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Central configurations in the collinear 5-body problem

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Abstract: We study the inverse problem of central configuration of collinear general 4- and 5-body problems. A central configuration for n-body problems is formed if the position vector of each particle with respect to the center of mass is a common scalar multiple of its acceleration. In the 3-body problem, it is always possible to find 3 positive masses for any given 3 collinear positions given that they are central. This is not possible for more than 4-body problems in general. We consider a collinear 5-body problem and identify regions in the phase space where it is possible to choose positive masses that will make the configuration central. In the symmetric case we derive a critical value for the central mass above which no central configurations exist. We also show that in general there is no such restriction on the value of the central mass.

Key words: Central configuration, n-body problem, inverse problem of central configuration

1. Introduction

Central configurations are one of the most important and fundamental topics in the study of few-body problems. Therefore, few-body problems in general and central configurations in particular have attracted a lot of attention over the years [4],[5],[10]. Studies on the central configuration of n-body problems (with $n \geq 4$) are limited due to the greater complexity of problems involving higher numbers of bodies. For $n \geq 4$, the main focus of the available literature is on the restricted problems; see, for example [2],[7], and [9]. This opens up a window to study the central configuration of a general 5-body problem. Hence, in this present study, we adapt a method presented in [6] to study the central configuration of general collinear 4- and 5-body problems.

Several methods and restriction techniques have been used to study the few-body problem. For example, Roberts discussed the relative equilibria for a special case of the 5-body problem in [8], which consists of 4 bodies, i.e. $(m_1, m_2,\ m_3, m_4) = (1, 1, 1, 1)$ at the vertices of a rhombus, with opposite vertices having the same mass, and a central body, i.e. m_5 at $-1/4$. Roberts showed the existence of a 1-parameter family of degenerate relative equilibria where the 4 equal masses are positioned at the vertices of a rhombus with the remaining body located at the center. Albouy and Llibre in [1] discussed the central configurations of the 1+4-body problem. They kept 4 equal masses on a sphere whose center is the 'big' mass. They found 4 symmetric central configurations and proved that they all have at least 1 plane of symmetry.

More recently in [3], Hampton and Jensen showed that in the 5-body problem the number of spatial central configurations is finite, except for some special cases. Ouyang and Xie in [6] considered the inverse problem

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of central configurations of collinear 4-bodies and identified possible conditions to choose positive masses while maintaining a central configuration. The authors established an expression for the 4 masses depending on the position x and the center of mass u, which give central configurations in the collinear 4-body problem. We model our problem on similar lines and propose a method to derive central configurations for a collinear 5-body problem. The proposed model has the fifth mass fixed at the center of mass. The rest of the paper is organized as follows. In Section 2, general equations are derived for the 5-body collinear central configurations. In Section 3, we use these equations to discuss the fully symmetric case of the proposed 5-body problem and derive a critical value for the central mass above which no central configurations are possible. In Section 4, we discuss the most general form of the proposed problem and derive its central configuration regions. Conclusions are given in Section 5.

2. General equations for 5-body collinear central configurations

The classical equation of motion for the n-body problem has the form

$$m_i \frac{d^2\vec{q}_i}{dt^2} = \sum_{j\neq i} \frac{m_i m_j \left(\vec{q}_i - \vec{q}_j\right)}{|\vec{q}_i - \vec{q}_j|^3} \qquad i = 1, 2, ..., n, \tag{1}$$

where the units are chosen so that the gravitational constant is equal to one and $q_i \in R^d (1 \leq d \leq 3), i = 1, 2, ..., n$ represents the positions in Euclidean space R^d of n masses m_i.

A *central configuration* $\mathbf{q} = (\vec{q}_1, \cdots, \vec{q}_n) \in R^{nd}$ is a particular configuration of the n bodies where the acceleration vector of each body is proportional to its position vector, and the constant of proportionality is the same for the n bodies. Therefore, a *central configuration* is a configuration that satisfies the equation

$$\sum_{j=1, j\neq i}^{n} \frac{m_j(\vec{q}_j - \vec{q}_i)}{|\vec{q}_j - \vec{q}_i|^3} = -\lambda(\vec{q}_i - \vec{c}), \qquad i = 1, 2, ..., n, \tag{2}$$

where λ is a scalar function that is the same for all particles and

$$\vec{c} = \frac{\sum\limits_{i=1}^{n} m_i \vec{q}_i}{\sum\limits_{i=1}^{n} m_i}, \qquad i = 1, 2, ..., n. \tag{3}$$

