Turkish Journal of Mathematics

http://journals.tubitak.gov.tr/math/

Research Article

Turk J Math
(2013) 37: 585 – 591
© TÜBİTAK
doi:10.3906/mat-1106-13

Finitistic Dimension Conjectures for representations of quivers

Sergio ESTRADA,[1] **Salahattin ÖZDEMİR**[2,*]
[1]Department of Applied Mathematics, University of Murcia, Murcia, Spain
[2]Department of Mathematics, Faculty of Sciences, Dokuz Eylül University, Buca, İzmir, Turkey

Received: 09.06.2011 • **Accepted:** 08.07.2012 • **Published Online:** 12.06.2013 • **Printed:** 08.07.2013

Abstract: Let R be a ring and Q be a quiver. We prove the first Finitistic Dimension Conjecture to be true for RQ, the path ring of Q over R, provided that R satisfies the conjecture. In fact, we prove that if the little and the big finitistic dimensions of R coincide and equal $n < \infty$, then this is also true for RQ and, both the little and the big finitistic dimensions of RQ equal $n+1$ when Q is non-discrete and n when Q is discrete. We also prove that RQ is a quasi-Frobenius ring if and only if R is quasi-Frobenius and Q is discrete.

Key words: Finitistic dimension conjecture, path ring, quasi-Frobenius ring, quiver representation

1. Introduction

In 1960 ([2]), Bass published the so-called *Finitistic Dimension Conjectures*: For a finite dimensional algebra Λ, (I) $\operatorname{findim}\Lambda = \operatorname{Findim}\Lambda$ and (II) $\operatorname{findim}\Lambda < \infty$, where

$$\operatorname{findim}\Lambda = \sup\{\operatorname{pd} M \mid \text{ M is a finitely generated left } \Lambda\text{-module with } \operatorname{pd} M < \infty\}$$

is the *(left) little finitistic dimension of* Λ, and

$$\operatorname{Findim}\Lambda = \sup\{\operatorname{pd} M \mid \text{ M is a left } \Lambda\text{-module with } \operatorname{pd} M < \infty\}$$

is the *(left) big finitistic dimension of* Λ. The first conjecture was proved to be false in [13]. In fact, it was proved that for any field k and any integer $n \geq 2$, there exist finite dimensional k-algebras Λ such that $\operatorname{findim}\Lambda = n$, while $\operatorname{Findim}\Lambda = n+1$. Also, it has been proved to be true, for instance, for left perfect rings when the little finitistic dimension is zero ([2]), and for Iwanaga-Gorenstein rings ([8]). However, the second conjecture still remains open. It has been proved to be true, for example, for finite dimensional monomial algebras ([7]), for Artin algebras with vanishing cube radical ([14]), or Artin algebras with representation dimension bounded by 3 ([9]).

Our goal in this paper is to provide a partial positive solution to the first Finitistic Dimension Conjecture. Let R be any ring and Q be any quiver. We prove that the path ring of Q over R, denoted by RQ, does satisfy the first Finitistic Dimension Conjecture provided that R satisfies the conjecture below, Theorem 3.9. In fact, we prove that if $\operatorname{Findim}(R) = \operatorname{findim}(R) = n(<\infty)$, then (i) $\operatorname{Findim}(RQ) = \operatorname{findim}(RQ) = n+1$ when Q is non-discrete, and (ii) $\operatorname{Findim}(RQ) = \operatorname{findim}(RQ) = n$ when Q is discrete. In particular, we infer from

*Correspondence: salahattin.ozdemir@deu.edu.tr
2010 *AMS Mathematics Subject Classification:* 16G20, 18G20.

Theorem 3.2 in [8] that if R is n-Gorenstein ring, that is a two-sided noetherian ring of finite self-injective dimension n on both sides, then RQ satisfies the first Finitistic Dimension Conjecture. Moreover, we prove that RQ is a quasi-Frobenius ring if and only if R is a quasi-Frobenius ring and Q is a discrete quiver, where R is called *quasi-Frobenius* if a left R-module is projective if and only if it is injective (or, equivalently, R is a 0-Gorenstein ring) (Proposition 3.10).

