Turk J Math
33 (2009) , 273 – 281.

doi:10.3906/mat-0803-16

Approximation by certain linear operators preserving $\mathbf{x^2}$

Lucyna Rempulska, Karolina Tomczak

Abstract

We investigate certain positive linear operators L_n preserving the functions $e_k(x) = x^k$, $k = 0, 1$, and modified operators L_n^* which preserve e_0 and e_2. We show that the error of approximation of f by $L_n^*(f)$ is smaller than for $L_n(f)$.

Key Words: Positive linear operator, polynomial weighted space, degree of approximation.

1. Introduction

1.1. Let as usual $N = \{1, 2, \ldots\}$, $N_0 = N \cup \{0\}$, and let I be the interval $[0, \infty)$ or $(0, \infty)$.

Similar to [5], let $p \in \mathbb{N}_0$,

$$w_0(x) := 1, \quad w_p(x) := (1 + x^p)^{-1} \quad \text{if} \quad p \geq 1, \tag{1.1}$$

for $x \in I$, and let $B_p \equiv B_p(I)$ be the set of all functions $f : I \to \mathbb{R}$ for which fw_p is bounded on I and the norm is given by the formula

$$\|f\|_p \equiv \|f(\cdot)\|_p := \sup_{x \in I} w_p(x)|f(x)|. \tag{1.2}$$

Moreover, let $C_p \equiv C_p(I)$ with $p \in \mathbb{N}_0$ be the set of all $f \in B_p$ for which fw_p is a uniformly continuous function on I. The norm in C_p is defined by (2).

The spaces B_p and C_p are called polynomial weighted spaces.

We see that if $p, q \in \mathbb{N}_0$ and $p < q$, then $B_p \subset B_q$, $C_p \subset C_q$ and $\|f\|_q \leq \|f\|_p$ for every $f \in B_p$.

1.2. It is known ([1–7, 14, 15]) that several classical positive linear operators, e.g. the Szász-Mirakyan, Baskakov, Post-Widder and Stancu operators, act from the space C_p to C_p for every $p \in \mathbb{N}_0$ and preserve the functions $e_0(x) = 1$ and $e_1(x) = x$ but does not preserve $e_2(x) = x^2$.

Recently O. Duman and M. A. Özarslan in papers [8, 9] have introduced modified Szász-Mirakyan operators S_n^* which preserve the functions e_0 and e_2. They have shown that the error of approximation of f, with the certain function space, by $S_n^*(f)$ is smaller than for the classical Szász-Mirakyan operators $S_n(f)$.

AMS Mathematics Subject Classification: 41A25, 41A36.

The similar problems were considered for the Bernstein polynomials and the MKZ type operators in [10] and [11].

1.3. The purpose of this paper is to extend the Duman-Özarslan idea ([8]) to certain sequences of linear positive operators L_n acting from the space C_p to B_p. In Section 2 we shall give definition of operators and their basic properties. The main theorems will be given in Section 3.

2. Definition and lemmas

2.1. Let $x \in I$ be a fixed point and let

$$\varphi_x(t) := |t - x| \quad \text{for} \quad t \in I. \tag{2.3}$$

We consider a sequence $(L_n)_{n_0}^{\infty}$, $n_0 \in \mathbb{N}$, of positive linear operators satisfying the following conditions:

(i) $L_n : C_p(I) \to B_p(I)$ for every $p \in \mathbb{N}_0$, $n \geq n_0$, and $L_n(f;0) = f(0)$ for every $f \in C_p([0,\infty))$;

(ii) For every $e_k(x) = x^k$, $k \in \mathbb{N}_0$, and $n \geq n_0$ there exists an algebraic polynomial $P_{n,k}$ of the order k such that $L_n(e_k;x) = P_{n,k}(x)$ and

$$L_n(e_0;x) = 1, \quad L_n(e_1;x) = x \quad \text{for} \quad x \in I,\, n \geq n_0; \tag{2.4}$$

(iii) There exist numbers $a, b \geq 0$, $a^2 + b^2 > 0$, and a numerical increasing and unbounded sequence $(\lambda_n)_{n_0}^{\infty}$ such that $\lambda_{n_0} > 0$ and

$$L_n(e_2;x) = x^2 + \frac{ax^2 + bx}{\lambda_n} \quad \text{for} \quad x \in I,\, n \geq n_0; \tag{2.5}$$

