When δ-semiperfect rings are semiperfect

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Abstract

Zhou defined δ-semiperfect rings as a proper generalization of semiperfect rings. The purpose of this paper is to discuss relative notions of supplemented modules and to show that the semiperfect rings are precisely the semilocal rings which are δ-supplemented. Module theoretic version of our results are obtained.

Key word and phrases: Semiperfect, δ-semiperfect, δ-supplemented.

1. Introduction

H. Bass characterized in [4] those rings R whose left R-modules have projective covers and termed them *left perfect rings*. He characterized them as those semilocal rings which have a left t-nilpotent Jacobson radical $\mathrm{Jac}(R)$. Bass's *semiperfect rings* are those whose finitely generated left (or right) R-modules have projective covers. Kasch and Mares transferred in [5] the notions of perfect and semiperfect rings to modules and characterized semiperfect modules by a lattice-theoretical condition as follows. A module M is called *supplemented* if for any submodule N of M there exists a submodule L of M minimal with respect to $M = N + L$. The left perfect rings are then shown to be exactly those rings whose left R-modules are supplemented while the semiperfect rings are those whose finitely generated left R-modules are supplemented. Equivalently it is enough for a ring R to be semiperfect if the left (or right) R-module R is supplemented. Recall that a submodule $N \leq M$ is said to be *small*, denoted by $N \ll M$, if $N + L \neq M$ for all proper submodules L of M, and that $L \leq M$, is said to be *essential* in M, denoted by $L \trianglelefteq M$, if $L \cap K \neq 0$ for each nonzero submodule $K \leq M$. A module M is said to be *singular* if $M \simeq N/L$ for some module N and a submodule $L \leq N$ with $L \trianglelefteq N$.

In [8], Zhou called a ring R δ-*semiperfect* if every finitely generated R-module M has a projective δ-cover P, i.e. P is a projective left R-module with a projection $p : P \to M$ onto M such that the kernel $\mathrm{Ker}(p)$ is δ-small in P, where a submodule $X \leq Y$ is said to be δ-*small* in Y (denoted by $X \ll_\delta Y$) if $X + Z \neq Y$ for each proper submodules $Z < Y$ with Y/Z singular. It is known that a ring R is δ-semiperfect if and only if it is a δ-supplemented module. Here a module M is called δ-supplemented if every submodule $L \leq M$ has a δ-supplement N in M, i.e. $M = N + L$ and $N \cap L \ll_\delta N$. For further properties of δ-semiperfect rings and δ-supplemented modules we refer to [6] and [8].

Zhou proved that δ-semiperfect rings properly contains semiperfect rings (see [8, Example 4.1]). An easy example of a ring that is δ-semiperfect, but not semilocal had been given by Zhou in [8] as follows: Let F be the field of two elements and $A = F^{\mathbb{N}}$ the (commutative) ring of sequences over F, whose operation are pointwise multiplication and pointwise addition. Note that the unit element 1_A of A is the sequence which is constant 1. Let $R \subseteq A$ be the subring generated by 1_A and all sequences that have only a finite number of entries non-zero. Then $\mathrm{Soc}(R)$ consists of all sequences that have only a finite number of entries non-zero and $R/Soc(R)$ is the only singular simple R-module. Moreover, $R/Soc(R) \simeq F$ is a field, i.e. $Soc(R)$ is an essential maximal ideal of R and R is δ-local (see below), hence δ-semiperfect. On the other hand, since A is von Neumann regular, R is von Neumann regular, i.e. $\mathrm{Jac}(R) = 0$ and R is not semilocal.

The purpose of this paper is to discuss the gap between supplemented and δ-supplemented modules and our main result is that an arbitrary associative unital ring R is semiperfect if and only if it is semilocal and δ-semiperfect. We characterize finitely generated δ-supplemented modules M as those which are sums of simple and δ-local modules or equivalently which satisfy the property that every maximal submodule of M has a δ-supplement. The notion of a δ-coclosed submodule is defined and it is shown that a submodule is a δ-supplement if and only if it is δ-coclosed and a weak δ-supplement.