Let us consider 5 collinear bodies of masses, m_0, m_1, m_2, m_3, and m_4. The mass m_0 is stationary at the center of mass of the system. We choose the coordinates for the rest of the 4 bodies as follows:

$$x_1 = -s - 1, x_2 = -1, x_3 = 1, \text{ and } x_4 = 1 + t \text{ where } s, t > 0. \tag{4}$$

Using (2) and (4), we obtain the following equations for central configurations.

$$\frac{m_2}{s^2} + \frac{m_0}{(1+s)^2} + \frac{m_3}{(2+s)^2} + \frac{m_4}{(2+s+t)^2} = \lambda(s + c + 1), \tag{5}$$

$$m_0 - \frac{m_1}{s^2} + \frac{m_3}{4} + \frac{m_4}{(t+2)^2} = \lambda(1 + c), \tag{6}$$

$$m_0 + \frac{m_1}{(s+2)^2} + \frac{m_2}{4} - \frac{m_4}{t^2} = \lambda(1 - c), \tag{7}$$

$$\frac{m_0}{(t+1)^2} + \frac{m_1}{(t+s+2)^2} + \frac{m_2}{(t+2)^2} + \frac{m_3}{t^2} = \lambda(t-c+1). \tag{8}$$

Let $\lambda = 1$. To solve equations (5)–(8) for m_1, m_2, m_3, and m_4, we use symbolic calculation in Mathematica.

$$\begin{aligned} m_1(m_0,s,t,c) &= \frac{N_{m_1}(m_0,s,t,c)}{D_m(s,t)},\ m_2(m_0,s,t,c) = \frac{N_{m_2}(m_0,s,t,c)}{D_m(s,t)}, \\ m_3(m_0,s,t,c) &= \frac{N_{m_2}(m_0,s,t,c)}{D_m(s,t)},\ m_4(m_0,s,t,c) = \frac{N_{m_4}(m_0,s,t,c)}{D_m(s,t)}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} N_{m_1}(m_0,s,t,c) &= A_3A_5s^2(t^5 - t^4(-5+c) - 4t^3(-2+c) - 4t^2(1+c) \\ &\quad -16t(1+c) - 16(1+c) + m_0(4t^2 + \frac{(2+t)^3(2+3t)}{(1+t)^2})), \end{aligned}$$

$$\begin{aligned} N_{m_2}(m_0,s,t,c) &= 4A_4s^2(A_3A_5(1+s+c) - (A_3-A_5)(1-c)t^2 - A_3t^3 \\ &\quad +m_0(-\frac{A_3A_5}{(1+s)^2} - A_5t^2 + \frac{A_3t^2}{(1+t)^2})), \end{aligned}$$

$$\begin{aligned} N_{m_3}(m_0,s,t,c) &= 4A_5t^2(A_3A_4(1+t-c) - (A_3-A_4)(1+c)s^2 - A_3s^3 \\ &\quad +m_0(-A_4s^2 + \frac{A_3s^2}{(1+s)^2} - \frac{A_3A_4}{(1+t)^2})), \end{aligned}$$

$$\begin{aligned} N_{m_4}(m_0,s,t,c) &= A_3A_4t^2(s^5 + 16(-1+c) + 16s(-1+c) + 4s^2(-1+c) \\ &\quad +4s^3(2+c) + s^4(5+c) + m_0(4s^2 + \frac{(2+s)^3(2+3s)}{(1+s)^2})), \end{aligned}$$

$$\begin{aligned} D_m(s,t) &= 256 + 512s + 384s^2 + 128s^3 + 16s^4 + (512 + 896s + 576s^2 \\ &\quad +160s^3 + 16s^4)t + (384 + 576s + 304s^2 + 64s^3 + 4s^4)t^2 \\ &\quad +(128 + 160s + 64s^2 + 16s^3 + 4s^4)t^3 \\ &\quad +(16 + 16s + 4s^2 + 4s^3 + s^4)t^4, \end{aligned}$$

$$A_3 = (s+t+2)^2, A_4 = (t+2)^2, A_5 = (s+2)^2.$$

Equations (9) are the general solutions for masses m_1, m_2, m_3, and m_4 with the mass m_0 arbitrary. These equations give regions of central configurations in the stm_0-space for fixed values of c. In other words, given values of s, t, and m_0, one can find values of m_1, m_2, m_3, and m_4 from equations (9), which will make the configurations central. The values of m_i obtained can also become negative, which is not useful for practical purposes. Therefore, we would like to find regions that will make the masses positive. In the next section we will analyze the special case where m_1, m_4, and m_2, m_3 are symmetric about the center of mass.

