Turk J Math
36 (2012) , 41 – 46.

doi:10.3906/mat-1006-327

# The character variety of a class of rational links

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**Abstract**

Let $G_n$ be the fundamental group of the exterior of the rational link $C(2n)$ in Conway's normal form, see [7]. A presentation for $G_n$ is given by $\langle a, b \mid (ab)^n = (ba)^n \rangle$ [3, Thm. 2.2]. We study the character variety in $SL(2,\mathbb{C})$ of the group $G_n$. In particular, we give the defining polynomial of the character variety of $G_n$. As an application, we show a well-known result that $G_n$ and $G_m$ are isomorphic only when $n = m$. Also as a consequence of the main theorem of this paper, we give a basis of the Kauffman bracket skein module of the exterior of the rational link $C(2n)$ modulo its $(A+1)$-torsion.

**Key Words:** Link group, character variety, $SL_2(\mathbb{C})$ representations, Kauffman bracket skein module

## 1. Introduction

Given a finitely generated group $G$ with a presentation $\langle x_1, \ldots, x_k | \, r_\alpha \rangle$, a group homomorphism $\rho : G \longrightarrow SL(2,\mathbb{C})$ is called a unimodular complex representation of dimension 2, that will be abbreviated in this paper by just a representation. Two representations $\rho, \rho^{'}$ are said to be equivalent if and only if $\rho(g) = P^{-1}\rho^{'}(g)P$ for some $P \in SL(2,\mathbb{C})$ and for every $g \in G$. A representation is reducible if it is equivalent to a representation into upper triangular matrices. Otherwise the representation is irreducible.

A representation is uniquely determined by a point in $SL(2,\mathbb{C})^k \subset \mathbb{C}^{4k}$. The latter inclusion defines affine coordinates via the correspondence $\rho \mapsto \{\rho(x_i)\}_{i=1}^{k}$. After substituting $k$ general matrices into the relators, we obtain a set of polynomials in these coordinates. The Hilbert basis theorem assures that the number of polynomials is finite. Therefore, the set of all these points $R(G)$ inherits the structure of affine algebraic variety. (see [8] for more details).

The character of a representation $\rho$ is the function $\chi_\rho : G \longrightarrow \mathbb{C}$ given by $\chi_\rho(g) = \mathrm{tr}(\rho(g))$. Equivalent representations clearly have the same character, and the converse is true only if one of the representations is irreducible, [2, Prop. 1.5.2]. Now define $t_g : R(G) \longrightarrow \mathbb{C}$ by $t_g(\rho) = \chi_\rho(g) = \mathrm{tr}(\rho(g))$ for every $g \in G$. It has been proved that the ring $T$ generated by all functions $\{t_g | \; g \in G\}$ is a finitely generated ring [2, Prop. 1.4.1] using the following well-known trace identities that will be used very often within this paper:

2000 *AMS Mathematics Subject Classification:* 14M99,57M27.

$$\operatorname{tr}(A) = \operatorname{tr}(A^{-1}) \tag{1}$$

$$\operatorname{tr}(AB) = \operatorname{tr}(BA) \tag{2}$$

$$\operatorname{tr}(B) = \operatorname{tr}(ABA^{-1}) \tag{3}$$

$$\operatorname{tr}(AB) = \operatorname{tr}(A)\operatorname{tr}(B) - \operatorname{tr}(AB^{-1}). \tag{4}$$

The last identity follows from the identity $B + B^{-1} = \operatorname{tr}(B)I$, which in turn follows from the Cayley-Hamilton theorem. Moreover, it has been proved that the set: $\{t_{x_i}, t_{x_i x_j}, t_{x_i x_j x_l} | \ 1 \leq i < j < l \leq k\}$ is a set of generators of $T$ ([4, Cor. 4.1.2]).

