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An identity between the m-spotty weight enumerators of a linear code and its dual*

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Abstract

The m-spotty byte error control codes provide a good source for detecting and correcting errors in semiconductor memory systems using high density RAM chips with wide I/O data (e.g. 8, 16, or 32 bits). m-spotty byte error control codes are very suitable for burst correction. Here, we introduce the m-spotty weights and m-spotty weight enumerator of linear codes over the ring $F_2 + uF_2$ and prove a MacWilliams type identity.

Key Words: Linear codes over $F_2 + uF_2$, m-spotty weights, MacWilliams identity

1. Introduction

Byte error control codes play an important role in computer memory systems that use chips with 4-bit I/O data [8]. Recently, high-density RAM chips with wide I/O data of 8, 16 and 32 bits have also found applications for byte error control codes. These chips are quite vulnerable to multiple random error bits while being exposed to strong electromagnetic waves, radio active particles, etc. As such in order to be able to correct multiple errors a new spotty byte error called m-spotty byte error is introduced in [11] for binary codes. Construction of codes correcting byte errors and properties of such codes are also investigated. Spotty byte error correcting codes further require lower number of check bits compared to the existing byte error control RS codes [11, 12, 14]. Some related work can be found in [6, 11, 14]. Recently, a MacWilliams identity has been proven for m-spotty byte error codes [10]. Most of the work on byte errors known to the author is applied over binary or extension fields of binary fields. A link between binary codes and quaternary (Z_4) codes is established in [4] where some binary nonlinear codes are represented as images of linear quaternary codes via a Gray map. Recently, Lee m-spotty weight enumerators over quaternary codes have been introduced and a MacWilliams type identity is proved by the author [9]. In [1], Bachoc considered linear codes over the ring $F_p + uF_p$ (p prime) and constructed modular lattices by making use of linear codes over these rings. Later, interest in linear codes over these rings has grown quite remarkably [2, 3]. Studying linear codes over special rings with algebraic structural properties leads to gaining insight into some linear or nonlinear codes over fields and their application to other algebraic structures. Here, in this paper, we introduce m-byte error control linear codes over the ring

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$R = F_2 + uF_2 = \{0, 1, u, 1+u\}$ with $u^2 = 0$ and establish a MacWilliams type identity for m-spotty weight enumerators.

The m-spotty Hamming distance for binary linear codes is shown to be metric [11]. A linear code C of length n over R is defined to be an R-submodule of R^n. The elements of C are called codewords.

The Hamming weight w of a codeword c is the number of nonzero entries of the codeword c and is denoted by $w(c)$. The Hamming distance between the codewords c and v is defined by $d(c, v) = w(c - v)$.

Now, we give the definition of m-spotty weight of a codeword c. Let $c = (c_{11}, c_{12}, \ldots, c_{1b}, \ldots, c_{n1}, c_{n2}, \ldots, c_{nb}) \in R^{bn}$ be a codeword of length $N = bn$. The i th byte of c will be denoted by $c_i = (c_{i1}, c_{i2}, \ldots, c_{ib})$. Spotty byte errors are introduced originally for binary codes [14].

Definition 1.1 *[14] An error e is called a spotty byte error or t/b-error if t or fewer bits within a b-byte are in error, where $1 \leq t \leq b$.*

Now, we extend the definition of m-spotty weights originally introduced in [11] for binary codes to codes over $F_2 + uF_2$.

Definition 1.2 *Let $e \in R^N$ be an error vector and $e_i \in R^b$ be the i th byte of e where $1 \leq i \leq n$. The number of t/b-errors in e, denoted by $w_M(e)$, and called m-spotty weight is defined as*

$$w_M(e) = \sum_{i=1}^{n} \left\lceil \frac{w(e_i)}{t} \right\rceil .$$

If $t = 1$, then $w_M(e) = w(e)$, the usual Hamming weight in this particular case.