(iv) For functions

$$T_{n;p}(x) := L_n\left(\varphi_x^p(t);x\right), \qquad x \in I,\, n \geq n_0,\, p \in \mathbb{N}_0. \tag{2.6}$$

there holds

$$\lim_{n\to\infty} \|T_{n;p}\|_p = 0. \tag{2.7}$$

Now, using L_n we define for $f \in C_p$, $p \in \mathbb{N}_0$, the following operators:

$$L_n^*(f;x) := L_n\left(f; u_n(x)\right), \qquad x \in I,\, n \geq n_0, \tag{2.8}$$

with

$$u_n(x) := \frac{-b + \sqrt{b^2 + 4\lambda_n(a+\lambda_n)x^2}}{2(a+\lambda_n)}. \tag{2.9}$$

From (8), (9) and properties (i)–(iv) of L_n we deduce that L_n^*, $n \geq n_0$, is a positive linear operator well defined on every space C_p, $p \in \mathbb{N}_0$.

From (8), (9) and (3)–(5) we deduce that

$$L_n^*(e_0;x)=1, \qquad L_n^*(e_1;x)=u_n(x), \qquad L_n^*(e_2;x)=x^2, \tag{2.10}$$

and

$$L_n\left(\varphi_x^2(t);x\right)=\frac{ax^2+bx}{\lambda_n}, \quad L_n^*(\varphi_x^2(t);x)=2x(x-u_n(x)), \tag{2.11}$$

for every $x\in I$ and $n\ge n_0$.

In this paper we shall denote by $M_k(p)$, $k\in\mathbb{N}$, suitable positive constants depending only on indicated parameter p.

2.2. Here we shall give some auxiliary results.

Lemma 1 *Let u_n and w_p be functions defined by (9) and (1). Then we have*

$$0\le u_n(x)\le x, \qquad 0\le 2x\left(x-u_n(x)\right)\le \left(ax^2+bx\right)/\lambda_n, \tag{2.12}$$

$$\sqrt{\frac{ax^2+bx}{\lambda_n}}-\sqrt{2x(x-u_n(x))}\ge\sqrt{\frac{ax^2+bx}{\lambda_n}}\frac{2ax+b}{4[2ax+b+2\lambda_n x]}, \tag{2.13}$$

$$0<\frac{w_p(x)}{w_p(u_n(x))}\le 1, \tag{2.14}$$

$$\left(w_p(x)\right)^2\le w_{2p}(x), \qquad \left(w_p(x)\right)^{-2}\le 2/w_{2p}(x), \tag{2.15}$$

for all $x\in I$, $n\ge n_0$ and $p\in\mathbb{N}_0$.

Proof. From (9) we deduce that $u_n(0)=0$ for $n\ge n_0$,

$$\begin{aligned}
0<u_n(x) &= \frac{2\lambda_n x^2}{b+\sqrt{b^2+4\lambda_n(a+\lambda_n)x^2}}<\frac{2\lambda_n x^2}{\sqrt{4\lambda_n^2x^2}}=x,\\
0<x-u_n(x) &= \frac{1}{2(a+\lambda_n)}\;\frac{[2(a+\lambda_n)x+b]^2-b^2-4\lambda_n(a+\lambda_n)x^2}{2(a+\lambda_n)x+b+\sqrt{b^2+4\lambda_n(a+\lambda_n)x^2}}\\
&= \frac{2(ax^2+bx)}{2(a+\lambda_n)x+b+\sqrt{b^2+4\lambda_n(a+\lambda_n)x^2}}<\frac{ax+b}{2\lambda_n},
\end{aligned}$$

and

$$\sqrt{(ax^2+bx)/\lambda_n}-\sqrt{2x(x-u_n(x))}=\frac{(ax^2+bx)/\lambda_n-2x(x-u_n(x))}{\sqrt{(ax^2+bx)/\lambda_n}+\sqrt{2x(x-u_n(x))}}$$

$$\ge\frac{1}{2\sqrt{(ax^2+bx)/\lambda_n}}\left(\frac{ax^2+bx}{\lambda_n}-\frac{4x(ax^2+bx)}{2(a+\lambda_n)x+b+\sqrt{b^2+4\lambda_n(a+\lambda_n)x^2}}\right)$$

$$=\sqrt{\frac{ax^2+bx}{\lambda_n}}\frac{2ax+b+\sqrt{b^2+4\lambda_n(a+\lambda_n)x^2}-2\lambda_n x}{2\left(2ax+b+2\lambda_n x+\sqrt{b^2+4\lambda_n(a+\lambda_n)x^2}\right)}$$

$$>\sqrt{\frac{ax^2+bx}{\lambda_n}}\frac{2ax+b}{4(2ax+b+2\lambda_n x)},$$

for every $x > 0$ and $n \geq n_0$.

