Turkish Journal of Mathematics

http://journals.tubitak.gov.tr/math/

Research Article

Turk J Math
(2015) 39
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doi:10.3906/mat-1306-60

Zero triple product determined generalized matrix algebras

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Received: 30.06.2013 • **Accepted:** 21.05.2014 • **Published Online:** 23.02.2015 • **Printed:** 20.03.2015

Abstract: In this paper, we prove that the generalized matrix algebra $\mathcal{G} = \begin{bmatrix} A & M \\ N & B \end{bmatrix}$ is a zero triple product (resp. zero Jordan triple product) determined if and only if A and B are zero triple products (resp. zero Jordan triple products) determined under certain conditions. Then the main results are applied to triangular algebras and full matrix algebras.

Key words: Zero triple product determined algebra, zero Jordan triple product algebra, generalized matrix algebra

1. Introduction

Let $\mathcal{R}$ be a commutative ring with identity and $\mathcal{A}$ be an associative $\mathcal{R}$-algebra. We define the *Jordan triple product* by $a \circ b \circ c = abc + cba$ for each $a, b, c \in \mathcal{A}$. These products on $\mathcal{A}$ are nonassociative and $\mathcal{A}$ becomes a Jordan triple algebra if we replace the original product by the Jordan triple product. We denote the $\mathcal{R}$-linear span of all elements of the form abc (resp. $a \circ b \circ c$), where $a, b, c \in \mathcal{A}$, by $\mathcal{A}^3$ (resp. $\mathcal{A} \circ \mathcal{A} \circ \mathcal{A}$).

The algebra $\mathcal{A}$ is called a *zero triple product determined algebra* if for every $\mathcal{R}$-module $\mathcal{W}$ and every $\mathcal{R}$-trilinear mapping $\phi\colon \mathcal{A} \times \mathcal{A} \times \mathcal{A} \longrightarrow \mathcal{W}$, the following holds: if $\phi(a, b, c) = 0$ whenever $abc = 0$, then there exists an $\mathcal{R}$-linear mapping $T\colon \mathcal{A}^3 \longrightarrow \mathcal{W}$ such that $\phi(a, b, c) = T(abc)$ for all $a, b, c \in \mathcal{A}$. If the ordinary product is replaced by the Jordan triple product and $\mathcal{A}^3$ is replaced by $\mathcal{A} \circ \mathcal{A} \circ \mathcal{A}$, then we shall say that $\mathcal{A}$ is a zero Jordan triple product determined algebra.

The question of characterizing linear maps that preserve zero product, Jordan product, commutativity, etc. on algebras has attracted the attention of many authors. However, these problems can be sometimes effectively solved by considering bilinear maps that preserve certain zero product properties (for instance, see [1–3, 6]). Motivated by these reasons, Brešar et al. [5] introduced the concept of zero product (resp. Jordan product, Lie product) determined algebras, which can be used to study the linear maps preserving zero product (resp. Jordan product, commutativity). For example, it is not difficult to check that any zero product preserving unital linear map on a zero product determined algebra is an algebra homomorphism. Then zero (resp. zero Jordan, zero Lie) product determined algebras were studied by many authors (see [9–11, 15] and the references therein). Recently, Yao and Zheng [17] extended the aforementioned products to more generalized forms, such as triple product and Jordan triple product, in order to get the concept of zero triple product (resp. zero Jordan triple product) determined algebras, which may be used to study the linear maps preserving zero triple product (resp. zero Jordan triple product). They showed that matrix algebra $\mathcal{M}_n(\mathcal{R})(n \geq 3)$ is always zero triple

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2010 *AMS Mathematics Subject Classification:* 15A04, 15A27, 15A78.

product determined, and $\mathcal{M}_n(\mathcal{F})(n \geq 3)$, where $\mathcal{F}$ is any field with $\mathrm{char}\mathcal{F} \neq 2$, is zero Jordan triple product determined.

Inspired by the aforementioned, we will study whether the generalized matrix algebra is zero triple (resp. zero Jordan triple) product determined. In particular, we prove that under certain conditions the generalized matrix algebra is zero triple product (resp. zero Jordan triple product) determined and the main results are then applied to triangular algebras and full matrix algebras.