There are two classes of spotty byte errors [14]. The first is the class of s-spotty (single spotty) byte errors which consists of errors of weight less than or equal to t in a byte where $t \leq b$. The other is the class of m-spotty (multiple spotty) byte errors where more than t errors occur in a byte of length b. In this particular case, if $k \geq t$ random errors have occurred in a byte of length b where $t \leq b$, then we say that multiple $\lceil \frac{k}{t} \rceil$ t/b errors have occurred. Otherwise, we say that single spotty or s-spotty errors have occurred in short. To illustrate the definitions we give an example:

Example 1.1 *Let $b = 6, n = 3$ and $t = 2$ and assume that $(00uu001100110u0u0u) \in R^{18}$ is a codeword. If the received word is $(0\underline{11}u001100110u\underline{1}u0\underline{1})$, then in the first and the last bytes s-spotty errors have occurred. On the other hand, if the received word is $(\underline{uu}1u\underline{111}\underline{u11}1\underline{u}0u\underline{11}0u)$, then in the first byte multiple $\lceil \frac{5}{2} \rceil = 3$ (triple) $2/6$ errors and in the second byte multiple $\lceil \frac{4}{2} \rceil = 2$ (double) $2/6$ errors and in the last byte an s-spotty error have occurred. (Here, the underline notation is used to note the error locations.)*

In a similar way, we define the m-spotty distance of two codewords c and v as $d_M(c, v) = \sum_{i=1}^{n} \lceil \frac{d(c_i, v_i)}{t} \rceil$. Further, it is also straightforward to show that this distance is a metric in R^N.

Let $c = (c_1, c_2, \ldots, c_N)$ and $v = (v_1, v_2, \ldots, v_N)$ be two elements of R^N. An inner product of the elements c and v is defined by $\langle c, v \rangle = \sum_{i=1}^{N} c_i v_i$.

Let C be a linear code. The set $C^{\perp} = \{v \in R^N | \langle c, v \rangle = 0 \text{ for all } c \in C\}$ is also a linear code and it is called the dual code of C.

The *m-spotty t/b-weight enumerator* of a linear code C is defined by

$$A(z) = \sum_{c \in C} z^{w_M(c)}.$$

Let $\alpha_i \in \mathbb{N}$ count the number of bytes with Hamming weight i. Then,

$$\sum_{j=0}^{b} \lceil j/t \rceil \cdot \alpha_j$$

gives the m-spotty t/b-weight of a codeword. Hence, we have

$$A(z) = \sum_{\alpha_0+\alpha_1+\cdots+\alpha_b=n} A_{\overline{\alpha}} \prod_{j=0}^{b} \left(z^{\lceil j/t \rceil} \right)^{\alpha_j}$$

where $\overline{\alpha} = (\alpha_0, \alpha_1, \ldots, \alpha_b)$ denotes the distribution of bytewise Hamming weights of a codeword and $A_{\overline{\alpha}}$ gives the number of codewords of weight distribution $\overline{\alpha}$.

2. The MacWilliams identity

The MacWilliams identity relates the weight enumerator of a code to its dual [7]. The MacWilliams identity has many important applications in algebraic coding theory. In this section, first we state several lemmas that will help on proving the main Theorem 2.1. We also give an example that illustrates the theorem.

The ring R has three ideals $\{0\}$, $\langle u \rangle = \{0, u\}$, and R. These are by definition the additive subgroups of R. As an additive group, R has four characters. In this paper, we always refer to the character χ defined by

$$\chi(a) = \begin{cases} 1, & a \in \{0, 1\}, \\ -1, & \text{otherwise.} \end{cases} \tag{1}$$

We note that χ is a nontrivial character, i.e. χ is not the identity map on the nonzero ideals of R.

We can readily obtain the following result by using the definition of the character χ in (1).