2. Preliminaries

A *quiver* is a directed graph whose edges are called arrows. As usual we denote a quiver by Q, understanding that $Q = (V, E)$ where V is the set of vertices and E is the set of arrows. An arrow of a quiver from a vertex v_1 to a vertex v_2 is denoted by $a : v_1 \to v_2$. In this case we write $s(a) = v_1$ as the initial (starting) vertex, and $t(a) = v_2$ as the terminal (ending) vertex. An arrow a in which $s(a) = t(a)$ is called a *loop*. A quiver is said to be *discrete* if it has no arrows. A *path* p of a quiver Q is a sequence of arrows $a_n \cdots a_2 a_1$ with $t(a_i) = s(a_{i+1})$. Thus $s(p) = s(a_1)$ and $t(p) = t(a_n)$. Two paths p and q can be composed, getting another path qp (or pq) whenever $t(p) = s(q)$ ($t(q) = s(p)$).

A quiver Q may be thought as a category in which the objects are the vertices of Q and the morphisms are the paths of Q.

A representation by modules X of a given quiver Q is a functor $X : Q \longrightarrow R\text{-}\mathcal{M}od$. Such a representation is determined by giving a module $X(v)$ to each vertex v of Q and a homomorphism $X(a) : X(v_1) \to X(v_2)$ to each arrow $a : v_1 \to v_2$ of Q. A morphism η between two representations X and Y is a natural transformation, so it will be a family η_v such that $Y(a) \circ \eta_{v_1} = \eta_{v_2} \circ X(a)$ for any arrow $a : v_1 \to v_2$ of Q. Thus, the representations of a quiver Q by modules over a ring R is a category, denoted by $(Q, R\text{-}\mathcal{M}od)$.

For a given quiver Q and a ring R, the path ring of Q over R, denoted by RQ, is defined as the free left R-module, whose base are the paths p of Q, and where the multiplication is the obvious composition between two paths. This is a ring with enough idempotents, so in fact it is a ring with local units (see [12, Ch.10, §49]). We denote by $RQ\text{-}\mathcal{M}od$ the category of unital RQ-modules (i.e. ${}_{RQ}M$ such that $RQM = M$). It is known that RQ is a projective generator of the category and that the categories $RQ\text{-}\mathcal{M}od$ and $(Q, R\text{-}\mathcal{M}od)$ are equivalent categories, and so $(Q, R\text{-}\mathcal{M}od)$ is a Grothendieck category with enough projectives.

For a given quiver Q, one can define a family of projective generators from an adjoint situation as it was shown in [10]. For every vertex $v \in V$ and the embedding morphism $\{v\} \subseteq Q$ the family $\{S_v(R) : v \in V\}$ is a family of projective generators of Q where the functor $S_v : R\text{-}\mathcal{M}od \longrightarrow (Q, R\text{-}\mathcal{M}od)$ is defined in [10, §28] as $S_v(M)(w) = \oplus_{Q(v,w)} M$ where $Q(v, w)$ is the set of paths of Q starting at v and ending at w. Then S_v is the left adjoint functor of the evaluation functor $T_v : (Q, R\text{-}\mathcal{M}od) \longrightarrow R\text{-}\mathcal{M}od$ given by $T_v(X) = X(v)$ for any representation $X \in (Q, R\text{-}\mathcal{M}od)$. There is also an algorithm for providing injective cogenerators in $(Q, R\text{-}\mathcal{M}od)$ due to [5].

Throughout the paper, by a representation of a quiver we will mean a representation by modules over a ring R. The letter R will usually denote a nontrivial associative ring with identity and not necessarily commutative. All modules will be unitary left R-modules, unless otherwise specified. The category of left R-modules will be denoted by $R\text{-}\mathcal{M}od$. By pd and id we denote the *projective dimension* and the *injective dimension* respectively. We refer to [6], [5] and [1] for any undefined notion used in the text.

3. Finitistic Dimension Conjectures over path rings

Any non-discrete quiver Q must contain proper or loop arrows. So the main idea of our proof is to consider first that Q has an arrow that is not a loop and then that Q has a loop.