2. δ-supplements

In this section we show that some of the technicalities on supplement submodules have their relative equivalent. Let $\mathcal{P}$ be the class of all singular simple R-modules. For a module M, as in [8], let

$$\delta(M) = \mathrm{Rej}(\mathcal{P}) = \bigcap\{N \leq M \mid M/N \in \mathcal{P}\} = \sum\{N \leq M \mid N \ll_\delta M\}.$$

Let S be a nonsingular simple module. Then it is easy to see that $\delta(S) = S$. Also note that if K is a maximal submodule which is essential in M, then M/K is singular, so $\delta(M) \leq K$.

We have the following basic Lemma.

Lemma 2.1 *[8, Lemma 1.2] A submodule $N \leq M$ is δ-small if and only if for all submodules $X \leq M$:*

if $X + N = M$, then $M = X \oplus Y$ for a projective semisimple submodule Y with $Y \leq N$.

A submodule $N \leq M$ is said to be *coclosed* if $N/K \ll M/K$ implies $K = N$ for each $K \leq N$. Every supplement submodule of a module M is coclosed. The notion of coclosed submodules is generalized as follows.

Definition 2.2 *Let M be an R-module and $N \leq M$. We call N a δ-coclosed submodule of M if, whenever N/X is singular and $N/X \ll_\delta M/X$ for some $X \leq N$, we have $X = N$.*

Supplements are coclosed and so are their δ-equivalents:

Lemma 2.3 *Let M be any module and $N \leq M$ be a δ-supplement in M. Then N is δ-coclosed.*

Proof. Let N be a δ-supplement of K in M. Then $N + K = M$ and $N \cap K \ll_\delta N$. Suppose N/X is singular and $N/X \ll_\delta M/X$ for some $X \leq N$. Then we have

$$N/X + (K + X)/X = M/X,$$

and

$$M/(K + X) \simeq N/(N \cap (K + X))$$

is singular as a factor module of the singular module N/X. Therefore we have $(K + X)/X = M/X$ as $N/X \ll_\delta M/X$. Then we get $K + X = M$, and so by modular law $N = N \cap K + X$. Since $N \cap K \ll_\delta N$ and N/X is singular, we have $X = N$. So N is a δ-coclosed submodule of M. $\square$

In the following proposition we give some properties of δ-coclosed submodules.

Proposition 2.4 *Let N be a δ-coclosed submodule of M. Then the following hold.*

(1) If $K \leq N \leq M$ and $K \ll_\delta M$ then $K \ll_\delta N$. Hence $\delta(N) = N \cap \delta(M)$.

(2) If X is a proper submodule of N such that $N/X \ll_\delta M/X$, then $N = X \oplus X'$ for some $X' \leq N$.

(3) If N is singular, then N is coclosed.

Proof. (1) Let $K \ll_\delta M$ and suppose $K + X = N$ for some $X \leq N$ with N/X singular. Then $N/X = (K + X)/X \ll_\delta M/X$ by [8, Lemma 1.3(2)]. So that $X = N$, because N is δ-coclosed.

Clearly $\delta(N) \leq N \cap \delta(M)$. Therefore we only need to prove that $N \cap \delta(M) \leq \delta(N)$. Let $x \in N \cap \delta(M)$. Then $Rx \ll_\delta M$, a nd so by the first part of the proof $Rx \ll_\delta N$, that is, $x \in \delta(N)$. Hence $\delta(N) = N \cap \delta(M)$.

(2) Let $X \leq N$ with $N/X \ll_\delta M/X$. Let $X' \leq N$ be the maximal submodule in N such that $X \cap X' = 0$. Then $X \oplus X' \trianglelefteq N$ by [3, Proposition 5.21(1)], and so $N/(X \oplus X')$ is singular. On the other hand, $N/(X \oplus X') \ll_\delta M/(X \oplus X')$. Since N is δ-coclosed, we have $N = X \oplus X'$, as desired.

(3) Since singular modules are closed under factor modules, this is clear. $\square$

Corollary 2.5 *Let N be a δ-supplement submodule of M. Then $\delta(N) = N \cap \delta(M)$.*

Proof. By Lemma 2.3 and Proposition 2.4(1). $\square$

Corollary 2.6 *For a module M and a submodule $N \leq M$, consider the following statements.*

(1) N is a δ-supplement submodule of M.

(2) N is δ-coclosed in M.