Lemma 2.1 *Let $H \neq \{0\}$ be an ideal of R. Then,*

$$\sum_{a \in H} \chi(a) = 0.$$

Lemma 2.2 *Let $a \in R$. Then,*

$$\sum_{r \in R} \chi(ar) = \begin{cases} 4, & a = 0 \\ 0, & a \neq 0 \end{cases}$$

Proof. If $a = 0$, then clearly $\chi(ar) = 0$ for all $r \in R$ and hence the result follows. Otherwise, if $a \neq 0$, and $a = u$, then $\sum_{r \in R} \chi(ur) = 2 \cdot \sum_{r \in \langle u \rangle} \chi(r) = 0$, by Lemma 2.1. If $a \neq 0$, and $a \neq u$, then $\sum_{r \in R} \chi(ar) = \sum_{r \in R} \chi(r) = 0$, by Lemma 2.1. □

Definition 2.1 *Let* $v = (v_1, v_2, ..., v_b) \in R^b$. *Then, the support of* v *is defined by* $\text{supp}(v) = \{i | v_i \neq 0\}$ *and the complement of* $\text{supp}(v)$ *is denoted by* $\overline{\text{supp}(v)}$.

Lemma 2.3 *Let* $c = (c_1, c_2, ..., c_b) \in R^b$, *with* $w(c) = j \neq 0$ *and* $k \in \{1, 2, \ldots, j\}$. *Then,*

$$\sum_{\substack{0 \leq w(v) \leq k \\ \text{supp}(v) \subseteq \text{supp}(c)}} \chi(\langle c, v \rangle) = 0.$$

Proof. Let $\{l_1, l_2, \ldots, l_k\} \subseteq \text{supp}(c)$. If we define a map $\phi : R^k \to R$ such that $\phi(v_1, v_2, \ldots, v_k) = c_{l_1} v_1 + \cdots + c_{l_k} v_k$. This is a ring homomorphism and the image $\text{Im}\phi = H$ is not zero since $w(c) \neq 0$. Further, H is the nonzero ideal of R generated by $\{c_{l_1}, \ldots, c_{l_k}\}$. Thus, by the first isomorphism theorem, $|R^k|/|\text{Ker}\phi| = H \neq \{0\}$. Let $|\text{Ker}\phi| = m$.

$$\sum_{\substack{w(v) \leq k \\ \text{supp}(v) \subseteq \text{supp}(c)}} \chi(\langle c, v \rangle) = \sum_{(v_{l_1}, \ldots, v_{l_k}) \in R^k} \chi\left(\sum_{i=1}^{k} c_{l_i} v_{l_i}\right) = m \sum_{h \in H} \chi(h) = 0$$

by Lemma 2.1. □

Now we introduce some auxiliary notations. Let $c = (c_1, c_2, ..., c_b) \in R^b$ and define

$$S_k(c) = \{v \in R^b | \text{supp}(v) \subseteq \text{supp}(c) \text{ and } k = |\text{supp}(v)|\} \text{ and}$$

$$\overline{S}_k(c) = \{v \in R^b | \text{supp}(v) \subseteq \overline{\text{supp}(c)} \text{ and } k = |\text{supp}(v)|\}.$$

Lemma 2.4 *Let* $c = (c_1, c_2, ..., c_b) \in R^b$ *and* $w(c) \neq 0$. *For all* k *positive integers, we let* $I_k = \{i_1, i_2, \ldots, i_k\} \subseteq \text{supp}(c)$ *and* $I_0 = \emptyset$. *Then, we have*

$$\sum_{\substack{v \in R^b \\ \text{supp}(v) = I_k}} \chi(\langle c, v \rangle) = (-1)^k.$$

Proof. We use the notation $R^* = R \setminus \{0\}$. We apply induction on k.