2. Preliminaries

Let us begin with the definition of generalized matrix algebras given by a Morita context. Let $\mathcal{R}$ be a commutative ring with identity. A *Morita context* consists of 2 $\mathcal{R}$-algebras A and B, 2 bimodules ${}_AM_B$ and ${}_BN_A$, and 2 bimodule homomorphisms called the pairings $\Phi_{MN} : M \underset{B}{\otimes} N \longrightarrow A$ and $\Psi_{NM} : N \underset{A}{\otimes} M \longrightarrow B$ satisfying the following commutative diagrams:

$$\begin{array}{ccc} M \underset{B}{\otimes} N \underset{A}{\otimes} M & \xrightarrow{\Phi_{MN}\otimes I_M} & A \underset{A}{\otimes} M \\ \Big\downarrow{\scriptstyle I_M\otimes\Psi_{NM}} & & \Big\downarrow{\scriptstyle \cong} \\ M \underset{B}{\otimes} B & \xrightarrow{\cong} & M \end{array} \quad \text{and} \quad \begin{array}{ccc} N \underset{A}{\otimes} M \underset{B}{\otimes} N & \xrightarrow{\Psi_{NM}\otimes I_N} & B \underset{B}{\otimes} N \\ \Big\downarrow{\scriptstyle I_N\otimes\Phi_{MN}} & & \Big\downarrow{\scriptstyle \cong} \\ N \underset{A}{\otimes} A & \xrightarrow{\cong} & N\,. \end{array}$$

Let us write this Morita context as $(A, B, M, N, \Phi_{MN}, \Psi_{NM})$. We refer the readers to [14] for the basic properties of Morita contexts. If $(A, B, M, N,\ \Phi_{MN}, \Psi_{NM})$ is a Morita context, then the set

$$\left[\begin{array}{cc} A & M \\ N & B \end{array}\right] = \left\{ \left[\begin{array}{cc} a & m \\ n & b \end{array}\right] \middle| a \in A, m \in M, n \in N, b \in B \right\}$$

forms an $\mathcal{R}$-algebra under matrix-like addition and matrix-like multiplication, where at least 1 of the 2 bimodules M and N is distinct from 0. Such an $\mathcal{R}$-algebra is usually called a *generalized matrix algebra* of order 2 and is denoted by

$$\mathcal{G} = \left[\begin{array}{cc} A & M \\ N & B \end{array}\right].$$

In a similar way, one can define a generalized matrix ring of order $n > 2$. It was shown that up to isomorphism, arbitrary generalized matrix algebra of order n $(n \geq 2)$ is a generalized matrix algebra of order 2 [12, Example 2.2]. If one of the modules M and N is zero, then $\mathcal{G}$ exactly degenerates to an *upper triangular algebra* or a *lower triangular algebra*. In this case, we denote the resulted upper triangular algebra (resp. lower triangular algebra) by

$$\mathcal{T}^{\mathcal{U}} = \left[\begin{array}{cc} A & M \\ O & B \end{array}\right] \quad \left(\text{resp. } \mathcal{T}_{\mathcal{L}} = \left[\begin{array}{cc} A & O \\ N & B \end{array}\right]\right).$$

Let us single out some classical examples of generalized matrix algebras that will be revisited in the sequel. We refer the readers to [4, 12, 16] for more details.

(a) full matrix algebras;

(b) upper and lower triangular matrix algebras;

(c) block upper and lower triangular matrix algebras.

It should be mentioned that our current generalized matrix algebras contain those generalized matrix algebras in the sense of Brown [7] as special cases. Let $\mathcal{M}_n(\mathcal{R})$ be the full matrix algebra consisting of all $n\times n$ matrices over $\mathcal{R}$. It is worth pointing out that the notion of generalized matrix algebras efficiently unifies triangular algebras with full matrix algebras. The distinguished feature of our systematic work is that we deal with all questions related to additive mappings of triangular algebras and full matrix algebras under a unified frame, which is the admired generalized matrix algebras frame (see [8,12,13,16]).

3. Zero triple product determined generalized matrix algebras

The purpose of this section is to verify whether a generalized matrix algebra is zero triple product determined, and then apply the main result to various triangular algebras and full matrix algebras.

The following is our main result in this section.

Theorem 3.1 *Let* $\mathcal{G}=\begin{bmatrix} A & M \\ N & B \end{bmatrix}$ *be a generalized matrix algebra consisting of algebras* A, B *and* (A,B)*-bimodules* M, N*. If* A *and* B *are both zero triple product determined algebras, then* $\mathcal{G}$ *is a zero triple product determined algebra.*

Conversely, let $\mathcal{G}$ *be a generalized matrix algebra with the bimodule homomorphisms* $\Phi_{MN}: M\underset{B}{\otimes} N\to A$ *and* $\Psi_{NM}: N\underset{A}{\otimes} M\to B$ *of* $\mathcal{G}$ *be trivial. If* $\mathcal{G}$ *is a zero triple product determined algebra, then* A *and* B *are both zero triple product determined algebras.*

