1(equi - stat)$ on I. So we get

$$T_n f_1 \to f_1(equi - stat) \text{ on } I.$$

3. Finally, $T_n(f_2;x) - f_2(x) = f_2(x)\left[\frac{n(n-1)g_n(x)}{(n+1)^2} - 1\right] + f_1(x)\frac{ng_n(x)}{(n+1)^2}$ then

$$|T_n(f_2;x) - f_2(x)| < \left|\frac{n(n-1)g_n(x)}{(n+1)^2} - 1\right| + \left|\frac{ng_n(x)}{(n+1)^2}\right|.$$

So we observe that

$$\left|\frac{n(n-1)g_n(x)}{(n+1)^2} - 1\right| \to 0(equi - stat) \text{ on } I \quad \text{and} \quad \left|\frac{ng_n(x)}{(n+1)^2}\right| \to 0(equi - stat) \text{ on } I. \tag{2.5}$$

Now given $\varepsilon > 0$, set

$$\Psi_n(x,\varepsilon) := |\{k \le n : |T_k f_2 - f_2| \ge \varepsilon\}|$$

and

$$\Psi_{1,n}(x,\varepsilon) := \left|\left\{k \le n : \left|\frac{n(n-1)g_n(x)}{(n+1)^2} - 1\right| \ge \frac{\varepsilon}{2}\right\}\right|,$$

$$\Psi_{2,n}(x,\varepsilon) := \left|\left\{k \le n : \left|\frac{ng_n(x)}{(n+1)^2}\right| \ge \frac{\varepsilon}{2}\right\}\right|.$$

By (2.5), it is obvious that $\Psi_n(x,\varepsilon) \le \Psi_{1,n}(x,\varepsilon) + \Psi_{2,n}(x,\varepsilon)$. Then, we get

$$\lim_{n\to\infty}\frac{\|\Psi_n(.,\varepsilon)\|_{C_B(I)}}{n} = 0$$

for every $\varepsilon > 0$. So, we get

$$T_n f_3 \to f_3(equi - stat) \text{ on } I.$$

Therefore, using (1), (2) and (3) in Theorem 2, we obtain that, for all $f \in H_w$,

$$T_n f \to f(equi - stat).$$

Since g_n is neither uniform nor statistically uniform convergent to $g = 1$ on $I = [0,\infty)$, the sequence $\{T_n f\}$ cannot uniformly converge to f on I in the ordinary sense or statistically sense.

3. Equi-Statistical Extension of the Korovkin Type Approximation Theorem

In this section, considering a sequence of positive linear operators defined on the space of all real valued continuous and bounded functions on a subset I^n of $\mathbb{R}^n$, the real n-dimensional space where $I^n := I \times I \times ... \times I$, we give an extension of Theorem 2.

We first consider the case of $m = 2$.

Let $I^2 := [0,\infty) \times [0,\infty)$. Then, the sup norm on $C_B\left(I^2\right)$ is given by,

$$\|f\|_{C_B(I^2)} := \sup_{(x,y)\in I^2} |f(x,y)|, \qquad \left(f \in C_B\left(I^2\right)\right).$$

Also, let H_{w_2} is the space of all real valued functions f defined on I^2 and satisfying

$$|f(u,v) - f(x,y)| \leq w_2\left(f; \left|\frac{u}{1+u} - \frac{x}{1+x}\right|, \left|\frac{v}{1+v} - \frac{y}{1+y}\right|\right) \tag{3.1}$$

where $w_2(f;\delta_1,\delta_2)$ is the modulus of continuity (for the functions of two variables) given by, for any $\delta_1, \delta_2 > 0$,

$$w_2(f;\delta_1,\delta_2) := \sup\left\{|f(u,v) - f(x,y)| : (u,v),(x,y) \in I^2, \text{ and } |u-x| \leq \delta_1, |v-y| \leq \delta_2\right\}.$$

It is clear that a necessary and sufficient condition for a function $f \in C_B\left(I^2\right)$ is

$$\lim_{\delta_1\to 0,\delta_2\to 0} w_2(f;\delta_1,\delta_2) = 0.$$

Now let f and f_n belong to H_{w_2}. Then we use the following notations:

$$\Psi_n(x,y,\varepsilon) \;:\; = |\{k \leq n : |f_k(x,y) - f(x,y)| \geq \varepsilon\}|, \qquad (x,y) \in I^2$$

$$\Phi_n(\varepsilon) \;:\; = \left|\left\{k \leq n : \|f_k - f\|_{C_B(I^2)} \geq \varepsilon\right\}\right|$$

where $\varepsilon > 0$ and $n \in \mathbb{N}$.