3. Fully symmetric collinear 4- and 5-body problems

Let us consider the $s = t$ case, where m_0 is kept stationary at the center of mass. The center of mass is taken to be at the origin. As a result, the pairs of masses (m_1, m_4) and (m_2, m_3) will be symmetric about the center of mass of the system. Furthermore, it can be shown that for $s = t$, $m_1 = m_4$ and $m_2 = m_3$. Therefore, we only need to analyze m_1 and m_2 as a function of $m_0 \geq 0$ and $t > 0$. The solutions of masses m_1 and m_2 derived from equations (9) are given below.

$$m_1 = \frac{N^*_{m_1}(m_0,t)}{D^*_m(t)} \text{ and } m_2 = \frac{N^*_{m_2}(m_0,t)}{D^*_m(t)}, \tag{10}$$

where

$$N^*_{m_1}(m_0,t) = 4t^2(2+t)^2(m_0\left(16+48t+52t^2+28t^3+7t^4\right) + (1+t)^2\left(-16-16t-4t^2+8t^3+5t^4+t^5\right)), \tag{11}$$

$$N^*_{m_2}(m_0,t) = 4t^2(2+t)^2(16+64t+100t^2+68t^3+17t^4) - 4m_0t^2(2+t)^2\left(16+16t+4t^2+4t^3+t^4\right), \tag{12}$$

$$D^*_m(t) = 256+1024t+1664t^2+1408t^3+656t^4+160t^5 + 24t^6+8t^7+t^8. \tag{13}$$

Lemma 1 *Suppose that* $P_1(t) = -16-16t-4t^2+8t^3+5t^4+t^5$*. Then for any* $t > 1.39681$*,* $P_1(t)$ *is always positive.*

Proof $P_1(t)$ is a polynomial of degree 5 in t and the sign of its coefficients changes only once; therefore, by Descartes' rule of signs, it can only have 1 real root, which is $t = 1.39681$. It can easily be shown that for $t > 1.39681$, $P_1(t)$ is always positive. For example, for $t = 1$, $P_1(t) < 0$, and for $t = 2$, $P_1(t) > 0$.

According to equation (13), D^*_m is positive for all values of $t > 0$. Therefore, we only need to analyze $N^*_{m_1}$ and $N^*_{m_2}$ for $m_0 \geq 0$ and $t > 0$.

In equation (11), the term $4t^2(2+t)^2$ is always positive; therefore, it does not have any effect on the sign of $N^*_{m_1}$. Similarly, the term $m_0(16+48t+52t^2+28t^3+7t^4)$ is also always positive. The only term in $N^*_{m_1}$ that can become negative is $(1+t)^2P_1(t)$. Therefore, by Lemma 1, $N^*_{m_1}$ will be positive for all $m_0 \geq 0$ and $t > 1.39681$. Hence, m_1 will also be positive for $m_0 \geq 0$ and $t > 1.39681$. For $0 < t \leq 1.39681$, the positivity of $N^*_{m_1}$ and hence m_1 is shown in Figure 1a, where m_1 is positive on the right side of the curve. It can be deduced from Figure 1a that m_1 is positive for $m_0 \geq 1$ and $t > 0$. □

Following the above procedure, for m_2 to be positive, we get the following relationship between m_0 and t:

$$m_0 < \frac{\left(16+64t+100t^2+68t^3+17t^4\right)}{\left(16+16t+4t^2+4t^3+t^4\right)}. \tag{14}$$

Careful analysis of (12) and (14) reveals that for m_2 to be positive, m_0 must be less than or equal to 17. This can also be seen in Figure 1b. In the white region of Figure 1b, it is not possible to find positive masses that will make the configuration central.

The common region where m_1 and m_2 are both positive is given in Figure 1c.