Proof Let 1_A (resp. 1_B) be the identity of the algebra A (resp. B) and I be the identity of the generalized matrix algebra $\mathcal{G}$. Let us write

$$P=\begin{bmatrix} 1_A & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad Q=\begin{bmatrix} 0 & 0 \\ 0 & 1_B \end{bmatrix}.$$

Then P and Q are standard idempotents in $\mathcal{G}$. □

Suppose that A and B are both zero triple product determined algebras. Let $\mathcal{W}$ be a $\mathcal{R}$-module, and $\phi\colon \mathcal{G}\times\mathcal{G}\times\mathcal{G}\longrightarrow\mathcal{W}$ be a $\mathcal{R}$ trilinear mapping such that for all $X,Y,Z\in\mathcal{G}$, $XYZ=0$ implies $\phi(X,Y,Z)=0$ (Here and throughout, X,Y,Z always denote the elements of $\mathcal{G}$).

For arbitrary elements $X,Y\in\mathcal{G}$, it is easy to compute that

$$\begin{aligned}(PXP)(QYQ)&=(QXQ)(PYP)=(PXQ)(PYQ)\\ &=(PXQ)(PYP)=(QXQ)(PYQ)=(QXP)(QYP)\\ &=(QXP)(QYQ)=(PXP)(QYP)=0.\end{aligned}$$

So we have

$$\begin{aligned}\phi(PXP,QYQ,Z)&=\phi(QXQ,PYP,Z)=\phi(PXQ,PYQ,Z)\\ &=\phi(PXQ,PYP,Z)=\phi(QXQ,PYQ,Z)=\phi(QXP,QYP,Z)\\ &=\phi(QXP,QYQ,Z)=\phi(PXP,QYP,Z)=0\end{aligned} \tag{3.1}$$

for all $X, Y, Z \in \mathcal{G}$.

From $(Q - QYP)I(PXP + QYPXP) = 0$, we obtain

$$\phi(QYP, I, PXP) = \phi(Q, I, QYPXP).$$

Letting $X = P$ in the above equation, we find

$$\phi(QYP, I, P) = \phi(Q, I, QYP). \tag{3.2}$$

Likewise, $(PXP + PXPYQ)I(Q - PYQ) = 0$ yields

$$\phi(PXP, I, PYQ) = \phi(PXPYQ, I, Q),$$

and setting $X = P$, we have

$$\phi(P, I, PYQ) = \phi(PYQ, I, Q). \tag{3.3}$$

Next we will complete the proof of this part via the following 7 steps.

Step 1. Let us define $\eta\colon A \times A \times A \longrightarrow \mathcal{W}$ by

$$\eta(a_1, a_2, a_3) = \phi\left(\begin{bmatrix} a_1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} a_2 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} a_3 & 0 \\ 0 & 0 \end{bmatrix}\right)$$

for all $a_1, a_2, a_3 \in A$. Therefore, η is a $\mathcal{R}$-trilinear mapping such that for all $a_1, a_2, a_3 \in A$, $a_1a_2a_3 = 0$ implies $\eta(a_1, a_2, a_3) = 0$. Since A is a zero triple product determined algebra, there is an $\mathcal{R}$-linear mapping $T_1\colon A \longrightarrow \mathcal{W}$ such that $\eta(a_1, a_2, a_3) = T_1(a_1a_2a_3)$ for all $a_1, a_2, a_3 \in A$. Therefore, $\eta(a_1, a_2, a_3) = \eta(a_1a_2a_3, 1_A, 1_A)$ and hence $\phi(PXP, PYP, PZP) = \phi(PXPYPZP, P, P)$, or we can write it as

$$\phi(PXP, PYP, PZP) = \phi(PXPYPZP, I, I).$$

Taking into account $PXP(PYP + PYPZQ)(Q - PZQ) = 0$ and $(PXP + PXPYPZQ)(Q - PYPZQ)I = 0$, we have

$$\phi(PXP, PYP, PZQ) = \phi(PXP, PYPZQ, I) = \phi(PXPYPZQ, I, I).$$

It is easy to verify that

$$\phi(PXP, PYP, QZP) = \phi(PXP, PYP, QZQ) = 0.$$

All these show that

$$\phi(PXP, PYP, Z) = \phi(PXPYPZ, I, I). \tag{3.4}$$

In a similar way, by the hypothesis that B is also a zero triple determined algebra, we get

$$\phi(QXQ, QYQ, Z) = \phi(QXQYQZ, I, I). \tag{3.5}$$

Step 2. Since $(PXP + PXPYQ)(Q - PYQ)Z = 0$, we obtain

$$\phi(PXP, PYQ, Z) = \phi(PXPYQ, I, Z).$$

Now, we have $(PXPYQ + PXPYQZP)I(QZP - P) = 0$ and $(PXPYQ + P)I(QZQ - PXPYQZQ) = 0$. Consequently