In the proof of Lemma 3.1, we use the following result: Let Q be the quiver consisting of a single vertex v and infinitely many loops $\alpha_1, \alpha_2, \ldots$. Then the defining basis of the path ring RQ is the set of all words on $\{\alpha_1, \alpha_2, \ldots\}$ with the empty word equal to the trivial path v; this is the identity of RQ. Thus, $RQ \cong R\{x_1, x_2, \ldots\}$ where the elements of $R\{x_1, x_2, \ldots\}$ are non-commuting polynomials in indeterminates $\{x_1, x_2, \ldots\}$ with coefficients in R. The isomorphism being induced by the R-linear map such that $v \mapsto 1_R$ and $\alpha_k \mapsto x_k$ for all $k = 1, 2, \ldots$

Lemma 3.1 *Let Q be any non-discrete quiver. If Q contains an arrow (respectively, a loop), then any projective representation over Q is also projective when it is restricted to the quiver $Q_1 \equiv\ v_1 \xrightarrow{a} v_2$ (respectively, Q_2, a quiver with one vertex and one loop).*

Proof Let P be a projective representation over Q. Then $\oplus_{t(a)=v} P(s(a)) \to P(v)$ is always a splitting monomorphism and $P(v)$ is a projective R-module for all $v \in V$ (for any quiver Q). So, in particular, for the quiver Q_1, $P(v_1) \xrightarrow{P(a)} P(v_2)$ will be a splitting monomorphism. Thus it is a projective representation over Q_1 (since Q_1 is left rooted) (see [3]). Now for the quiver Q_2, we may assume that Q contains a vertex with infinitely many loops, and that Q_2 is one of these loops. Then we have that

$$(Q, R\text{-}\mathcal{M}od) \cong R\{x_1, x_2, \ldots\} - \mathrm{Mod},$$

where the elements of $R\{x_1, x_2, \ldots\}$ are non-commuting polynomials in indeterminates $\{x_1, x_2, \ldots\}$ with coefficients in R. Without loss of generality we may assume that $(Q_2, R\text{-}\mathcal{M}od) \cong R[x_1]\text{-}\mathcal{M}od$. Now if P is a projective representation over Q, that is, a projective $R\{x_1, x_2, \ldots\}$-module, then $P \oplus L = R\{x_1, x_2, \ldots\}^{(I)}$ for some representation L over Q and an index set I. But,

$$R\{x_1, x_2, \ldots\} \cong \bigoplus_{p \in {}_{x_1}<X>} R[x_1] \cdot p$$

as $R[x_1]$- modules, where $X = \{x_1, x_2, \ldots\}$, $<X>$ is the free monoid of words on X, and ${}_{x_1}<X>$ is the submonoid of $<X>$ of words which does not start by x_1 (notice that if p is the empty word, then we set $R[x_1] \cdot p = R[x_1]$). Since $R[x_1] \cdot p \cong R[x_1]$ as $R[x_1]$-modules, then $\oplus_{p \in {}_{x_1}<X>} R[x_1] \cdot p$ is $R[x_1]$-projective. So P is $R[x_1]$-projective as a direct summand, or equivalently P is a projective representation in $(Q_2, R\text{-}\mathcal{M}od)$. □

Lemma 3.2 *Let $Q_1 = (V_1, E_1)$ be a subquiver of $Q = (V, E)$. Assume that every projective representation over Q is also projective when it is restricted to Q_1. Let M be any representation of Q_1. If $\mathrm{pd}_{Q_1}(M) = n$, then $\mathrm{pd}_Q(\widetilde{M}) \geq n$, where $\widetilde{M}$ is the following representation of Q: $\widetilde{M}(v) = M(v)\ \forall v \in V_1$, $\widetilde{M}(v) = 0\ \forall v \in V - V_1$, and $\widetilde{M}(a) = M(a)\ \forall a \in E_1$, $\widetilde{M}(a) = 0\ \forall a \in E - E_1$.*