Inequalities (14) and (15) are obvious for each $p \in \mathbb{N}_0$ by (1) and (12). □

Lemma 2 *For each $p \in \mathbb{N}_0$, there exists $M_1(p) = \text{const.} > 0$ such that for the above operators L_n and L_n^* there holds*

$$\|L_n^*(1/w_p)\|_p \leq \|L_n(1/w_p)\|_p \leq M_1(p), \qquad n \geq n_0, \tag{2.16}$$

and

$$\|L_n^*(f)\|_p \leq \|L_n(f)\|_p \leq M_1(p)\|f\|_p, \tag{2.17}$$

for every $f \in C_p$ and $n \geq n_0$.

Formulas (8) and (9) and inequality (17) show that L_n^*, $n \geq n_0$, is linear positive operator acting from the space C_p to B_p for every $p \in \mathbb{N}_0$.

Proof. The inequalities (16) and (17) with $M_1(p) = 1$ are obvious for $p = 0$ by (1), (2), (4) and (12).

If $p \in \mathbb{N}$, then by (1)–(3) and properties of L_n, we have

$$L_n(1/w_p(t);x) = 1 + L_n(e_p;x) \leq 1 + 2^p x^p + 2^p L_n\left(\varphi_x^p(t);x\right) \text{ for } x \in I, n \geq n_0. \tag{2.18}$$

Next from (7) it results that there exists $M_2(p) = \text{const.} > 0$ such that $\|T_{n;p}\|_p \leq M_2(p)$ for $n \geq n_0$, which by (1), (2), (6) and (18) implies that

$$\|L_n(1/w_p)\|_p \leq 2^p\left(1 + M_2(p)\right) \text{ for } n \geq n_0.$$

Thus the inequality (16) for L_n is proved.

From (8), (14) and (16) for L_n we deduce that

$$\begin{aligned} w_p(x)L_n^*\left(1/w_p(t);x\right) &= \frac{w_p(x)}{w_p\left(u_n(x)\right)} w_p\left(u_n(x)\right) L_n\left(1/w_p(t);u_n(x)\right) \\ &\leq \|L_n(1/w_p)\|_p \leq M_1(p) \text{ for } x \in I, n \geq n_0. \end{aligned}$$

Analogously for $f \in C_p$, $p \in \mathbb{N}_0$, we get

$$w_p(x)\left|L_n^*\left(f(t);x\right)\right| \leq \|L_n(f)\|_p \quad \text{for} \quad x \in I, n \geq n_0. \tag{2.19}$$

Moreover, for $f \in C_p$, $p \in \mathbb{N}_0$, we have

$$|L_n(f;x)| \leq L_n\left(|f|;x\right), \; x \in I, \; n \geq n_0,$$

which by (2) implies that

$$\|L_n(f)\|_p \leq \|f\|_p \, \|L_n(1/w_p)\|_p, \; n \geq n_0. \tag{2.20}$$

Now from (19), (20) and (16) immediately follows (17). □

For L_n and L_n^* there holds the following:

Lemma 3 *Let* $f, g \in C_p$ *with a fixed* $p \in \mathbb{N}_0$. *Then*

$$|L_n(f(t)g(t);x)| \le \left(L_n(f^2(t);x)\right)^{1/2} \left(L_n(g^2(t);x)\right)^{1/2} \quad \text{for } x \in I, n \ge n_0,$$

and

$$\|L_n(fg)\|_{2p} \le \|L_n(f^2)\|_{2p}^{1/2} \, \|L_n(g^2)\|_{2p}^{1/2} \quad \text{for } n \ge n_0.$$