$$\phi(PXPYQ, I, QZP) = \phi(PXPYQZP, I, I),$$

$$\phi(PXPYQ, I, QZQ) = \phi(I, I, PXPYQZQ) = \phi(PXPYQZQ, I, I),$$

where we used (3.3) in the last step. We can also get

$$\phi(PXPYQ, I, PZQ) = \phi(PXPYQ, I, PZP) = 0.$$

The last 3 relations imply that

$$\phi(PXP, PYQ, Z) = \phi(PXPYQ, I, Z) = \phi(PXPYQZ, I, I). \tag{3.6}$$

Step 3. From $(QXP + QXPYQ)(PYQ - Q)Z = 0$ we can see that

$$\phi(QXP, PYQ, Z) = \phi(QXPYQ, I, Z).$$

Furthermore, it follows from Eq. (3.5) that

$$\phi(QXPYQ, I, QZQ) = \phi(QXPYQZQ, I, I).$$

Using $(QXPYQ + QXPYQZP)I(P - QZP) = 0$ we obtain

$$\phi(QXPYQ, I, QZP) = \phi(QXPYQZP, I, I).$$

It is easy to check that

$$\phi(QXPYQ, I, PZQ) = \phi(QXPYQ, I, PZP) = 0.$$

Therefore,

$$\phi(QXP, PYQ, Z) = \phi(QXPYQ, I, Z) = \phi(QXPYQZ, I, I). \tag{3.7}$$

Step 4. Since $(PXQ + PXQYP)(QYP - P)Z = 0$, we have

$$\phi(PXQ, QYP, Z) = \phi(PXQYP, I, Z).$$

A direct computation yields $(PXQYP + PXQYPZQ)I(Q - PZQ) = 0$; this implies that

$$\phi(PXQYP, I, PZQ) = \phi(PXQYPZQ, I, I).$$

By Eq. (3.4) we know that

$$\phi(PXQYP, I, PZP) = \phi(PXQYPZP, I, I).$$

We can also get

$$\phi(PXQYP, I, QZQ) = \phi(PXQYP, I, QZP) = 0.$$

Therefore,

$$\phi(PXQ, QYP, Z) = \phi(PXQYPZ, I, I). \tag{3.8}$$

Step 5. From $(QXQ + QXQYP)(P - QYP)Z = 0$ we get

$$\phi(QXQ, QYP, Z) = \phi(QXQYP, I, Z).$$

Using $(QXQYP + QXQYPZQ)I(PZQ - Q) = 0$, we have

$$\phi(QXQYP, I, PZQ) = \phi(QXQYPZQ, I, I).$$

Applying $(Q - QXQYP)I(PZP + QXQYPZP) = 0$ and revisiting the relation (3.2), we obtain

$$\phi(QXQYP, I, PZP) = \phi(I, I, QXQYPZP) = \phi(QXQYPZP, I, I).$$

Obviously,

$$\phi(QXQYP, I, QZQ) = \phi(QXQYP, I, QZP) = 0,$$

and so

$$\phi(QXQ, QYP, Z) = \phi(QXQYP, I, Z) = \phi(QXQYPZ, I, I). \tag{3.9}$$

Step 6. By a simple computation we have $(Q-QXP)(PYP+QXPYP)Z = 0$ and $(Q+QXPYP)(P-QXPYP)Z = 0$. It follows that

$$\phi(QXP, PYP, Z) = \phi(Q, QXPYP, Z) = \phi(QXPYP, P, Z).$$

This gives

$$\phi(QXP, PYP, Z) = \phi(QXPYP, I, Z).$$

Note that $(QXPYP + QXPYPZQ)I(PZQ - Q) = 0$, we have

$$\phi(QXPYP, I, PZQ) = \phi(QXPYPZQ, I, I).$$

Since $(Q - QXPYP)I(PZP + QXPYPZP) = 0$, we get from (3.2) that

$$\phi(QXPYP, I, PZP) = \phi(I, I, QXPYPZP) = \phi(QXPYPZP, I, I).$$

It is not difficult to check that

$$\phi(QXPYP, I, QZQ) = \phi(QXPYP, I, QZP) = 0.$$

Now we obtain

$$\phi(QXP, PYP, Z) = \phi(QXPYPZ, I, I). \tag{3.10}$$

Step 7. From $(P + PXQ)(QYQ - PXQYQ)Z = 0$ and $(P + PXQYQ)(Q - PXQYQ)Z = 0$ we can get