For $k = 0$ i.e. $I_0 = \emptyset$, we have

$$\sum_{\substack{v \in R^b \\ \text{supp}(v) = I_0}} \chi(\langle c, v \rangle) = \sum_{w_H(v) = 0} \chi(0) = \chi(0) = 1.$$

For $k = 1$, we have

$$\sum_{\substack{v \in R^b \\ \text{supp}(v) = I_1}} \chi(\langle c, v \rangle) = \sum_{\substack{i_1 \in I \\ v_{i_1} \in R^*}} \chi(c_{i_1} v_{i_1}) = \sum_{v_{i_1} \in R} \chi(c_{i_1} v_{i_1}) - 1 = -1.$$

Now, we assume that the identity holds true for $k = r$, i.e.

$$\sum_{\substack{v \in R^b \\ \text{supp}(v) = I_r}} \chi(\langle c, v \rangle) = (-1)^r.$$

For $k = r+1$, suppose $\text{supp}(v) = \{i_1, i_2, ..., i_r, i_{r+1}\}$. Then

$$\sum_{\substack{v\in R^b \\ \text{supp(v)}=\text{I}_{\text{r+1}}}} \chi(\langle c, v\rangle) = \sum_{v_{i_1}, v_{i_2}, ..., v_{i_r}, v_{i_{r+1}} \in R^*} \chi\left(\sum_{j=1}^{r} c_{i_j} v_{i_j} + c_{i_{r+1}} v_{i_{r+1}}\right)$$

$$= \sum_{v_{i_1}, v_{i_2}, ..., v_{i_r}, v_{i_{r+1}} \in R^*} \chi\left(\sum_{j=1}^{r} c_{i_j} v_{i_j}\right) \chi\left(c_{i_{r+1}} v_{i_{r+1}}\right)$$

$$= \sum_{v_{i_1}, v_{i_2}, ..., v_{i_r} \in R^*} \chi\left(\sum_{j=1}^{r} c_{i_j} v_{i_j}\right) \sum_{v_{i_{r+1}} \in R^*} \chi\left(c_{i_{r+1}} v_{i_{r+1}}\right)$$

$$= (-1)^r (-1) = (-1)^{r+1}.$$

□

Corollary 2.1 *Let $c = (c_1, c_2, ..., c_b) \in R^b$ and $w(c) = j \neq 0$. For all $0 \leq k \leq j$, we have*

$$\sum_{v\in S_k(c)} \chi(\langle c, v\rangle) = (-1)^k \binom{j}{k}.$$

Proof.

$$\sum_{v\in S_k(c)} \chi(\langle c, v\rangle) = \sum_{I_k \subseteq \text{supp(c)}} \sum_{\text{supp(v)}=\text{I}_\text{k}} \chi(\langle c, v\rangle) = \sum_{I_k \subseteq \text{supp(c)}} (-1)^k = \binom{j}{k} (-1)^k.$$

□

Lemma 2.5 *Let $c = (c_1, c_2, ..., c_b) \in R^b$ and $w(c) = j \neq 0$. For all $0 \leq k \leq j$, we have*

$$\sum_{v\in \overline{S}_k(c)} \chi(\langle c, v\rangle) = 3^k \binom{b-j}{k}.$$

Proof. Since $v \in \overline{S}_k(c)$ with $\text{supp(v)} \subseteq \overline{\text{supp(c)}}$ we have $\chi(\langle c, v\rangle) = 1$. Further, since $k = |\text{supp}(v)|$, there are $\binom{b-j}{k}$ ways of choosing a subset of size k from the complement of support of c of size k. For each subset of size k, the sum of characters equals to 3^k. Hence, the result follows. □

Lemma 2.6 *Let $c = (c_1, c_2, ..., c_b) \in R^b$ with $w(c) = j$, $0 \leq j_1 \leq j$ and $0 \leq j_2 \leq b-j$. We define $S_{j_1,j_2}(c) = \{v \in R^b | j_1 = |\text{supp}(v) \cap \text{supp}(c)| \text{ and } j_2 = |\text{supp}(v) \cap \overline{\text{supp}(c)}|\}$. Then,*

$$\sum_{v\in S_{j_1,j_2}(c)} \chi(\langle c, v\rangle) = (-1)^{j_1} 3^{j_2} \binom{j}{j_1} \binom{b-j}{j_2}.$$