(3) For all $X \leq N$, $X \ll_\delta M$ implies $X \ll_\delta N$.

Then $(1) \Rightarrow (2) \Rightarrow (3)$ hold. If N has a weak δ-supplement *in M, i.e. $N + K = M$ and $N \cap K \ll_\delta M$ for some submodule $K \leq M$, then $(3) \Rightarrow (1)$ holds.*

Proof. $(1) \Rightarrow (2)$ By Lemma 2.3.

$(2) \Rightarrow (3)$ By Proposition 2.4(1).

$(3) \Rightarrow (1)$ Suppose N has a weak δ-supplement in M. Then $N + L = M$ and $N \cap L \ll_\delta M$. Then $N \cap L \ll_\delta N$ by (3), i.e N is a δ-supplement of L in M. $\square$

3.   On the structure of δ-supplemented modules

A module M is said to be *local* if M has a largest proper submodule. It is easy to see that, M is local if and only if $\text{Rad}(M)$ is a maximal submodule of M and $\text{Rad}(M) \ll M$ (see [7, 41.4]).

Definition 3.1 *Let M be an R-module. M is said to be δ-local if $\delta(M) \ll_\delta M$ and $\delta(M)$ is a maximal submodule of M.*

It is easy to see that every simple module is local, and a simple module is δ-local if and only if it is singular. Let S be a nonsingular simple module and S' be a singular simple module. Then S is local but not δ-local, since $\delta(S) = S$. On the other hand, let $M = S \oplus S'$, then clearly M is not local. Since $\delta(S) = S$ and $\delta(S') = 0$, we have $\delta(M) = \delta(S) \oplus \delta(S') = S$. Clearly $\delta(M)$ is maximal, and nonsingularity of S implies $\delta(M) \ll_\delta M$, so M is δ-local.

The following lemma is elementary, we include it for completeness.

Lemma 3.2 *Let M be a module and H a local submodule of M. Then H is a supplement of each proper submodule $K \leq M$ with $H + K = M$.*

Proof.   Since K is a proper submodule of M and $K + H = M$, we have $K \cap H$ is a proper submodule of H. Therefore $K \cap H \ll H$, since H is local. That is, H is a supplement of K in M.   □

Lemma 3.3 *Any δ-local module is δ-supplemented.*

Proof.   Let M be a δ-local module and N be a proper submodule of M. Since $\delta(M)$ is a maximal submodule of M, we have either $N \leq \delta(M)$ or $\delta(M) + N = M$. If $N \leq \delta(M)$ then M is a δ-supplement of N in M. Now suppose $N + \delta(M) = M$. Since $\delta(M) \ll_\delta M$, we have $N \oplus Y = M$ for some semisimple submodule $Y \leq \delta(M)$ by Lemma 2.1. Clearly, Y is a δ-supplement of N in M. Therefore M is δ-supplemented.   □

Lemma 3.4 *Let M be an R-module and let K be a maximal submodule with $\text{Soc}(M) \leq K$. If L is a δ-supplement of K in M, then L is δ-local.*

Proof.   By hypothesis, we have $K + L = M$ and $K \cap L \ll_\delta L$. We claim that $K \cap L$ is an essential submodule of L. Really, if $(K \cap L) \cap T = 0$ for some nonzero submodule $T \leq L$, then $L = (K \cap L) \oplus T$ and $L/(K \cap L) \simeq T$ is simple. We get $M = K + L = K + T$, and so $T \not\leq K$; giving a contradiction since $\text{Soc}(M) \leq K$. Therefore $\delta(L) \leq K \cap L$. Hence $\delta(L) = K \cap L$.   □

A submodule $N \leq M$ is called *cofinite* if M/N is finitely generated. M is called *cofinitely δ-supplemented* if every cofinite submodule of M has a δ-supplement in M. In case M is finitely generated, clearly every submodule of M is cofinite; so M is δ-supplemented if and only if M is cofinitely δ-supplemented. Therefore by [1, Proposition 2.5], if a finitely generated module M is a sum of δ-supplemented modules then M is δ-supplemented.

Proposition 3.5 *For a finitely generated module M, the following are equivalent.*

(1) M is δ-supplemented.

(2) every maximal submodule of M has a δ-supplement.