Identical inequalities there hold for operators L_n^*. □

Applying (4) and (10)–(16), we can derive the following

Corollary 1 *Let* φ_x *and* w_p *be defined by (3) and (1). Then*

$$\begin{aligned} L_n\left(\varphi_x(t);x\right) \le \left(L_n(\varphi_x^2(t);x)\right)^{1/2} &= \left(\left(ax^2+bx\right)/\lambda_n\right)^{1/2}, \\ L_n^*\left(\varphi_x(t);x\right) \le \left(L_n^*(\varphi_x^2(t);x)\right)^{1/2} &= (2x(x-u_n(x))^{1/2} \le \left(\left(ax^2+bx\right)/\lambda_n\right)^{1/2}, \end{aligned}$$

and

$$\begin{aligned} w_p(x)L_n(\varphi_x(t)/w_p(t);x) &\le \sqrt{2}\|L_n\left(1/w_{2p}\right)\|_{2p}^{1/2}\left(L_n(\varphi_x^2(t);x)\right)^{1/2}, \\ w_p(x)L_n^*(\varphi_x(t)/w_p(t);x) &\le \sqrt{2}\|L_n\left(1/w_{2p}\right)\|_{2p}^{1/2}\left(L_n^*(\varphi_x^2(t);x)\right)^{1/2}, \end{aligned}$$

for every $x \in I$ *and* $n \ge n_0$.

3. Theorems

Here we shall prove two approximation theorems for the above operators. Let C_p^1, $p \in \mathbb{N}_0$, be the set of all functions $f \in C_p$ having first derivative $f' \in C_p$.

Theorem 1 *For each* $p \in \mathbb{N}_0$, *there exists* $M_3(p) = \text{const.} > 0$ *such that for every* $f \in C_p^1$ *there holds*

$$w_p(x)\,|L_n(f;x) - f(x)| \le M_3(p)\,\|f'\|_p \sqrt{(ax^2+bx)/\lambda_n} \tag{3.21}$$

and

$$w_p(x)\,|L_n^*(f;x) - f(x)| \le M_3(p)\,\|f'\|_p \sqrt{2x\,(x-u_n(x))}, \tag{3.22}$$

for $x \in I$ *and* $n \ge n_0$.

Proof. We shall prove only (22) because the proof of (21) is identical by the above lemmas.

For a fixed $f \in C_p^1$ and $x \in I$ we can write

$$f(t) - f(x) = \int_x^t f'(u)\,du, \quad t \in I,$$

and

$$\left|\int_x^t f'(u)\,du\right| \le \|f'\|_p \left|\int_x^t \frac{du}{w_p(u)}\right| \le \|f'\|_p\,(1/w_p(t) + 1/w(x))\,|t-x|.$$

Using now L_n^* and (10), (3) and Corollary 1, we get

$$\begin{aligned} w_p(x)\left|L_n^*\left(f(t);x\right)-f(x)\right| &\leq w_p(x)L_n^*\left(\left|\int_x^t f'(u)\,du\right|;x\right) \\ &\leq \|f'\|_p\left(w_p(x)L_n^*\left(\varphi_x(t)/w_p(t);x\right)+L_n^*\left(\varphi_x(t);x\right)\right) \\ &\leq \|f'\|_p\left(\sqrt{2}\|L_n\left(1/w_{2p}\right)\|_{2p}^{1/2}+1\right)\sqrt{2x\left(x-u_n(x)\right)}, \end{aligned}$$

which by (16) implies (22). □

Theorem 2 *For each $p\in\mathbb{N}_0$, there exists $M_4(p)=\text{const.}>0$ such that for every $f\in C_p$, $x\in I$ and $n\geq n_0$, we have*

$$w_p(x)\left|L_n(f;x)-f(x)\right|\leq M_4(p)\omega\left(f;\sqrt{(ax^2+bx)/\lambda_n}\right)_p \tag{3.23}$$

and

$$w_p(x)\left|L_n^*(f;x)-f(x)\right|\leq M_4(p)\omega\left(f;\sqrt{2x(x-u_n(x))}\right)_p\leq M_4(p)\omega\left(f;\sqrt{(ax^2+bx)/\lambda_n}\right)_p \tag{3.24}$$

where $\omega(f;\cdot)_p$ is the modulus of continuity of $f\in C_p$, i.e.

$$\omega(f;t)_p:=\sup_{0\leq h\leq t}\|\Delta_h f(\cdot)\|_p \quad \textit{for} \quad t\geq 0, \tag{3.25}$$

and $\Delta_h f(x)=f(x+h)-f(x)$.