Proof. Let $c^{(1)}$ and $c^{(2)}$ denote the partial vectors consisting of the first j and the last $b-j$ entries of c, respectively. Then,

$$\begin{aligned}
&\sum_{v\in S_{j_1,j_2}(c)} \chi\left(\langle c,v\rangle\right) = \\
&= \sum_{v\in S_{j_1,j_2}(c)} \chi\left(\sum_{i\in \mathrm{supp}(v)\cap \mathrm{supp}(c)} c_i v_i + \sum_{r\in \mathrm{supp}(v)\cap \overline{\mathrm{supp}(c)}} c_r v_r\right) \\
&= \sum_{v\in S_{j_1,j_2}(c)} \chi\left(\sum_{i\in \mathrm{supp}(v)\cap \mathrm{supp}(c)} c_i v_i\right)\cdot \chi\left(\sum_{r\in \mathrm{supp}(v)\cap \overline{\mathrm{supp}(c)}} c_r v_r\right) \\
&= \sum_{v\in S_{j_1}(c^{(1)})} \chi\left(\left\langle c^{(1)}, v^{(1)}\right\rangle\right)\cdot \sum_{v\in S_{j_2}(c^{(2)})} \chi\left(\left\langle c^{(2)}, v^{(2)}\right\rangle\right) = (-1)^{j_1}\binom{j}{j_1}3^{j_2}\binom{b-j}{j_2}.
\end{aligned}$$

In the last line of the equations above, both Corollary 2.1 and Lemma 2.5 are applied. □

Lemma 2.7 *Let $c=(c_1,c_2,...,c_b)\in R^b$ and $w(c)=j$. Then,*

$$\sum_{v\in R^b} \chi\left(\langle c,v\rangle\right) z^{\lceil w_M(v)/t\rceil} = \sum_{j_1=0}^{j}\sum_{j_2=0}^{b-j} (-1)^{j_1}3^{j_2}\binom{j}{j_1}\binom{b-j}{j_2} z^{\lceil (j_1+j_2)/t\rceil}.$$

Proof. Since the sum $\sum_{v\in R^b} \chi\left(\langle c,v\rangle\right) z^{\lceil w_M(v)/t\rceil}$ runs over all $v\in R^b$, we can split the sum according to the set $S_{j_1,j_2}(c)$ where j_1 and j_2 run through all possible cases. Hence, by Lemma 2.6, we have

$$\sum_{v\in R^b} \chi\left(\langle c,v\rangle\right) z^{\lceil w_M(v)/t\rceil} = \sum_{j_1=0}^{j}\sum_{j_2=0}^{b-j}\sum_{v\in S_{j_1,j_2}(c)} \chi\left(\langle c,v\rangle\right) z^{\lceil (j_1+j_2)/t\rceil}.$$

□

Lemma 2.8 *Let C be a linear code over R and $C^{\perp}$ its dual code and*

$$\hat{f}(u) = \sum_{v\in R^{nb}} \chi\left(\langle u,v\rangle\right) f(v).$$

Then,

$$\sum_{v\in C^{\perp}} f(v) = \frac{1}{|C|}\sum_{u\in C}\hat{f}(u).$$

Proof.

$$\begin{aligned}\sum_{u\in C}\hat{f}(u) &= \sum_{u\in C}\sum_{v\in R^{nb}}\chi(\langle u,v\rangle)f(v)\\ &= \sum_{u\in C}\sum_{v\in C^{\perp}}\chi(\langle u,v\rangle)f(v)+\sum_{u\in C}\sum_{v\in R^{nb}\setminus C^{\perp}}\chi(\langle u,v\rangle)f(v)\\ &= |C|\sum_{v\in C^{\perp}}f(v)+\sum_{v\in R^{nb}\setminus C^{\perp}}\sum_{u\in C}\chi(\langle u,v\rangle)f(v).\end{aligned}$$