(3) $M = H_1 + H_2 + \cdots + H_n$ where H_i is either simple or δ-local.

Proof. $(1) \Rightarrow (2)$ Clear.

$(2) \Rightarrow (3)$ Let $\Lambda(M) \le M$ be the sum of all δ-supplement submodules of maximal submodules $N \le M$ with $\mathrm{Soc}(M) \le N$. Then by Lemma 3.4 $\Lambda(M)$ is a sum of δ-local submodules of M. We claim that $M = \mathrm{Soc}(M) + \Lambda(M)$. Suppose the contrary. Then $\mathrm{Soc}(M) + \Lambda(M) \le K$ for some maximal submodule $K \le M$, because M is finitely generated. By (2) K has a δ-supplement L in M. Since $\mathrm{Soc}(M) \le K$, L is δ-local by Lemma 3.4. Hence $L \le \Lambda(M) \le K$, a contradiction. Therefore $M = \mathrm{Soc}(M) + \Lambda(M)$. Since M is finitely generated, M is a finite sum of simple submodules and δ-local submodules, as desired.

$(3) \Rightarrow (1)$ By Lemma 3.3, δ-local modules are δ-supplemented, and clearly simple modules are also δ-supplemented. Therefore M is δ-supplemented as a finite sum of δ-supplemented modules. $\square$

By [7, 41.6], a finitely generated module is supplemented if and only if it is a (finite) sum of local modules. Hence we can conclude from Proposition 3.5 that if every δ-local submodule of a module M with finitely generated socle is local, then M is supplemented if and only if it is δ-supplemented.

4. When δ-supplemented modules are supplemented

We will turn to the problem of characterising when a δ-semiperfect ring is semiperfect. Recall that a module M is called semilocal if $M/\mathrm{Rad}(M)$ is semisimple.

For any module M, let $X(M) = \mathrm{Soc}(M)/(\mathrm{Soc}(M) \cap \mathrm{Rad}(M))$.

Lemma 4.1 *Let R be a ring and M a finitely generated, δ-supplemented left R-module. Then M is semilocal if and only if $\mathrm{Soc}(M)/\mathrm{Soc}(M) \cap \mathrm{Rad}(M)$ is finitely generated.*

Proof. If M is semilocal (and finitely generated), then $M/\mathrm{Rad}(M)$ is semisimple artinian. Moreover

$$X(M) = \mathrm{Soc}(M)/(\mathrm{Soc}(M) \cap \mathrm{Rad}(M)) \simeq (\mathrm{Soc}(M) + \mathrm{Rad}(M))/\mathrm{Rad}(M) \subseteq M/\mathrm{Rad}(M)$$

implies that $X(M)$ is semisimple artinian; so $X(M)$ finitely generated.

To show the converse we use induction on the length of $X(M) = \mathrm{Soc}(M)/(\mathrm{Soc}(M) \cap \mathrm{Rad}(M))$. Suppose $X(M) = 0$, i.e. $\mathrm{Soc}(M) \subseteq \mathrm{Rad}(M)$. Then $\mathrm{Rad}(M) = \delta(M)$ and hence $M/\delta(M)$ is semisimple.

Assume that any finitely generated δ-supplemented module N with $X(N)$ of length $n \ge 0$ is semilocal and let M be a finitely generated δ-supplemented module with $X(M)$ having length $n+1$. Since $\mathrm{Soc}(M) \nsubseteq \mathrm{Rad}(M)$, there exists a simple direct summand $E \subseteq M$ with $M = E \oplus N$ for some $N \subseteq M$. Morever $\mathrm{Rad}(M) = \mathrm{Rad}(N)$ and $\mathrm{Soc}(M) = E \oplus \mathrm{Soc}(N)$. Hence

$$X(M) = \mathrm{Soc}(M)/(\mathrm{Soc}(M) \cap \mathrm{Rad}(M)) \simeq E \oplus (\mathrm{Soc}(N)/\mathrm{Soc}(N) \cap \mathrm{Rad}(N)) = E \oplus X(N).$$

Thus N is a finitely generated δ-supplemented module (direct summands of δ-supplemented modules are δ-supplemented) and $X(N)$ has length n. By induction hypothesis N is semilocal and hence $M = E \oplus N$ is semilocal. $\square$

It is shown in [6, Theorem 3.3] that, δ-semiperfect rings are exactly those rings R that are δ-supplemented as a left (or right) R-module. Similarly, a ring R is semiperfect if and only if R is supplemented as a left (or right) R-module (see [7, 42.6]).