We choose elements $g_1, \ldots, g_m \in G$ such that $T$ is generated by $\{t_{g_i} | \ 1 \leq i \leq m\}$ and define the map $t : R(G) \longrightarrow \mathbb{C}^m$ by $t(\rho) = (t_{g_1}(\rho), \ldots, t_{g_m}(\rho)) = (\chi_\rho(g_1), \ldots, \chi_\rho(g_m))$. Now it is clear that each character is uniquely determined by the point $(\operatorname{tr}(\rho(g_1)), \ldots, \operatorname{tr}(\rho(g_m))) \in \mathbb{C}^m$, where $m = \frac{k(k^2+5)}{6}$. The set of all these points is an algebraic variety (see [2, Cor. 1.4.5] for the proof) that is called the character variety of $G$ and is denoted by $X(G)$. It is a simple exercise to show that $X(G)$ is well defined up to an isomorphism. So it is an invariant of the group $G$. The coordinate ring of the character variety $X(G)$, $\mathbb{C}[X(G)]$, is the quotient of the polynomial ring $\mathbb{C}[t_{g_i}], 1 \leq i \leq m$ by the ideal generated by the defining polynomials of this variety.

The Kauffman bracket skein module of an oriented 3-manifold $M$, $\mathcal{K}(M)$, is an algebraic invariant of $M$ that is defined in terms of framed links. It is defined to be the quotient of the module freely generated by equivalence classes of framed links in $M$ over $\mathbb{Z}[A, A^{-1}]$ by the smallest submodule containing Kauffman relations (see [1, 11] for more details). Till recently, this module was topologically unexplained. The topological meaning of this module was given by Bullock in [1] at a special value of $A$. He showed that this module, after we set $A = -1$ and we tensor it with $\mathbb{C}$, $\mathcal{K}_{-1}(M)$, has a natural algebra structure over $\mathbb{C}$. Moreover, it is canonically isomorphic to the coordinate ring of the character variety of the fundamental group of $M$ after factoring it by its nilradical.

We state the main results of this paper and delay their proof for the next section, but we recall the definition of Chebyshev polynomials of the first kind; see [6].

**Definition 1.1** *The* $k^{\text{th}}$ *Chebyshev polynomial of the first kind* $S_k(x)$ *is defined inductively by* $S_0(x) = 1, S_1(x) = x$ *and* $S_k(x) = xS_{k-1}(x) - S_{k-2}(x)$.

For a given representation $\rho$ of the group $G_n = < a, b \mid (ab)^n = (ba)^n >$, we write $\operatorname{tr}(x)$ to denote $\operatorname{tr}(\rho(x))$ for any word $x$ in $a$ and $b$. Also, we abbreviate $\operatorname{tr}(a), \operatorname{tr}(b)$ and $\operatorname{tr}(ab)$ by $t_1, t_2$ and $t_3$ respectively.

**Theorem 1.2** *The character variety* $X(G_n)$ *is an algebraic subvariety of the variety* $\mathbb{C}^3$, *and the defining polynomial is given by the equality* $\operatorname{tr}((ab)^n a^{-1} b^{-1}) - \operatorname{tr}((ba)^{n-1}) = 0$. *Furthermore, we have the factorization* $\operatorname{tr}((ab)^n a^{-1} b^{-1}) - \operatorname{tr}((ba)^{n-1}) = (t_3^2 + t_2^2 + t_1^2 - t_3 t_2 t_1 - 4) S_{n-1}(t_3)$. *The first factor determines the character variety for abelian representations, and the second factor determines the character variety for nonabelian representations.*

As an application to the above theorem, we show the following well-known result.

**Corollary 1.3** *If $G_n$ and $G_m$ are isomorphic as groups, then $n = m$.*

**Theorem 1.4** *If $L$ is the rational link $C(2n)$, then $\mathcal{K}(S^3 \setminus L)/N$ is a free module over $\mathbb{Z}[A, A^{-1}]$ with a basis $\{x^i x'^j y^k |\ i, j \geq 0,\ 0 \leq k \leq n\}$, where $N$ is the $(A+1)$-torsion submodule. Here $x, x'$, and $y$ represents the conjugacy classes of $a, b$ and $ab$ in the fundamental group of the exterior of $L$ respectively.*

## 2. Proofs

Let $F$ be the free group of rank 2 on the generators $a$ and $b$ and $L$ be the rational link $C(2n)$. The Fricke-Vogt theorem states that $X(F) = \mathbb{C}^3$ (see [5] for more details). We prove the first theorem of this paper as a sequence of the following lemmas.