Proof Suppose for the contrary that $r = \mathrm{pd}_Q(\widetilde{M}) < n$. Then there exists an exact sequence in $(Q, R\text{-}\mathcal{M}od)$

$$0 \to P_r \to \cdots \to P_1 \to P_0 \to \widetilde{M} \to 0,$$

with P_i, $0 \leq i \leq r$ being projective representations over Q. Now by the assumption,

$$0 \to P_r \mid_{Q_1} \to \cdots \to P_1 \mid_{Q_1} \to P_0 \mid_{Q_1} \to \widetilde{M} \mid_{Q_1} = M \to 0$$

is a projective resolution of M in $(Q, R\text{-}\mathcal{M}od)$ with $r < n$. This contradicts with $\mathrm{pd}_{Q_1}(M) = n$. □

Lemma 3.3 *Let M be a module such that $\mathrm{pd}_R(M) = n$. Then:*

1. $\mathrm{pd}_Q(\overline{M}) = n + 1$, *where $\overline{M} \equiv M \to 0$ is a representation of the quiver $Q \equiv \bullet \to \bullet$.*
2. $\mathrm{pd}_{R[x]}(M) = n + 1$, *where $R[x]$ is a polynomial ring and M is considered as an $R[x]$-module (via $xM = 0$).*

Proof

1. Let us fix the notation $\overline{X} \equiv X \to 0$ for any module X in this proof. We show that $\mathrm{pd}_Q(\overline{M}) = n + 1$ by induction on n. For $n = 0$, M will be projective. So from the commutative diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & 0 & \longrightarrow & M & \xrightarrow{id} & M & \longrightarrow & 0 \\ & & \downarrow & & \downarrow{\scriptstyle id} & & \downarrow{\scriptstyle 0} & & \\ 0 & \longrightarrow & M & \xrightarrow{id} & M & \longrightarrow & 0 & \longrightarrow & 0 \end{array}$$

it follows that $\mathrm{pd}_Q(\overline{M}) = 1$. (Indeed, $\overline{M}$ cannot be projective since $M \xrightarrow{0} 0$ cannot be a splitting epimorphism.) Now suppose that $\mathrm{pd}_R(M) = n$. Then there exist sequences of modules $0 \to P_n \to \cdots \to P_1 \to K \to 0$ and $0 \to K \to P_0 \to M \to 0$, where P_i is a projective module for every $0 \leq i \leq n$ and $K = \mathrm{Ker}(P_0 \to M)$. Since $\mathrm{pd}_R(K) = n - 1$, then by induction hypothesis $\mathrm{pd}_Q(\overline{K}) = n$. Since P_0 is a projective module, we know that $\mathrm{pd}_Q(\overline{P_0}) = 1$. Therefore from the short exact sequence of representations $0 \to \overline{K} \to \overline{P_0} \to \overline{M} \to 0$ we get the long exact sequence of homology

$$\cdots \to 0 = \mathrm{Ext}^n_Q(\overline{P_0}, X) \to \mathrm{Ext}^n_Q(\overline{K}, X) \to \mathrm{Ext}^{n+1}_Q(\overline{M}, X) \to$$

$$\to \mathrm{Ext}^{n+1}_Q(\overline{P_0}, X) = 0 \to \mathrm{Ext}^{n+1}_Q(\overline{K}, X) = 0 \to \mathrm{Ext}^{n+2}_Q(\overline{M}, X) \to$$

$$\to \mathrm{Ext}^{n+2}_Q(\overline{P_0}, X) = 0 \to \cdots .$$

So, we have $\mathrm{Ext}^{n+j}_Q(\overline{M}, X) = 0$, $\forall j \geq 2$ and for any representation X. Since $\mathrm{pd}_Q(\overline{K}) = n$, then $0 \neq \mathrm{Ext}^n_Q(\overline{K}, X) \cong \mathrm{Ext}^{n+1}_Q(\overline{M}, X)$, and this implies that $\mathrm{pd}_Q(\overline{M}) = n + 1$.