Definition 4 (f_n) *is said to be statistically pointwise convergent to* f *on* I *if* $st - \lim_{n\to\infty} f_n(x,y) = f(x,y)$ *for each* $(x,y) \in I^2$, *i.e., for every* $\varepsilon > 0$ *and for each* $(x,y) \in I^2$, $\lim_{n\to\infty} \frac{\Psi_n(x,y,\varepsilon)}{n} = 0$. *Then, it is denoted by* $f_n \to f$ $(stat)$ *on* I^2.

Definition 5 (f_n) *is said to be equi-statistically convergent to* f *on* I^2 *if for every* $\varepsilon > 0$, $\lim_{n\to\infty} \frac{\Psi_n(x,y,\varepsilon)}{n} = 0$ *uniformly with respect to* $(x,y) \in I^2$, *which means that* $\lim_{n\to\infty} \frac{\|\Psi_n(.,.,\varepsilon)\|_{C_B(I^2)}}{n} = 0$ *for every* $\varepsilon > 0$. *In this case, we denote this limit by* $f_n \to f$ $(equi - stat)$ *on* I^2.

Definition 6 (f_n) *is said to be statistically uniform convergent to* f *on* I^2 *if* st-$\lim_{n\to\infty} \|f_n - f\|_{C_B(I^2)} = 0$, *or* $\lim_{n\to\infty} \frac{\Phi_n(\varepsilon)}{n} = 0$. *This limit is denoted by* $f_n \rightrightarrows f$ $(stat)$ *on* I^2.

Lemma 2 *$f_n \rightrightarrows f$ on I^2 (in the ordinary sense) implies $f_n \rightrightarrows f$ (stat) on I^2, which also implies $f_n \to f$ (equi − stat) on I^2. Furthermore, $f_n \to f$ (equi − stat) on I implies $f_n \to f$ (stat) on I^2; and $f_n \to f$ on I^2 (in the ordinary sense) implies $f_n \to f$ (stat) on I^2.*

However, one can construct an example which guarantees that the converses of Lemma 2 are not always true. Such an example is in the following:

Example 2 *Define $g_n, n \in \mathbb{N}$ by the formula*

$$g_n(x,y) := \begin{cases} 0, & (x,y) = \left(\frac{1}{n}, \frac{1}{n}\right) \\ 1, & (x,y) \neq \left(\frac{1}{n}, \frac{1}{n}\right) \end{cases} . \tag{3.2}$$

Since $g_n : [0,\infty) \times [0,\infty) \to \mathbb{R}$ is continuous and

$$|g_n(u,v) - g_n(x,y)| = \begin{cases} 0, & (x,y) = (u,v) = \left(\frac{1}{n}, \frac{1}{n}\right) \\ 0, & (x,y) \neq (u,v) \neq \left(\frac{1}{n}, \frac{1}{n}\right) \\ 1, & (x,y) = \left(\frac{1}{n}, \frac{1}{n}\right), (u,v) \neq \left(\frac{1}{n}, \frac{1}{n}\right) \\ 1, & (x,y) \neq \left(\frac{1}{n}, \frac{1}{n}\right), (u,v) = \left(\frac{1}{n}, \frac{1}{n}\right) \end{cases}$$

for all $(x,y), (u,v) \in [0,\infty) \times [0,\infty)$. Then we have

$$|g_n(u,v) - g_n(x,y)| \le w_2\left(g_n; \left|\frac{u}{1+u} - \frac{x}{1+x}\right|, \left|\frac{v}{1+v} - \frac{y}{1+y}\right|\right).$$

So $g_n \in H_{w_2}$. Then observe that $g_n \to g = 1$(equi − stat) on I^2, but (g_n) does not usual uniform convergent and statistically uniform convergent to the function $g = 1$ on I^2.

Let L is a positive linear operator mapping H_{w_2} into $C_B(I^2)$. Also, we denote the value of Lf at a point $(x,y) \in I^2$ is denoted by $L(f(u,v); x,y)$ or simply $L(f;x,y)$.

Now we have the following result.