**Lemma 2.1** *Let $R, A, B$ be three matrices in $SL(2, \mathbb{C})$ such that $\mathrm{tr}(A) = t_1, \mathrm{tr}(B) = t_2$, and $\mathrm{tr}(AB) = t_3 \neq (st) + (st)^{-1}$, where $s + s^{-1} = t_1, t + t^{-1} = t_2, \mathrm{tr}(R) = 2, \mathrm{tr}(RA) = t_1$, and $\mathrm{tr}(RB) = t_2$, then $R = I$ (identity matrix).*

**Proof.** If $R \neq I$ then we choose an equivalent representation so that $R = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Now $\mathrm{tr}(RA) = t_1$ and $\mathrm{tr}(RB) = t_2$ implies $A = \begin{pmatrix} s & u \\ 0 & s^{-1} \end{pmatrix}$, and $B = \begin{pmatrix} t & v \\ 0 & t^{-1} \end{pmatrix}$, where $s, t, s^{-1}, t^{-1}, u, v \in \mathbb{C}$. Hence $\mathrm{tr}(AB) = st + (st)^{-1}$. □

**Remark 2.2** *The above lemma and its proof are generalizations of [12, Lem. 2] and its proof.*

**Lemma 2.3** *For any representation of $F$, we have*

$$\mathrm{tr}((ab)^n a^{-1} b^{-1}) - \mathrm{tr}((ba)^{n-1}) = (t_3^2 + t_2^2 + t_1^2 - t_3 t_2 t_1 - 4) S_{n-1}(t_3).$$

**Proof.** We show this by induction on $n$. It is clear that the statement is true for $n = 1$ and $n = 2$. Now using the trace identities, we obtain the relation

$$\begin{aligned}
\mathrm{tr}((ab)^n a^{-1} b^{-1}) - \mathrm{tr}((ba)^{n-1}) &= \mathrm{tr}(ab)\, \mathrm{tr}((ab)^{n-1} a^{-1} b^{-1}) - \mathrm{tr}((ab)^{n-2} a^{-1} b^{-1}) \\
&\qquad - \mathrm{tr}(ab)\, \mathrm{tr}((ba)^{n-2}) + \mathrm{tr}((ba)^{n-3}) \\
&= \mathrm{tr}(ab)(\mathrm{tr}((ab)^{n-1} a^{-1} b^{-1}) - \mathrm{tr}((ba)^{n-2})) \\
&\qquad - (\mathrm{tr}((ab)^{n-2} a^{-1} b^{-1}) - \mathrm{tr}((ba)^{n-3})) \\
&= (t_3 S_{n-2}(t_3) - S_{n-3}(t_3))(t_3^2 + t_2^2 + t_1^2 - t_3 t_2 t_1 - 4) \\
&= (t_3^2 + t_2^2 + t_1^2 - t_3 t_2 t_1 - 4) S_{n-1}(t_3).
\end{aligned}$$

The last two equalities follows from the induction hypothesis and the inductive definition of Chebyshev polynomials, respectively. □

**Lemma 2.4** *For any representation of $F$ and $m \geq 1$, we have*

$$\operatorname{tr}((ba)^n(ba)^m) - \operatorname{tr}((ab)^n(ba)^m) = S_{n-1}(t_3)S_{m-1}(t_3)(t_3^2 + t_2^2 + t_1^2 - t_3t_2t_1 - 4).$$

**Proof.** We show this by induction on $m$. For $m = 1$

$$\begin{aligned}\operatorname{tr}((ba)^n(ba)) - \operatorname{tr}((ab)^n(ba)) &= \operatorname{tr}(ab)\operatorname{tr}((ba)^n) - \operatorname{tr}((ba)^{n-1}) \\ &\qquad - \operatorname{tr}(ab)\operatorname{tr}((ab)^n) + \operatorname{tr}((ab)^n a^{-1}b^{-1}) \\ &= \operatorname{tr}((ab)^n a^{-1}b^{-1}) - \operatorname{tr}((ba)^{n-1}) \\ &= S_{n-1}(t_3)(t_3^2 + t_2^2 + t_1^2 - t_3t_2t_1 - 4).\end{aligned}$$