Recall that projective δ-supplemented modules M are δ-lifting in the sense of [6], i.e for every submodule N of M there exists a decomposition $M = D_1 \oplus D_2$ such that $D_1 \subseteq N$ and $N \cap D_2 \ll_\delta D_2$.

Proposition 4.2 *A projective semilocal, δ-supplemented module with small radical is supplemented.*

Proof. Let $S = \operatorname{Soc}(M) = D \oplus (S \cap \operatorname{Rad}(M))$. Since M is semilocal, there exists $N \subseteq M$ such that $D + N = M$ and $D \cap N \subseteq \operatorname{Rad}(M)$. But since $D \cap \operatorname{Rad}(M) = 0$, $M = D \oplus N$ with D semisimple and $\operatorname{Rad}(M) = \operatorname{Rad}(N)$. Note that

$$\operatorname{Soc}(N) = S \cap N = (D \oplus S \cap \operatorname{Rad}(M)) \cap N = (D \cap N) \oplus (S \cap \operatorname{Rad}(M) = S \cap \operatorname{Rad}(N) \subseteq \operatorname{Rad}(N).$$

Hence if $K \subseteq N$ is a maximal submodule, then N/K must be singular, since otherwise N/K would be isomorphic to a simple direct summand of N which is impossible as $\operatorname{Soc}(N) \subseteq \operatorname{Rad}(N)$. Thus $\operatorname{Rad}(N) = \delta(N)$. By [6, 3.2] N is δ-lifting since it is δ-supplemented and projective. Hence for any submodule $L \subseteq N$ there exist $A, B \subseteq N$ such that $N = A \oplus B$ and $A \subseteq L$ and $L \cap B \ll_\delta B$. In particular, $L \cap B \subseteq \delta(B) \subseteq \delta(N) = \operatorname{Rad}(N)$. As M has a small radical, so has N and hence $L \cap B \ll N$. But since B is a direct summand of N, $L \cap B \ll B$. This shows that B is a supplement of L in N, i.e. N is a supplemented module. We showed that $M = D \oplus N$ is the direct sum of two supplemented modules. Hence M is supplemented as a (finite) sum of supplemented modules. $\square$

Corollary 4.3 *Let R be a ring with $J = \operatorname{Jac}(R)$ and $S = \operatorname{Soc}({}_R R)$. Then the following statements are equivalent.*

(1) R is semiperfect.

(2) R is δ-semiperfect and semilocal.

(3) R is δ-semiperfect and $S/S \cap J$ is finitely generated.

Proof. (1) $\Rightarrow$ (2) is clear, (2) $\Leftrightarrow$ (3) follows from Lemma 4.1 and (2) $\Rightarrow$ (1) follows from Proposition 4.2. $\square$

Remark 4.4 *In particular any ring R with finitely generated left socle (e.g. R is left noetherian) is semiperfect if and only if it is δ-semiperfect. There are δ-semiperfect rings which are not semilocal and hence not semiperfect (see [8, Example 4.1]).*

We finish this section by showing that the last remark also holds for modules, i.e. finitely generated modules with finitely generated socle are supplemented if and only if they are δ-supplemented.

Lemma 4.5 *Let M be a module and $K \leq M$ be a maximal submodule of M. Suppose $\operatorname{Soc}(M)$ is finitely generated and K has a δ-supplement H in M. Then K has a supplement in M contained in H.*

Proof. By hypothesis, H is a δ-supplement of K in M, that is, $K + H = M$ and $K \cap H \ll_\delta H$. In particular, $K \cap H \leq \delta(H)$. Since

$$M/K = (H + K)/K \simeq H/(K \cap H)$$

is simple, $K \cap H$ is a maximal submodule of H. Therefore, we have either $\delta(H) = H$ or $\delta(H) = K \cap H$. First, suppose that $\delta(H) = H$. Then $K \cap H$ is not essential in H. So there exists a submodule T of H such that $H = (K \cap H) \oplus T$. In this case, $M = K + H = K \oplus T$, so T is a supplement of K in M and T is contained in H.