Now, for fixed $v\in R^{nb}\setminus C^{\perp}$ and for all $c\in C$ let $\phi_v(c)=\langle c,v\rangle$. Since ϕ_v is an R-module homomorphism, $\phi_v(C)$ is a nonzero ideal of R. Hence, by Lemma 2.1, $\sum_{u\in C}\chi(\langle u,v\rangle)=0$. Therefore, the second double sum in the last line of the equations equals zero, hence we get the required result. □

Theorem 2.1 *Let C be a linear code. The relation between the m-spotty t/b- weight enumerators of C and its dual is given by*

$$\sum_{\alpha_0+\alpha_1+\cdots+\alpha_b=n}A_{\overline{\alpha}}^{\perp}\prod_{j=0}^{b}\left(z^{\lceil j/t\rceil}\right)^{\alpha_j}=\frac{1}{|C|}\sum_{\alpha_0+\alpha_1+\cdots+\alpha_b=n}A_{\overline{\alpha}}\prod_{j=0}^{b}\left(F_j(z)\right)^{\alpha_j}$$

where

$$F_j(z)=\sum_{j_1=0}^{j}\sum_{j_2=0}^{b-j}(-1)^{j_1}3^{j_2}\binom{j}{j_1}\binom{b-j}{j_2}z^{\lceil(j_1+j_2)/t\rceil}.$$

Proof. In Lemma 2.8, we set $f(v)=\prod_{i=1}^{n}z^{\lceil w(v_i)/t\rceil}$ where v_i represents the i th byte of v. Then,

$$\begin{aligned}\hat{f}(c) &= \sum_{v\in R^{nb}}\chi(\langle c,v\rangle)\prod_{i=1}^{n}z^{\lceil w(v_i)/t\rceil}\\ &= \sum_{v_1\in R^b}\sum_{v_2\in R^b}\cdots\sum_{v_n\in R^b}\chi(\langle c_1,v_1\rangle)\chi(\langle c_2,v_2\rangle)\cdots\chi(\langle c_n,v_n\rangle)\prod_{i=1}^{n}z^{\lceil w(v_i)/t\rceil}.\end{aligned}$$

Hence,

$$\hat{f}(c)=\prod_{i=1}^{n}\left(\sum_{v_i\in R^b}\chi(\langle c_i,v_i\rangle)z^{\lceil w(v_i)/t\rceil}\right).$$

By Lemma 2.7, we have

$$\hat{f}(c)=\prod_{i=1}^{n}\left(\sum_{j_1=0}^{k_i}\sum_{j_2=0}^{b-k_i}(-1)^{j_1}3^{j_2}\binom{k_i}{j_1}\binom{b-k_i}{j_2}z^{\lceil(j_1+j_2)/t\rceil}\right)$$

where $k_i = w(c_i)$. Thus,

$$\hat{f}(c) = \prod_{j=0}^{b} \left(\sum_{j_1=0}^{j} \sum_{j_2=0}^{b-j} (-1)^{j_1} 3^{j_2} \binom{j}{j_1} \binom{b-j}{j_2} z^{\lceil (j_1+j_2)/t \rceil} \right)^{\alpha_j(c)}$$

where $\alpha_j(c) = |\{i|\ w(c_i) = j\}|$.

$$\sum_{v \in C^{\perp}} f(v) = \frac{1}{|C|} \sum_{c \in C} \prod_{j=0}^{b} (F_j(z))^{\alpha_j(c)}.$$

Therefore,

$$\sum_{v \in C^{\perp}} f(v) = \frac{1}{|C|} \sum_{\substack{\alpha_0, \alpha_1, \ldots, \alpha_b \geq 0 \\ \alpha_0 + \alpha_1 + \cdots + \alpha_b = n}} A_{\overline{\alpha}} \prod_{j=0}^{b} (F_j(z))^{\alpha_j}.$$

□

Here, we give a moderate example in order to illustrate the theorem.