Proof. Similar to [5], we use the Steklov function f_h of $f\in C_p$, i.e.

$$f_h(x):=\frac{1}{h}\int_0^h f(x+t)\,dt, \quad x\in I, h>0. \tag{3.26}$$

We have $f_h\in C_p^1$ and by (26) and (25) we get

$$\|f_h-f\|_p\leq\omega(f;h)_p \tag{3.27}$$

and

$$\|f_h'\|_p\leq h^{-1}\omega(f;h)_p, \quad \text{for} \quad h>0. \tag{3.28}$$

By the above properties of f_h and (10) we can write for $f\in C_p$ and L_n^*:

$$|L_n^*(f;x)-f(x)|\leq|L_n^*(f(t)-f_h(t);x|+|L_n^*(f_h(t);x)-f_h(x)|+|f_h(x)-f(x)|, \tag{3.29}$$

for $x\in I$, $n\geq n_0$ and $h>0$. Next, by (17) and (27) we have

$$\|L_n^*\left(f-f_h\right)\|_p\leq M_1(p)\|f-f_h\|_p\leq M_1(p)\omega(f;h)_p. \tag{3.30}$$

Theorem 1 for f_h and (28) imply that

$$w_p(x)\,|L_n^*(f_h(t);x) - f_h(x)| \le M_3(p)\|f_h'\|_p\sqrt{2x(x-u_n(x))} \le M_3(p)h^{-1}\sqrt{2x(x-u_n(x))}\,\omega(f;h)_p. \qquad (3.31)$$

Combining (29)-(31) and (27), we get

$$w_p(x)\,|L_n^*(f;x) - f(x)| \le \omega(f;h)_p\left(1 + M_1(p) + M_3(p)h^{-1}\sqrt{2x(x-u_n(x))}\right), \qquad (3.32)$$

for each $x \in I$, $n \ge n_0$ and $h > 0$. Putting $h = \sqrt{2x\,(x-u_n(x))}$ with $x > 0$ to (32), we obtain the desired estimation for $x > 0$.
If $x = 0$, then (24) follows by the property (i) of L_n and (8) and (9).

The proof of (23) is analogous. □

From (23), (24) and $\lim\limits_{n\to\infty} \lambda_n = +\infty$ we can derive the following two corollaries.

Corollary 2 *If $f \in C_p$, $p \in \mathbb{N}_0$, then*

$$\lim_{n\to\infty} L_n(f;x) = f(x) = \lim_{n\to\infty} L_n^*(f;x)$$

at every $x \in I$. This convergence is uniform on every interval $[x_1, x_2] \subset I$.

Corollary 3 *The error of approximation of a function $f \in C_p$, $p \in \mathbb{N}_0$, ($f(x) \ne e_k(x)$ for $k = 0,1$) by $L_n^*(f)$, $n \ge n_0$, is smaller than by $L_n(f)$.*

4. Applications

We present four examples of well-known positive linear operators satisfying the conditions (i)–(iv) given in Section 2 for operators L_n. For these operators we can consider modified operators of the type L_n^* defined by (8) and (9).

4.1. The Szász-Mirakyan operators ([5], [15])

$$S_n(f;x) := e^{-nx}\sum_{k=0}^{\infty}\frac{(nx)^k}{k!}f\left(\frac{k}{n}\right), \quad x \ge 0, n \in \mathbb{N},$$

satisfy conditions (i)–(iv) and $S_n(e_2;x) = x^2 + \frac{x}{n}$, i.e. the condition (5) there holds with $a = 0$, $b = 1$ and $\lambda_n = n$ for $n \in \mathbb{N}$. From (8) and (9) there results that the modified operators S_n^* preserving e_0 and e_2 are defined by the formula $S_n^*(f;x) := S_n\left(f;u_n(x)\right)$ with

$$u_n(x) = \frac{-1 + \sqrt{1 + 4n^2x^2}}{2n},$$

for $f \in C_p$, $x \ge 0$ and $n \in \mathbb{N}$.

4.2. The Baskakow operators ([5])

$$V_n(f;x) := (1+x)^{-n} \sum_{k=0}^{\infty} \binom{n+k-1}{k} \left(\frac{x}{1+x}\right)^k f\left(\frac{k}{n}\right), \; x \geq 0, n \in \mathbb{N},$$

satisfy conditions (i)–(iv) and the formula (5) there holds with $a = b = 1$ and $\lambda_n = n$ for $n \in \mathbb{N}$, because $V_n(e_2;x) = x^2 + x(1+x)/n$. Now the modified operators $V_n^*(f;x) := V_n(f;u_n(x))$ are connected with