Theorem 3 *Let $\{L_n\}$ be a sequence of positive linear operators from H_{w_2} into $C_B(I^2)$. Then, for any $f \in H_{w_2}$,*

$$L_n f \to f \text{ (equi − stat)} \tag{3.3}$$

is satisfied if the following holds:

$$L_n f_i \to f_i \text{ (equi − stat)}, \qquad (i = 0,1,2,3), \tag{3.4}$$

where

$$f_0(u,v) = 1,\ f_1(u,v) = \frac{u}{1+u},\ f_2(u,v) = \frac{v}{1+v},\ f_3(u,v) = \left(\frac{u}{1+u}\right)^2 + \left(\frac{v}{1+v}\right)^2.$$

Proof. Using the similar technique in proof of Theorem 2, we can obtain the proof. □

Now we give an example such that Theorem 3 works but the case of classical and statistical (Theorem 2.1 of [8]) do not work as Remark 1.

Remark 2 Suppose that $I = [0,\infty)$ and $I^2 = [0,\infty) \times [0,\infty)$. We consider the following positive linear operators defined on H_{w_2}:

$$T_n(f;x,y) = \frac{g_n(x,y)}{(1+x)^n(1+y)^n} \sum_{k=0}^{n}\sum_{l=0}^{n} f\left(\frac{k}{n-k+1}, \frac{l}{n-l+1}\right)\binom{n}{k}\binom{n}{l} x^k y^l,$$

where $f \in H_{w_2}$, $(x,y) \in I^2$, $n \in \mathbb{N}$ and $g_n(x,y)$ is given by (3.2). If $g_n(x,y) = 1$ than T_n turn out to be the operators of Bleimann, Butzer and Hahn [2] (of two variables). From [8], we can see that

$$\begin{aligned}
T_n(f_0;x,y) &= g_n(x,y), \\
T_n(f_1;x,y) &= \frac{ng_n(x,y)}{n+1}\left(\frac{x}{1+x}\right), \\
T_n(f_2;x,y) &= \frac{ng_n(x,y)}{n+1}\left(\frac{y}{1+y}\right), \\
T_n(f_3;x,y) &= \frac{n(n-1)g_n(x,y)}{(n+1)^2}\frac{x^2}{(1+x)^2} + \frac{ng_n(x,y)}{(n+1)^2}\frac{x}{1+x} \\
&\quad + \frac{n(n-1)g_n(x,y)}{(n+1)^2}\frac{y^2}{(1+y)^2} + \frac{ng_n(x,y)}{(n+1)^2}\frac{y}{1+y}.
\end{aligned}$$

Then, as in the previous section, it is easy to check that the conditions in (3.4) hold. So, by Theorem 3, we obtain that, for all $f \in H_{w_2}$

$$T_n f \to f(equi-stat) \text{ on } I^2.$$

Since the function sequence $g_n(x,y)$ is not usual uniform convergent and statistically uniform convergent to the function $g = 1$ on I^2, $\{T_n f\}$ is not usual uniform convergent and statistically uniform convergent to f.

Now replace I^2 by $I^n := [0,\infty) \times ... \times [0,\infty)$ and consider the modulus of continuity $w_n(f;\delta_1,...,\delta_n)$ (for the function f of $n-$variables) given by, for any $\delta_1,...,\delta_n > 0$,

$$\begin{aligned}
& w_n(f;\delta_1,...,\delta_n) := \sup\{|f(u_1,...,u_n) - f(x_1,...,x_n)| : (u_1,...,u_n), (x_1,...,x_n) \in I^n \\
& \text{and } |u_i - x_i| \leq \delta_i,\ (i = 0,1,...,n)\}.
\end{aligned}$$

Then, let H_{w_n} is the space of all real valued functions f defined on I^n and satisfying

$$|f(u_1,...,u_n) - f(x_1,...,x_n)| \leq w_n\left(f; \left|\frac{u_1}{1+u_1} - \frac{x_1}{1+x_1}\right|, ..., \left|\frac{u_n}{1+u_n} - \frac{x_n}{1+x_n}\right|\right).$$

Therefore, using the similar technique in proof of Theorem 3 and definition of equi-statistically convergence on H_{w_n}, we can get the following result immediately.

Theorem 4 *Let $\{L_n\}$ be a sequence of positive linear operators from H_{w_n} into $C_B(I^n)$. Then, for any $f \in H_{w_n}$,*

$$L_n f \to f \ (equi-stat)$$

is satisfied if the following holds:

$$L_n f_i \to f_i \ (equi - stat), \qquad (i = 0, 1, ..., n+1),$$

where

$$\begin{aligned} f_0(u_1, ..., u_n) &= 1, \ \ f_i(u_1, ..., u_n) = \frac{u_i}{1+u_i}, \quad (i = 1, 2, ..., n) \\ f_{n+1}(u_1, ..., u_n) &= \sum_{k=1}^{n} \left(\frac{u_k}{1+u_k} \right)^2. \end{aligned}$$

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