Now for $m > 1$, we have

$$\begin{aligned}\operatorname{tr}((ba)^n(ba)^m) - \operatorname{tr}((ab)^n(ba)^m) &= \operatorname{tr}(ab)\operatorname{tr}((ba)^n(ba)^{m-1}) - \operatorname{tr}((ba)^n(ba)^{m-2}) \\ &\qquad - \operatorname{tr}(ab)\operatorname{tr}((ab)^n(ba)^{m-1}) + \operatorname{tr}((ab)^n(ba)^{m-2}) \\ &= \operatorname{tr}(ab)(\operatorname{tr}((ba)^n(ba)^{m-1}) - \operatorname{tr}((ab)^n(ba)^{m-1})) \\ &\qquad - (\operatorname{tr}((ba)^n(ba)^{m-2}) - \operatorname{tr}((ab)^n(ba)^{m-2})) \\ &= (S_{n-1}(t_3)(t_3^2 + t_2^2 + t_1^2 - t_3t_2t_1 - 4))(t_3S_{m-2}(t_3) - S_{m-3}(t_3)).\end{aligned}$$

The last equality follows by the induction hypothesis. Finally, the result follows by the inductive definition of the Chebyshev polynomial. □

**Lemma 2.5** *For any representation of $F$, we have*

$$\operatorname{tr}((ab)^n(a^{-1}b^{-1})^n) - 2 = (t_3^2 + t_2^2 + t_1^2 - t_3t_2t_1 - 4)S_{n-1}^2(t_3).$$

**Proof.**

$$\begin{aligned}\operatorname{tr}((ab)^n(a^{-1}b^{-1})^n) - 2 &= \operatorname{tr}((ab)^n(a^{-1}b^{-1})^n - I) \\ &= \operatorname{tr}((ab)^n - (ba)^n)(a^{-1}b^{-1})^n) \\ &= \operatorname{tr}((a^{-1}b^{-1})^n)\operatorname{tr}((ab)^n - (ba)^n) - \operatorname{tr}((ab)^n(ba)^n - (ba)^{2n}) \\ &= \operatorname{tr}((ba)^n(ba)^n) - \operatorname{tr}((ab)^n(ba)^n).\end{aligned}$$

The result follows by applying Lemma 2.4 for $m = n$. □

Now we are ready to connect all these lemmas and give a proof of the main theorem. If $\rho$ is a representation of $G_n$ where $\rho(a) = A$ and $\rho(b) = B$, then it is clear that $\operatorname{tr}((ab)^n a^{-1}b^{-1}) - \operatorname{tr}((ba)^{n-1}) = 0$. We must prove the converse: if $t_1, t_2$, and $t_3$ satisfy $\operatorname{tr}((ab)^n a^{-1}b^{-1}) - \operatorname{tr}((ba)^{n-1}) = 0$, then there exist matrices $A$ and $B$ such that $\operatorname{tr}(A) = t_1, \operatorname{tr}(B) = t_2$, and $\operatorname{tr}(AB) = t_3$ such that $(AB)^n(A^{-1}B^{-1})^n = I$. We have one of the two factors $t_3^2 + t_2^2 + t_1^2 - t_3t_2t_1 - 4$ or $S_{n-1}(t_3)$ equal to zero.

1. If the first factor is zero, then the matrices $A = \begin{pmatrix} s & 0 \\ 0 & s^{-1} \end{pmatrix}$ and $B = \begin{pmatrix} t & 0 \\ 0 & t^{-1} \end{pmatrix}$, where $s + s^{-1} = t_1$ and $t + t^{-1} = t_2$ satisfy the above requirements.

2. If the second factor is zero, then consider $R = (AB)^n(A^{-1}B^{-1})^n$, then $\operatorname{tr}(R) = 2$ by using Lemma 2.5. To show that that $R = I$, it is enough to satisfy the conditions of Lemma 2.1. We can rewrite $R = AWA^{-1}W^{-1}$, where $W = A^{-1}(AB)^n$. We have $\operatorname{tr}(RA^{-1}) = \operatorname{tr}(AWA^{-1}W^{-1}A^{-1}) = \operatorname{tr}(A^{-1}) = \operatorname{tr}(A) = t_1$. By trace identities $\operatorname{tr}(RA)+\operatorname{tr}(RA^{-1}) = \operatorname{tr}(A)\operatorname{tr}(R) = 2t_1$. Hence, $\operatorname{tr}(RA) = t_1$. Similarly, we can rewrite $R = UBU^{-1}B^{-1}$, where $U = (AB)^nB^{-1}$. Now we have $\operatorname{tr}(RB) = \operatorname{tr}(UBU^{-1}) = \operatorname{tr}(B) = t_2$. At the end, since all the conditions of Lemma 2.1 are satisfied, then $R = I$.