$$u_n(x) = \frac{-1+\sqrt{1+4n(n+1)x^2}}{2(n+1)}.$$

4.3. The Post-Wildder operators ([7, 12])

$$P_n(f;x) := \int_0^{\infty} f(t)p_n(x,t)\,dt, \;\; x > 0, n \in \mathbb{N}, p_n(x,t) = \frac{(n/x)^n\, t^{n-1}}{(n-1)!} \exp(-nt/x),$$

have properties (i)–(iv) and $a = 1$, $b = 0$ and $\lambda_n = n$ in the formula (5), because $P_n(e_2;x) = x^2 + x^2/n$. Hence the operators $P_n^*(f;x) := P_n(f;u_n(x))$ are modified by

$$u_n(x) = \sqrt{\frac{n}{n+1}}x.$$

4.4. The Stancu operators ([14, 13])

$$L_n(f;x) := \int_0^{\infty} f(t)s_n(x,t)\,dt, \;\; x > 0, n \geq 2, s_n(x,t) = \frac{t^{nx-1}}{B(nx,n+1)(1+t)^{nx+n-1}},$$

with the Euler beta function B, satisfy conditions (i)–(iv) and $L_n(e_2;x) = x^2 + \frac{x(x+1)}{n-1}$, for $n \geq 2$, i.e. the formula (5) there holds with $a = b = 1$ and $\lambda_n = n-1$ for $n \geq 2$. Now modified Stancu operators $L_n^*(f;x) := L_n(f;u_n(x))$ are conected with

$$u_n(x) = \frac{-1+\sqrt{1+4n(n-1)x^2}}{2n}.$$

Applying Theorem 1 and Theorem 2, we can estimate the error of approximation of functions $f \in C_p$ by classical operators S_n, V_n, P_n, L_n and modified operators S_n^*, V_n^*, P_n^* and L_n^*.

References

[1] Abel U.: Asymptotic approximation with Stancu beta operators, Rev. Anal. Numér. Théor. Approx., 27(1), 5-13 (1998).

[2] Agratini O.: Linear operators that preserve some test functions, Int. J. Math. Math. Sci. Art. ID 94136 11 (2006).

[3] Agratini O.: On the iterates of a class of summation-type linear positive operators, Comput. Math. Appl. 55, 1178-1180 (2008).

[4] Baskakov V. A.: An example of sequence of linear positive operators in the space continuous functions, Dokl. Akad. Nauk SSSR, 113, 249-251 (1957).

[5] Becker M.: Global approximation theorems for Szász-Mirakyan and Baskakov operators in polynomial weight spaces, Indiana Univ. Math. J., 27(1), 127-142 (1978).

[6] De Vore R. A. and Lorentz G. G.: Constructive Approximation, Springer-Verlag, Berlin, New York, 1993.

[7] Ditzian Z. and Totik V.: Moduli of Smoothness, Springer-Verlag, New-York, 1987.

[8] Duman O. and Özarslan M. A.: Szász-Mirakyan type operators providing a better error estimation, Applied Math. Letters 20(12), 1184-1188 (2007).

[9] Duman O., Özarslan M. A. and Aktuğlu H.: Better error estimation for Szász-Mirakjan-Beta operators, J. Comput. Anal. Appl. 10, 53-59 (2008).

[10] King I. P.: Positive linear operators which preserve x^2, Acta Math. Hungar., 99(3), 203-208 (2003).

[11] Özarslan M. A. and Duman O.: MKZ type operators providing a better estimation on [1/2,1), Canadian Math. Bull. 50, 434-439 (2007).

[12] Rempulska L. and Skorupka M.: On strong approximation applied to Post-Widder operators, Anal. Theor. Applic., 22(2), 172-182 (2006).

[13] Rempulska L. and Skorupka M.: Approximation properties of modified Stancu beta operators, Rev. Anal. Numér. Théor. Approx., 35(2), 189-197 (2006).

[14] Stancu D.D.: On the beta approximating operators of second kind, Rev. Anal. Numér. Théor. Approx., 24(1-2), 231-239 (1995).

[15] Szász O.: Generalization of S. Bernstein's polynomials to the infinite interval, J. Res. Nat. Bur. Standards Sect. B, 45(1), 239-245 (1950).

L. REMPULSKA, K. TOMCZAK
Institute of Mathematics
Poznan University of Technology
Piotrowo 3a,
60-965 Poznan-POLAND
e-mail: lrempuls@math.put.poznan.pl

Received 13.03.2008