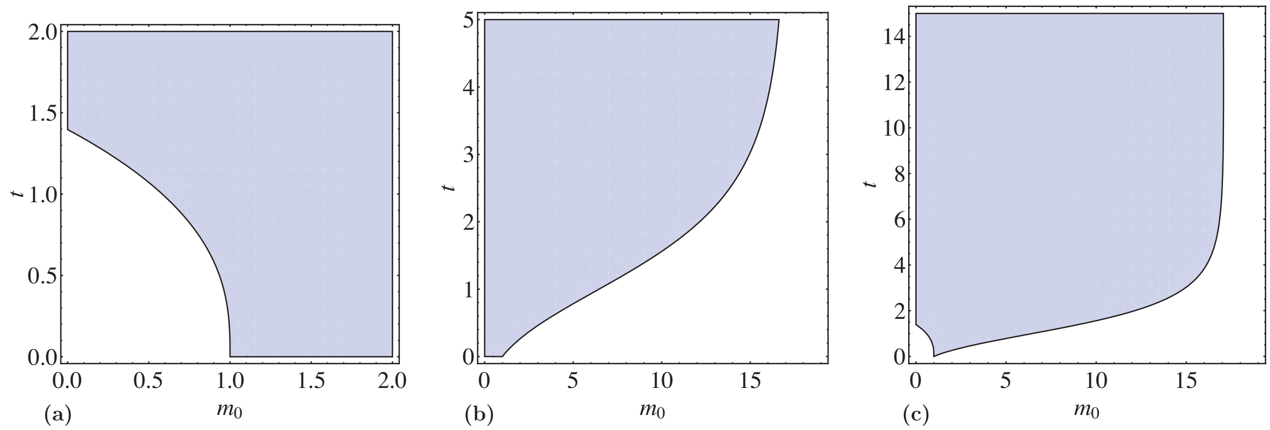

Figure 1. a) Solution space where m_1 is positive; b) solution space where m_2 is positive; c) solution space where both m_1 and m_2 are positive.

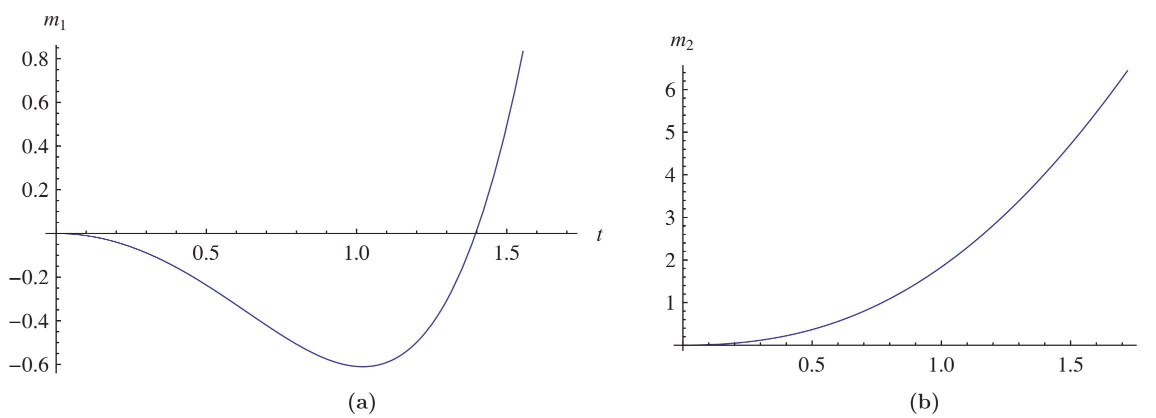

Figure 2. a) Solution space for m_1 when $m_0=0$ and $s=t$; b) solution space for m_2 when $m_0=0$ and $s=t$.

In the special case when $m_0 = 0$, which is the 4-body symmetric case, the expressions for m_1 and m_2 reduce to

$$m_1 = \frac{4t^2(1+t)^2(2+t)^2 P_1(t)}{D_m}, \tag{15}$$

$$m_2 = \frac{4t^2(2+t)^2\left(16+64t+100t^2+68t^3+17t^4\right)}{D_m}. \tag{16}$$

In this case, the solutions for m_1 and m_2 are very easy to analyze. The only term in m_1 that can become negative is $P_1(t)$. Hence, by Lemma 1, $m_1 > 0$ for $t > 1.39681$. This is shown numerically in Figure 2a. As m_2 is positive for all values of t (see Figure 2b), both m_1 and m_2 will be positive for $t > 1.39681$.

4. General collinear 4- and 5-body problems

In this section, we find regions in the stm_0-space where m_1, m_2, m_3, and m_4 are all positive. We will analyze the 4 masses individually, both analytically and numerically. Finally, an intersection of all 4 regions will be given, which will show the regions where central configurations are possible for positive masses. In the complement of these regions, no central configurations are possible for positive masses. We leave out the analysis of when $m_0 = 0$, which is the collinear 4-body case of this 5-body problem, as it was discussed in detail by Ouyang and Xie in [6].