Finally, Lemma 2.3 takes care of the factorization of $\operatorname{tr}((ab)^n a^{-1}b^{-1}) - \operatorname{tr}((ba)^{n-1})$ in $\mathbb{C}[t_1, t_2, t_3]$. This completes the proof of the theorem.

**Remark 2.6** *The above proof of the main theorem imitates the proof of [9, Thm. 3.3.1]. We can give another proof by combining the result of [4, Thm. 3.2] and Lemmas 2.3, 2.4, and 2.5.*

Now we proceed to prove Corollary 1.3. It is a well-know fact that the $k^{\text{th}}$ Chebyshev polynomial has $k$ distinct roots; see [6]. Therefore, we conclude $X(G_n)$ and $X(G_m)$ have different number of irreducible components if $n \neq m$. Hence Corollary 1.3 follows since the character variety is defined up to an isomorphism.

We note that the $t_3$ degree of $\operatorname{tr}((ab)^n a^{-1}b^{-1}) - \operatorname{tr}((ba)^{n-1})$ is $n+1$ since the $k^{\text{th}}$ Chebyshev polynomial is of degree $k$.

**Lemma 2.7** *[10, Lemm. 1.1] If $D_2$ is the two-punctured disk, then as an element of the Kauffman bracket skein module $\mathcal{K}(D_2 \times I) \cong \mathbb{Z}[A, A^{-1}][x, x', y]$ the closure of a braid on $2k$ strands is a polynomial having $y$-degree $k$ with coefficient of the form $\pm A^m, m \in \mathbb{Z}$ and hence it is invertible in $\mathbb{Z}[A, A^{-1}]$.*

For what follows, we let $r = (ab)^n a^{-1}b^{-1} - (ba)^{n-1}$ and $D(t_1, t_2, t_3) = t_3^2 + t_2^2 + t_1^2 - t_3t_2t_1 - 4$. We recall the maps $\Phi$ and $\Psi$ defined in [1]. The map $\Phi : \mathcal{K}_{-1}(S^3 \setminus L) \to \mathbb{C}[X(G_n)] = \mathbb{C}[t_1, t_2, t_3]/ < D(t_1, t_2, t_3)S_{n-1}(t_3) >$ is defined by $\Phi(K)(\chi_\rho) = -\chi_\rho(K) = -\operatorname{tr}(\rho(K))$. Also the map $\Psi : \mathbb{C}[t_a, t_b, t_{ab}] \to \mathcal{K}_{-1}(S^3 \setminus L)$ is defined by $\Psi(t_w) = -K_w$, for any word $w$ in $a$ and $b$, where $K_w$ is any knot in the homotopy class of the unique unoriented curve that corresponds to $w$ in $\mathcal{K}_{-1}(S^3 \setminus L)$.

**Proof of Theorem 1.4.** The conjugacy classes of $(ab)^n a^{-1}b^{-1}$ and $(ba)^{n-1}$ represent two equal skein elements of the module $\mathcal{K}(S^3 \setminus L)$. As $S^3 \setminus L$ is obtained by gluing two copies of $D_2 \times I$, then the above two elements can be considered two equal skein elements in $\mathcal{K}(D_2 \times I)$ that can be represented as closure of two braids of $2n+2$ and $2n-2$ strands, respectively in a natural way. After we expand these two elements in terms of the basis elements of $\mathcal{K}(D_2 \times I)$, we obtain $y^{n+1} =$ terms of smaller degree. Hence the set $\{x^i x'^j y^k | i, j \geq 0, 0 \leq k \leq n\}$ spans $\mathcal{K}(S^3 \setminus L)$ over $\mathbb{Z}[A, A^{-1}]$, where $x, x'$, and $y$ represents the conjugacy classes of $a, b$ and $ab$ in the fundamental group of the exterior of $L$ respectively.

It is clear that $\mathcal{K}_{-1}(D_2 \times I)$ projects onto $\mathcal{K}_{-1}(S^3 \setminus L)$ and the conjugacy class of $r$ in $\mathcal{K}_{-1}(S^3 \setminus L)$ is equal to zero. So the conjugacy class of $r$ as an element of $\mathcal{K}_{-1}(D_2 \times I)$ is in the kernel of the projection.