2. We show that $\mathrm{pd}_{R[x]}(M) = n+1$ by induction on n. If $n = 0$, then by [11, Lemma 9.27] $\mathrm{pd}_{R[x]}(M[x]) = \mathrm{pd}_R(M) = 0$, that is, $M[x]$ is $R[x]$-projective, where $M[x] = R[x] \otimes_R M$. Moreover, we have a short exact sequence of $R[x]$-modules $0 \to M[x] \to M[x] \to M \to 0$ (see [11, Lemma 9.29]). So, $\mathrm{pd}_{R[x]}(M) = 1$ since M cannot be $R[x]$-projective (otherwise the sequence would split, but it is impossible). Now suppose that $\mathrm{pd}_R(M) = n$ where $n \geq 2$. Then we have a short exact sequence of R-modules $0 \to K \to P \to M \to 0$

where P is projective and $\mathrm{pd}_R(K) = n-1$. So by induction hypothesis, we know that $\mathrm{pd}_{R[x]}(K) = n$ and $\mathrm{pd}_{R[x]}(P) = 1$ (via $xK = 0$ and $xP = 0$). In fact, we can consider this short exact sequence over $R[x]$-modules, and from this sequence we get the long exact sequence of $R[x]$-modules:

$$\cdots \to 0 = \mathrm{Ext}^n(P,Y) \to \mathrm{Ext}^n(K,Y) \to \mathrm{Ext}^{n+1}(M,Y) \to \mathrm{Ext}^{n+1}(P,Y) = 0$$

$$\to \mathrm{Ext}^{n+1}(K,Y) = 0 \to \mathrm{Ext}^{n+2}(M,Y) \to \mathrm{Ext}^{n+2}(P,Y) = 0 \to \cdots .$$

Then we obtain that $\mathrm{Ext}^{n+j}(M,Y) = 0$ $\forall j \geq 2$ and for any $R[x]$-module Y. Since $\mathrm{pd}_{R[x]}(K) = n$, then $0 \neq \mathrm{Ext}^n(K,Y) \cong \mathrm{Ext}^{n+1}(M,Y)$. Hence $\mathrm{pd}_{R[x]}(M) = n+1$.

□

Lemma 3.4 *Let Q be any quiver. If* $\mathrm{Findim}(R) = n$*, then* $\mathrm{Findim}(RQ) \leq n+1$.

Proof Let X be any representation in $(Q, R\text{-}\mathcal{M}od)$ such that $\mathrm{pd}_Q(X) < \infty$. Then it is clear that $\mathrm{pd}_R(X(v)) < \infty$, and so $\mathrm{pd}_R(X(v)) \leq n$ for any vertex v of Q. Now from the exact sequence

$$0 \to K \to P_{n-1} \to \cdots \to P_1 \to P_0 \to X \to 0$$

of representations of Q where P_i is a projective representation $\forall 0 \leq i \leq n-1$ and $K = \mathrm{Ker}(P_{n-1} \to P_{n-2})$, we obtain an exact sequence of modules

$$0 \to K(v) \to P_{n-1}(v) \to \cdots \to P_1(v) \to P_0(v) \to X(v) \to 0$$

for any vertex v of Q. Then $K(v)$ must be a projective module since $\mathrm{pd}_R(X(v)) \leq n$. Moreover, by the argument given in the proof of [10, Corollary 28.3], we have the short exact sequence

$$0 \to \oplus_{a\in E} S_{t(a)}(K(s(a))) \to \oplus_{v\in V} S_v(K(v)) \to K \to 0$$

for the representation K (in fact, it exists for every representation in $(Q, R\text{-}\mathcal{M}od)$). Since the functor S preserves projectives, it follows that $\mathrm{pd}_Q(X) \leq n+1$, and this implies that $\mathrm{Findim}(RQ) \leq n+1$. □

Proposition 3.5 *Let Q be a non-discrete quiver. If* $\mathrm{Findim}(R) = n$*, then* $\mathrm{Findim}(RQ) = n+1$.