Now, let $\delta(H) = K \cap H$. If $K \cap H \ll H$, then H is a supplement of K in M. Suppose $K \cap H = \delta(H)$ is not small in H, that is, $\delta(H) + L_1 = H$ for some proper submodule $L_1 \lneq H$. Then by [8, Lemma 1.2], $H = L_1 \oplus Y_1$ for some semisimple submodule $Y_1 \leq \delta(H)$. Since L_1 is a direct summand of H, we have

$$\delta(L_1) = L_1 \cap \delta(H) = L_1 \cap H \cap K = L_1 \cap K$$

and $\delta(L_1) \ll_\delta L_1$. We also have

$$K + H = K + L_1 + Y_1 = K + L_1.$$

Therefore L_1 is a δ-supplement of K.

Since L_1 is a proper submodule of H and Y_1 is a (nonzero) semisimple module contained in H, we have $\mathrm{Soc}(L_1) \lneq \mathrm{Soc}(H)$. Now, if $\delta(L_1) \ll L_1$, then L_1 is a supplement of K in M, and we are done. Suppose $\delta(L_1)$ is not small in L_1. Then $L_1 = \delta(L_1) + L_2$ for some $L_2 \lneq L_1$. Arguing as above, we get L_2 is a δ-supplement of K in M with $\mathrm{Soc}(L_1) \gneq \mathrm{Soc}(L_2)$. Continuing in this way, if none of the L_i's is a supplement of K we shall get a strictly descending chain of submodules $\mathrm{Soc}(L_1) \geq \mathrm{Soc}(L_2) \geq \cdots$ of $\mathrm{Soc}(M)$. This will contradict the fact that $\mathrm{Soc}(M)$ is finitely generated (see [3, Corollary 10.16]). Therefore K has a supplement in M. $\square$

Corollary 4.6 *Let M be a finitely generated module. Suppose $\mathrm{Soc}(M)$ is finitely generated. Then M is supplemented if and only if M is δ-supplemented.*

Proof. Necessity is clear. Sufficiency is a direct consequence of Proposition 3.5 and Lemma 4.5. $\square$

Corollary 4.7 *Let M be a module with finitely generated socle. Then M is cofinitely supplemented if and only if M is cofinitely δ-supplemented.*

Proof. Necessity is clear. Conversely suppose M is cofinitely δ-supplemented. Let K be a maximal submodule of M. If $\mathrm{Soc}(M)$ is not contained in K, then we have $K + \mathrm{Soc}(M) = M$ by maximality of K in M. Then $K + S = M$ for some simple submodule of M. Since S is simple and $S \nleq K$, we have $K \oplus S = M$, and hence S is a supplement of K in M.

Now, if $\mathrm{Soc}(M) \leq K$ and H is a δ-supplement of K in M, then K has a supplement in M by Lemma 4.5. Hence M is cofinitely supplemented by [2, Theorem 2.8]. $\square$

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References

[1] Al-Takhman, K.: Cofinitely δ-supplemented and Cofinitely δ-semiperfect modules. Int. J. Algebra. 1(12), 601-613 (2007).

[2] Alizade, R., Bilhan, G., Smith, P. F.: Modules whose maximal submodules have supplements. Comm. Algebra. 29, 2389-2405 (2001).

[3] Anderson, F. W., Fuller, K. R.: Rings and categories of modules. New York. Springer 1992.

[4] Bass, H.: Finitistic dimension and a homological generalization of semiprimary rings. Trans. Amer. Math. Soc. 95, 466–488 (1960).

[5] Kasch, F., Mares, E. A.: Eine Kennzeichnung semi-perfekter Moduln. Nagoya Math. J. 27, 525-529 (1966).

[6] Koşan, M. T.: δ-lifting and δ-supplemented modules. Algebra Colloq. 14(1), 53-60 (2007).

[7] Wisbauer, R.: Foundations of Modules and Rings. Gordon and Breach 1991.

[8] Zhou, Y.: Generalizations of perfect, semiperfect and semiregular rings. Algebra Colloq. 7(3), 305-318 (2000).

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