The general solutions for masses m_1, m_2, m_3, and m_4 with the mass m_0 arbitrary are given by equations (9) in Section 2. These equations have only one symmetry with $s \neq t$. The common denominator $D_m(s,t)$ of m_i (where $i = 1, 2, 3, 4$) is a polynomial in s and t with positive coefficients. Therefore, $D_m(s,t) > 0$ for all $s, t > 0$. Hence, we only need to analyze the numerators $N_{m_i}(m_0, s, t, c)$. We will analyze them one by one.

The only part of $N_{m_1}(m_0, s, t, c)$ and $N_{m_4}(m_0, s, t, c)$, that can become negative is:

$$Neg_{m_1}(t,c) = t^5 - t^4(-5+c) - 4t^3(-2+c) - 4t^2(1+c) - 16t(1+c) - 16(1+c),$$

$$Neg_{m_4}(s,c) = s^5 + s^4(5+c) + 4s^3(2+c) + 4s^2(-1+c) + 16s(-1+c) + 16(-1+c).$$

$Neg_{m_1}(t,c)$ is a polynomial of degree 5 in t; its coefficients change sign only once for $-1 < c < 1$ and are all positive for $c < -1$. Therefore, by Descartes' rule of signs it will have only 1 real positive root, which is $t = 1.39681$ for $c = 0$. $Neg_{m_1}(t,0)$ is positive for $t > 1.39681$ and hence N_{m_1} is also positive. It can easily be shown that $Neg_{m_1}(t,c) > 0$ when $t > t_0$, where t_0 is obtained by solving the monotonically increasing function $c(t)$ for fixed values of c.

$$c(t) = \frac{-16 - 16t - 4t^2 + 8t^3 + 5t^4 + t^5}{16 + 16t + 4t^2 + 4t^3 + t^4}$$

It is straightforward to show that $c(t)$ is a monotonically increasing function by showing that $\frac{dc(t)}{dt} > 0$ for all t. This means that m_1 is positive for all $m_0 \geq 0$ and $t > t_0$. When $Neg_{m_1}(t,c) < 0$, it does not automatically mean that $N_{m_1}(t,c) < 0$. For $t < t_0$, we must have

$$m_0 > \frac{(1+t)^2\left(16 + 16t + 4t^2 - 8t^3 - 5t^4 - t^5 + \left(16 + 16t + 4t^2 + 4t^3 + t^4\right)c\right)}{16 + 48t + 52t^2 + 28t^3 + 7t^4}.$$

For the behavior of m_1 when $m_0 > 0$ and $c = 0$, please see Figure 3a. At $c = 0$, $Neg_{m_4}(s,c)$ has similar behavior as that of $Neg_{m_1}(t,c)$. The region where $Neg_{m_4}(s,c) > 0$ is bounded below by $c(s)$, which is a monotonically increasing function of s, i.e for all $m_0 \geq 0$ and $s > s_0$, m_4 is greater than zero.

$$c(s) = \frac{16 + 16s + 4s^2 - 8s^3 - 5s^4 - s^5}{16 + 16s + 4s^2 + 4s^3 + s^4}.$$

The value of s_0 is obtained in the same way as t_0. When $Neg_{m_4}(s,c) < 0$, it does not automatically mean that $N_{m_4}(s,c) < 0$. For $s < s_0$, we must have

$$m_0 > \frac{(1+s)^2\left(16 + 16s + 4s^2 - 8s^3 - 5s^4 - s^5 - \left(16 + 16s + 4s^2 + 4s^3 + s^4\right)c\right)}{16 + 48s + 52s^2 + 28s^3 + 7t^4}.$$

For the behavior of m_4 when $m_0 > 0$ and $c = 0$, please see Figure 3b.

The expression $m_2(m_0, s, t, c)$, which gives the values of m_2, is a complicated function of $m_0, s, t,$ and c. To understand the behavior of m_2 we initially take $c = 0$. After some simplifications, we see that m_2 can be written as:

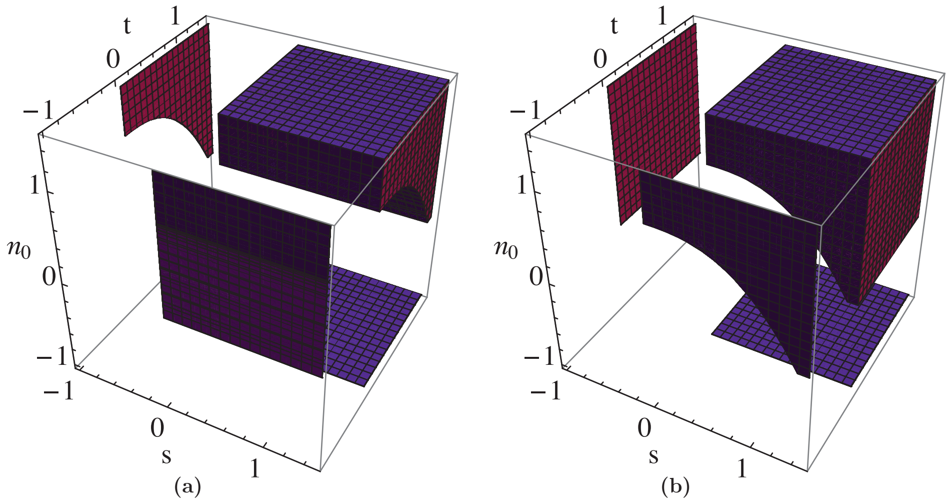

Figure 3. a) Solution space for $m_1 > 0$ when $m_0 > 0$ is arbitrary and $s \neq t$; b) solution space for $m_4 > 0$ when $m_0 > 0$ is arbitrary and $s \neq t$.

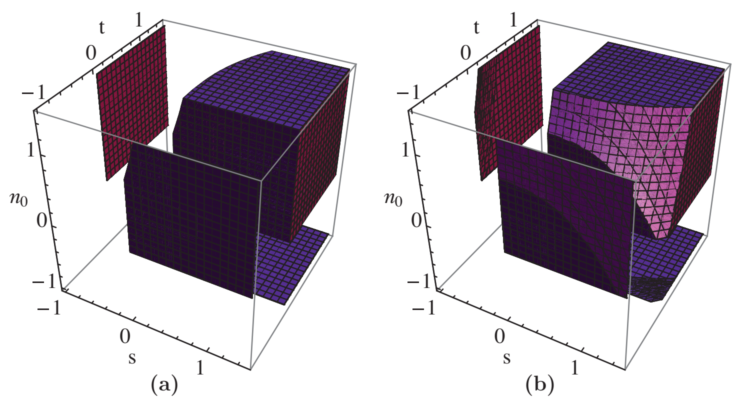

Figure 4. a) Solution space for $m_2 > 0$ when $m_0 > 0$ is arbitrary and $s \neq t$; b) solution space for $m_3 > 0$ when $m_0 > 0$ is arbitrary and $s \neq t$.

$$m_2(m_0, s, t) = \frac{4A_4 s^2}{D_m(s,t)} \left(Neg_{m_2}(s,t) - \frac{m_0 C_{m_0}(s,t)}{(1+s)^2(1+t)^2} \right), \tag{17}$$

where

$$\begin{aligned} Neg_{m_2}(s,t) &= (1+s)(2+s)^4 + 2\,(s+1)\,(s+2)^3\,t + (s+1)\,(s+2)^2\,t^2 \\ &\quad - (s+4)\,(s+2)\,t^3 - (5+2s)t^4 - t^5. \end{aligned}$$

$$\begin{aligned} C_{m_0}(s,t) &= (s+2)^4 + 2\,(s+3)\,(s+2)^3\,t + \left(13+8s+s^2\right)(s+2)^2\,t^2 \\ &\quad +2\,(s+2)\left(7+8s+4s^2+s^3\right)t^3 \\ &\quad + \left(7+14s+13s^2+6s^3+s^4\right)t^4. \end{aligned}$$

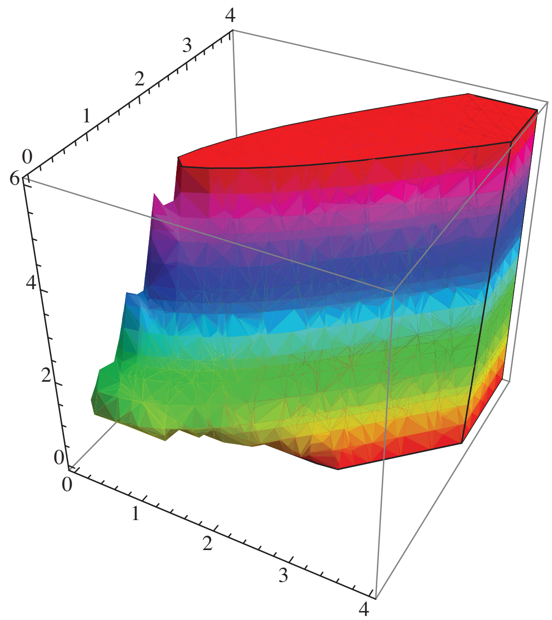

Figure 5. Solution space for $m_1 > 0$, $m_2 > 0$, $m_3 > 0$, and $m_4 > 0$ when $m_0 > 0$ is arbitrary and $s \neq t$.