Proof By Lemma 3.4, $\mathrm{Findim}(RQ) \leq n+1$. Now let M be an R-module such that $\mathrm{pd}_R(M) = n$. Since Q is a non-discrete quiver, we can assume that it contains an arrow, say Q_1 or a loop, say Q_2. So in this case, by Lemma 3.3, there exists a representation M_1 of Q_1 (resp., M_2 of Q_2) such that $\mathrm{pd}_{Q_1}(M_1) = n+1$ (resp., $\mathrm{pd}_{Q_2}(M_2)$ = n+1). (Notice that $(Q_2, R\text{-}\mathcal{M}od) \cong R[x]$-Mod.) Thus by Lemmas 3.1 and 3.2, it follows that $\mathrm{pd}_Q(\widetilde{M_1}) \geq n+1$ (resp. $\mathrm{pd}_Q(\widetilde{M_2}) \geq n+1$), where $\widetilde{M_1}, \widetilde{M_2}$ are the representations given in Lemma 3.2. Hence $\mathrm{Findim}(RQ) \geq n+1$.

□

Corollary 3.6 *Let Q be a non-discrete quiver. If R is ring such that* $\mathrm{Findim}(R) = \mathrm{findim}(R) = n(< \infty)$*, then* $\mathrm{Findim}(RQ) = \mathrm{findim}(RQ) = n+1$.

Corollary 3.7 *Let Q be a non-discrete quiver. If R is n-Gorenstein ring, then* $\mathrm{Findim}(RQ) = \mathrm{findim}(RQ) = n+1$.

Proof Since R is an n-Gorenstein ring, $\mathrm{Findim}(R) = \mathrm{findim}(R) = n$ by [8, Theorem 3.2]. So the result follows by Corollary 3.6. □

Remark 3.8 *If the quiver Q is discrete, then a representation X of Q is projective if and only if $X(v)$ is a projective module for all $v \in V$. So, if* $\mathrm{Findim}(R) = \mathrm{findim}(R) = n$, *then it is immediate that* $\mathrm{Findim}(RQ) = \mathrm{findim}(RQ) = n$.

Theorem 3.9 *Let Q be any quiver. If R is a ring such that* $\mathrm{Findim}(R) = \mathrm{findim}(R) = n(< \infty)$, *then* $\mathrm{Findim}(RQ) = \mathrm{findim}(RQ)(< \infty)$.

Proof By Corollary 3.6 and Remark 3.8. □

Recall that a ring R is called *quasi-Frobenius* (briefly, a *QF-ring*) if a left R-module is projective if and only if it is injective. This notion can be extended to the path ring of any quiver Q. So we call RQ a quasi-Frobenius ring if a representation of $(Q, R\text{-}\mathcal{M}od)$ is projective if and only if it is injective.

Proposition 3.10 *Let Q be a quiver. Then RQ is a QF-ring if and only if R is a QF-ring and Q is discrete.*

Proof ($\Rightarrow$) Suppose on the contrary that Q is not discrete. Then we have two cases:

1. Let Q contain an arrow $v_1 \xrightarrow{a} v_2$. We know that $\oplus_{v\in V} S_v(R)$ is a projective generator of $(Q, R\text{-}\mathcal{M}od)$. So, the induced representation

$$S_v(R)(v_1) \xrightarrow{S_v(R)(a)} S_v(R)(v_2) \equiv \oplus_{Q(v,v_1)} R \longrightarrow \oplus_{Q(v,v_2)} R$$

   of the quiver $v_1 \xrightarrow{a} v_2$ is projective (see [3, Theorem 3.1]). But since $Q(v, v_1) \subsetneqq Q(v, v_2)$, $S_v(R)(a)$ cannot be a splitting epimorphism, and so the representation cannot be injective (see [4, Theorem 4.2]).

2. Let Q contain loops with one vertex. Then $(Q, R\text{-}\mathcal{M}od) \cong R\{x_1, x_2, \ldots\}$-Mod, where the elements of $R\{x_1, x_2, \ldots\}$ are non-commuting polynomials in indeterminates $\{x_1, x_2, \ldots\}$ with coefficients in R. So $R\{x_1, x_2, \ldots\}$ is a projective module over itself, but not injective since it is not divisible (for instance, x_1 doesn't have an inverse).