Example 2.1 *Let*

$$G = \begin{bmatrix} 1 & 0 & u & 1 & u & 0 \\ 0 & 1 & 1 & u & 0 & u \end{bmatrix}$$

be the generator matrix of a linear code C of length 6. C is a free submodule and it has 16 codewords. The dual of C is a linear code of length 6 also and it has $4^4 = 256$ codewords. The necessary computations in order to apply the main theorem are listed in Tables 1 and 2.

Table 1. The codewords and bitwise Hamming weights where $n = 2$ and $b = 3$.

Codewords	$(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$	Codewords	$(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$
$v_1 = (0,0,0,0,0,0)$	(2,0,0,0)	$v_9 = (1,0,u,1,u,0)$	(0,0,2,0)
$v_2 = (0,u,u,0,0,0)$	(1,0,1,0)	$v_{10} = (1,u,0,1,u,0)$	(0,0,2,0)
$v_3 = (0,1,1,u,0,u)$	(0,0,2,0)	$v_{11} = (1,1,1+u,1+u,u,u)$	(0,0,0,2)
$v_4 = (0,1+u,1+u,u,0,u)$	(0,0,2,0)	$v_{12} = (1,1+u,1,1+u,u,u)$	(0,0,0,2)
$v_5 = (u,0,0,u,0,0)$	(0,2,0,0)	$v_{13} = (1+u,0,u,1+u,u,0)$	(0,0,2,0)
$v_6 = (u,u,u,u,0,0)$	(0,1,0,1)	$v_{14} = (1+u,u,0,1+u,u,0)$	(0,0,2,0)
$v_7 = (u,1,1,0,0,u)$	(0,1,0,1)	$v_{15} = (1+u,1,1+u,1,u,u)$	(0,0,0,2)
$v_8 = (u,1+u,1+u,0,0,u)$	(0,1,0,1)	$v_{16} = (1+u,1+u,1,1,u,u)$	(0,0,0,2)

Hence, by Theorem 2.1, the m-spotty 2/3-weight enumerator of the dual code is

$$W_{C^{\perp}}(z) = \sum_{\alpha_0+\alpha_1+\alpha_2+\alpha_3=2} A_{\overline{\alpha}} \prod_{j=0}^{3} (F_j(z))^{\alpha_j} = \frac{1}{|C|} \left(16 + 96z + 1376z^2 + 1632z^3 + 976z^4\right).$$

Therefore,

$$W_{C^{\perp}}(z) = 1 + 6z + 86z^2 + 102z^3 + 61z^4.$$

Table 2. The codewords and the corresponding terms.

Codewords	$F_0(z)F_1(z)F_2(z)F_3(z)$
v_1	$F_0(z)F_0(z) = 1 + 72z + 1350z^2 + 1944z^3 + 729z^4$
v_2	$F_2(z)F_0(z) = 1 + 32z - 114z^2 + 81z^4$
$v_i \quad (i = 3, 4, 9, 10, 13, 14)$	$F_2(z)F_2(z) = 1 - 8z + 22z^2 - 24z^3 + 9z^4$
v_5	$F_1(z)F_1(z) = 1 + 16z + 46z^2 - 144z^3 + 81z^4$
$v_i \quad (i = 6, 7, 8)$	$F_1(z)F_3(z) = 1 + 8z - 10z^2 - 8z^3 + 9z^4$
$v_i \quad (i = 11, 12, 15, 16)$	$F_3(z)F_3(z) = 1 - 2z^2 + z^4$

$F_0(z) = 1 + 36z + 27z^2$,$F_1(z) = 1 + 8z - 9z^2$, $F_2(z) = 1 - 4z + 3z^2$, and $F_3(z) = 1 - z^2$.

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