The coefficient of m_0 in m_2 above is always negative. Other than the coefficient of m_0, which is always negative, the term that can become negative is given by $Neg_{m_2}(s,t)$. Consider $Neg_{m_2}(s,t)$ to be a polynomial in t with variable coefficients. Given $s > 0$, the coefficients of t^0, t, t^2 are positive and the coefficients of t^3, t^4, t^5 are negative. Therefore, by Descartes' rule of signs $Neg_{m_2}(s,t)$ will have only 1 positive root for each value of s, which will determine a smooth monotone increasing function $t = f(s)$. The function $f(s) \cong s + 1.4$ will determine a boundary between the negative and positive values of $Neg_{m_2}(s,t)$. If $t > f(s)$, $Neg_{m_2}(s,t)$ will be negative and hence m_2 will also be negative, because the second part of m_2 that involves m_0 is always negative. For $t < f(s)$, $Neg_{m_2}(s,t)$ is always positive, but it does not guarantee that $Neg_{m_2}(s,t)$ and hence m_2 will also be positive. For m_2 to be positive, we must also have

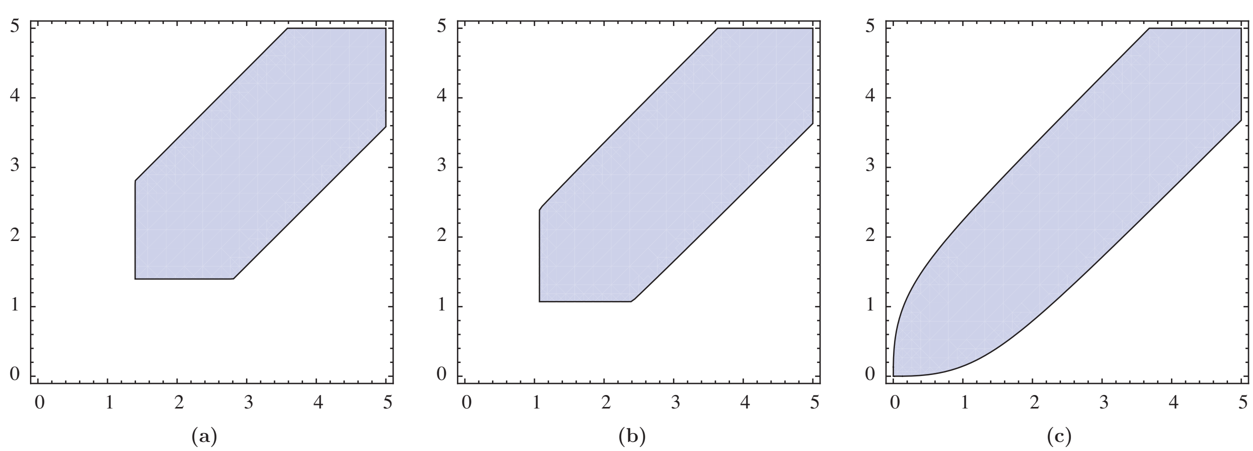

Figure 6. Solution space for $m_i > 0$, $i = 1,2,3,4$ when a) $m_0 = 0$, b) $m_0 = 0.5$, c) $m_0 = 1$.