So in each case we have a contradiction with RQ to be a QF-ring. Hence Q must be a discrete quiver. Now it is easy to notice that a representation P of $(Q, R\text{-}\mathcal{M}od)$ is projective (resp., injective) if and only if $P(v)$ is a projective left R-module, for each $v \in V$ (resp., an injective left R-module, for each $v \in V$). For instance this can be derived from the fact that a discrete quiver is, in particular, left rooted and right rooted, so we can use the characterization of projective (resp., injective) representations given in [3, Theorem 3.1] (resp., [4, Theorem 4.2]). Hence, if M is a projective (resp., an injective) R-module then we can easily construct a projective (resp., injective) representation X in $(Q, R\text{-}\mathcal{M}od)$ by the assignment $X(v) = M$ and $X(w) = 0$, $\forall w \in V, w \neq v$ (where v is any fixed vertex of Q). Then by the hypothesis X will be an injective (resp., a projective) representation in $(Q, R\text{-}\mathcal{M}od)$, that is, M will be an injective (resp., a projective) left R-module. So R is a QF-ring.

($\Leftarrow$) It follows from the previous observation on projective (resp., injective) representations of discrete quivers. $\square$

Acknowledgments

The first author has been partially supported by MTM 2010-20940-C02-02 by the fundacion Seneca GERM and by the Junta de Andalucia and FEDER funds. The second author has been partially supported by the Council of Higher Education (YÖK) and by the Scientific Technological Research Council of Turkey (TÜBİTAK).

The second author wishes to acknowledge the hospitality of the University of Murcia during his stay as a visiting researcher in the Department of Applied Mathematics. The authors wish to thank the referee for his/her valuable suggestions.

References

[1] Assem, I., Simson, D., Skowroński, A.: Elements of the representation theory of associative algebras. Vol. 1, vol. 65 of London Mathematical Society Student Texts. Cambridge: Cambridge University Press, ISBN 978-0-521-58423-4; 978-0-521-58631-3; 0-521-58631-3. Techniques of representation theory 2006.

[2] Bass, H.: Finitistic dimension and a homological generalization of semi-primary rings. Trans. Amer. Math. Soc. 95, 466–488 (1960).

[3] Enochs, E., Estrada, S.: Projective representations of quivers. Comm. Algebra. 33(10), 3467–3478 (2005).

[4] Enochs, E., Estrada, S., García Rozas, J. R.: Injective representations of infinite quivers. Applications. Canad. J. Math. 61, 315–335 (2009).

[5] Enochs, E. E., Herzog, I.: A homotopy of quiver morphisms with applications to representations. Canad. J. Math. 51, 294–308 (1999).

[6] Enochs, E. E., Jenda, O. M. G.: Relative homological algebra. vol. 30 of de Gruyter Expositions in Mathematics. Berlin: Walter de Gruyter & Co., ISBN 3-11-016633-X, 2000.

[7] Green, E. L., Kirkman, E., Kuzmanovich, J.: Finitistic dimensions of finite-dimensional monomial algebras. J. Algebra. 136(1), 37–50 (1991).

[8] Hügel, A. L., Herbera, D., Trlifaj, J.: Tilting modules and Gorenstein rings. Forum Math. 18(2), 211–229 (2006).

[9] Igusa, K., Todorov, G.: On the finitistic global dimension conjecture for Artin algebras. In Representations of algebras and related topics. vol. 45 of Fields Inst. Commun. Providence, RI: Amer. Math. Soc. 201–204 (2005). Graduate Texts in Mathematics. New York: Springer-Verlag 1999.

[10] Mitchell, B.: Rings with several objects. Advances in Math. 8, 1–161 (1972).

[11] Rotman, J.: An Introduction to Homological Algebra. New York: Academic Press 1979.

[12] Wisbauer, R.: Foundations of Module and Ring Theory. Reading: Gordon and Breach 1991.

[13] Zimmermann-Huisgen, B.: Homological domino effects and the first finitistic dimension conjecture. Invent. Math. 108(2), 369–383 (1992).

[14] Zimmermann-Huisgen, B.: Bounds on global and finitistic dimension for finite-dimensional algebras with vanishing radical cube. J. Algebra. 161(1), 47–68 (1993).