Hence we obtain a projection $\widetilde{p}:\mathcal{K}_{-1}(D_2\times I)/<r>\rightarrow\mathcal{K}_{-1}(S^3\setminus L)$ and an inclusion $\widetilde{i}:\mathcal{K}_{-1}(S^3\setminus L)\rightarrow\mathcal{K}_{-1}(D_2\times I)/<r>$. Now the map $\Phi\circ\widetilde{p}:\mathcal{K}_{-1}(D_2\times I)/<r>\rightarrow\mathbb{C}[t_1,t_2,t_3]/<D(t_1,t_2,t_3)S_{n-1}(t_3)>$ is an algebra isomorphism since the algebra homomorphism $\widetilde{i}\circ\Psi:\mathbb{C}[t_a,t_b,t_{ab}]/<t_r>\rightarrow\mathcal{K}_{-1}(D_2\times I)/<r>$ is its inverse as $\Psi(t_r)=-K_r=0\in\mathcal{K}_{-1}(S^3\setminus L)$ under the identification of $\mathbb{C}[X(G_n)]$ and $\mathbb{C}[t_a,t_b,t_{ab}]/<t_r>$.

Now if we combine $\mathcal{K}_{-1}(D_2\times I)/<r>\cong\mathbb{C}[X(G_n)]$ from above and $\mathcal{K}_{-1}(S^3\setminus L)/\sqrt{0}\cong\mathbb{C}[X(G_n)]$ [1, Thm. 10], we get $\mathcal{K}_{-1}(D_2\times I)/<r>\cong\mathcal{K}_{-1}(S^3\setminus L)/\sqrt{0}$. Therefore, we get $\mathcal{K}_{-1}(D_2\times I)/<r>\cong\mathcal{K}_{-1}(S^3\setminus L)$ since the first module projects onto the second module. Hence the set $\{x^i x'^j y^k|i,j\geq 0, 0\leq k\leq n\}$ is linearly independent over $\mathbb{C}$ in $\mathcal{K}_{-1}(S^3\setminus L)$, then it is a basis for $\mathcal{K}(S^3\setminus L)/N$.

**Corollary 2.8** *The algebra $\mathcal{K}_{-1}(S^3\setminus L)$ has trivial nilradical.*

## References


[1] Bullock, D.: Rings of $SL_2(\mathbb{C})$-characters and the Kauffman bracket skein module. Comment. Math. Helv. 72, 521–542 (1997).

[2] Culler, M., Shalen, P.: Varieties of Group Representations and Splitting of 3-Manifolds. Ann. Math. 2nd Ser 117 (1), 109–146 (1983).

[3] Gaebler, R.: Alexander polynomials of two-bridge knots and links. thesis (2004).

[4] González-Acuña, F., José María Montesinos-Amilibia.: On the character variety of group representations in $SL(2,\mathbb{C})$ and $PSL(2,\mathbb{C})$. Math. Z. 214(4), 627–652 (1993).

[5] Goldman, W.: Trace coordinates on Fricke spaces of some simple hyperbolic surfaces,″Handbook of Teichmüller theory, vol. II,″ (A. Papadopoulos, ed.) Chapter 15, 611–684, European Mathematical Society 2009. math.GT/0901.1404.

[6] Hsiao, H.: On factorization of Chebyshev's polynomials of the first kind. Bulletin of the Institute of Mathematics Academia Sinica 12(1), 89–94 (1984).

[7] Kawauchi, K.: A Survey of Knot Theory. Birkhäuser Verlag (1996).

[8] Lubotzky, A., Magid, A.: Varieties of representations of finitely generated groups. Mem. Amer. Math. Soc. 58(336):xi+117, (1985).

[9] Le, T.: Varieties of representations and their cohomology-jump subvarieties for knot groups. Russian Acad. Sci. Math. 18(1), 187–209 (1994).

[10] Le, T.: The colored Jones polynomial and the A-polynomial of knots. Adv. Math. 207(2), 782–804 (2006).

[11] Przytycki, J., Sikora, A.: On Skein Algebras And $SL_2(\mathbb{C})$-Character Varieties. Topology 39, 115–148 (2000).

[12] Whittemore, A.: On representations of the group of Listing's knot by subgroups of $SL(2,\mathbb{C})$. Proc. Amer. Math. Soc. 40, 378–382 (1973).



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Received: 03.06.2010