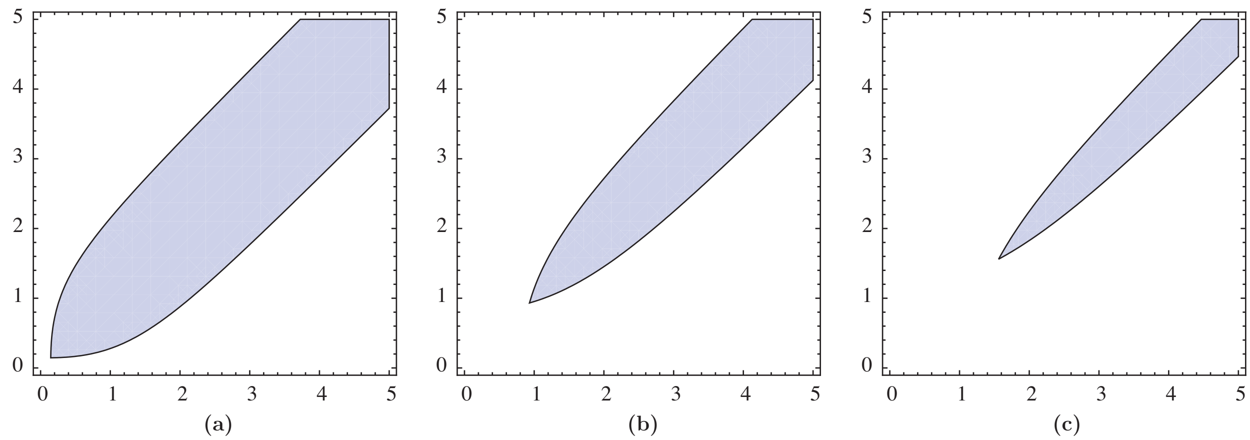

Figure 7. Solution space for $m_i > 0$, $i = 1, 2, 3, 4$ when a) $m_0 = 1.5$, b) $m_0 = 6$, c) $m_0 = 10$.

$$m_0(s,t) < \frac{(1+s)^2(1+t)^2 Neg_{m_2}(s,t)}{C_{m_0}(s,t)}. \tag{18}$$

In the special case of $s = t$, the above inequality gives an upper bound of 17.0 on m_0, as has been shown in Section 3, but no such bound on m_0 exists in the general case. The above inequality will give an upper bound of m_0 for each value of s and t. Therefore, it can be concluded that for all $t < f(s)$ we can find a suitable $m_0 > 0$ that will make m_2 positive. Conversely, for all $m_0 > 0$, we can find $s, t > 0$, which will make m_2 positive. Please refer to Figure 4a for regions in stm_0-space where m_2 is positive. In the general case when $c \neq 0$, the coefficient of m_0 is always negative; therefore, we only need to analyze $Neg_{m_2}(s,t,c)$, which is given below.

$$\begin{aligned} Neg_{m_2}(s,t,c) \quad = \quad & -t^5 - t^4(5+2s-c) - t^3(2+s)(4+s-2c) \\ & + t^2\left((2+s)^2(1+s+c)\right) + t\left(2(2+s)^3(1+s+c)\right) \\ & + (2+s)^4(1+s+c). \end{aligned}$$

Like $Neg_{m_2}(s,t)$, $Neg_{m_2}(s,t,c)$ is also a polynomial in t with variable coefficients as functions of s and c. By careful analysis of $Neg_{m_2}(s,t,c)$, it can be seen that the coefficients of t change sign at most once for each value of s and c. For some values of s and c, none of the coefficients of t changes sign. By Descartes' rule of signs, $Neg_{m_2}(s,t,c)$ will have at most 1 positive root for each value of s and c, which will determine a smooth monotone increasing function $t = f(s,c)$. The function $f(s,c)$ will define a boundary between the positive and negative values of m_2 provided that m_0 satisfies the following inequality:

$$m_0(s,t) < \frac{(1+s)^2(1+t)^2 Neg_{m_2}(s,t,c)}{C_{m_0}(s,t)}. \tag{19}$$

As $m_2(s,t,c) = m_3(t,s,-c)$, the analysis of m_3 will be similar to the analysis of m_2. For example, the upper bound on m_0 is given by

$$m_0(s,t) < \frac{(1+s)^2(1+t)^2 Neg_{m_3}(s,t,c)}{C_{m_0}(s,t)}, \tag{20}$$

where $Neg_{m_3}(s,t,c) = Neg_{m_2}(t,s,-c)$. The above inequalities will give an upper bound of m_0 for fixed values of s, t, and c.

See Figures 3b and 4b for the regions where m_4 and m_3 are positive. Numerically, regions of central configuration for the general collinear 5-body problem are given in Figure 5. Cross-sections of the region in Figure 5 are given in Figures 6 and 7. In Figures 3–7, c is taken to be zero.

5. Conclusions

We model a general collinear 5-body problem where 4 of the masses are arranged on a line with the fifth mass stationary at the center of mass. We form expressions for m_i, $i = 1, 2, 3, 4$ as functions of s, t, and m_0, which give central configurations in the 5-body problem. In the fully symmetric case of this 5-body problem, regions in the tm_0-plane are identified where no central configurations are possible if we take all the 5 masses to be positive. Conversely, in the complement of the region mentioned above, it is always possible to choose positive masses. It is also shown that for $m_0 > 17$ no central configurations exist unless we allow for some of the masses to become negative. Similarly, we analyze m_i, $i = 1, 2, 3, 4$ in the general collinear 5-body problem. We identify regions in the stm_0-space where no central configurations are possible if we restrict all the masses to being positive. In the complement of these regions, it is always possible to choose positive masses.

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