$$\phi(PXQ, QYQ, Z) = \phi(P, PXQYQ, Z) = \phi(PXQYQ, I, Z).$$

By $(PXQYQ + PXQYQZP)I(QZP - P) = 0$ we obtain

$$\phi(PXQYQ, I, QZP) = \phi(PXQYQZP, I, I).$$

Using $(P + PXQYQ)I(QZQ - PXQYQZQ) = 0$ and applying (3.3) again yield

$$\phi(PXQYQ, I, QZQ) = \phi(P, I, PXQYQZQ) = \phi(PXQYQZQ, I, I).$$

Moreover,

$$\phi(PXQYQ, I, PZP) = \phi(PXQYQ, I, PZQ) = 0.$$

Hence we have

$$\phi(PXQ, QYQ, Z) = \phi(PXQYQZ, I, I). \tag{3.11}$$

In view of Eqs. (3.1) and ((3.4)–(3.11)) we obtain

$$\begin{aligned}
&\phi(PXP+PXQ+QXQ+QXP, PYP+PYQ+QYQ+QYP, Z)\\
=&\phi(PXPYPZ, I, I)+\phi(PXPYQZ, I, I)+\phi(PXQYQZ, I, I)+\phi(QXQYQZ, I, I)\\
+&\phi(QXPYPZ, I, I)+\phi(QXPYQZ, I, I)+\phi(PXQYPZ, I, I)+\phi(QXQYPZ, I, I)\\
=&\phi(XYZ, I, I).
\end{aligned}$$

If we define the $\mathcal{R}$-linear mapping $T:\mathcal{G}\to\mathcal{W}$ by $T(Z)=\phi(Z,I,I)$ for all $Z\in\mathcal{G}$, then T satisfies all the requirements in the definition of zero product determined algebras. Thus, $\mathcal{G}$ is a zero triple product determined algebra.

Conversely, assume $\mathcal{G}$ is a zero triple product determined algebra with the bimodule homomorphisms $\Phi_{MN}: M\underset{B}{\otimes}N\to A$ and $\Psi_{NM}: N\underset{A}{\otimes}M\to B$ of $\mathcal{G}$ be trivial. Let $\mathcal{W}$ be a $\mathcal{R}$-module and $\psi: A\times A\times A\to\mathcal{W}$ be a $\mathcal{R}$-trilinear map such that for all $a_1,a_2,a_3\in A, a_1a_2a_3=0$ implies $\psi(a_1,a_2,a_3)=0$. Define a $\mathcal{R}$-trilinear map $\phi:\mathcal{G}\times\mathcal{G}\times\mathcal{G}\to\mathcal{W}$ by $\phi(X',Y',Z')=\psi(a_1,a_2,a_3)$ where $X'=\begin{pmatrix} a_1 & m_1\\ n_1 & b_1\end{pmatrix}, Y'=\begin{pmatrix} a_2 & m_2\\ n_2 & b_2\end{pmatrix}, Z'=\begin{pmatrix} a_3 & m_3\\ n_3 & b_3\end{pmatrix}\in\mathcal{G}$. If $XYZ=0$, then $\phi(X,Y,Z)=0$. Since $\mathcal{G}$ is a zero triple product determined algebra, there is a $\mathcal{R}$-linear map $T:\mathcal{G}\to\mathcal{W}$ such that $\phi(X',Y',Z')=T(X'Y'Z')$ for all $X',Y',Z'\in\mathcal{G}$. Define a $\mathcal{R}$-linear map $T_1: A\to\mathcal{W}$ by $T_1(a)=T\begin{pmatrix} a & 0\\ 0 & 0\end{pmatrix}$ for all $a\in A$. Therefore, $\psi(a_1,a_2,a_3)=T_1(a_1a_2a_3)$ for all $a_1,a_2,a_3\in A$. Thus, A is a zero triple product determined algebra. Similarly, we can show that B is a zero triple product determined algebra. The proof is completed.

As a direct consequence of this theorem we have the following corollary.

Corollary 3.2 *$\mathcal{T}^{\mathcal{U}}$ (resp. $\mathcal{T}_{\mathcal{L}}$) is a zero triple product determined algebra if and only if A and B are zero triple product determined algebras.*

3.1. Full matrix algebras

Let $\mathcal{R}$ be a commutative ring with identity, A be a torsion free or $i(i\le k)$-torsion free unital algebra over $\mathcal{R}$, and $M_n(A)$ be the algebra of $n\times n$ matrices with $n\ge 2$. Then the *full matrix algebra* $M_n(A)(n\ge 2)$ can be represented as a generalized matrix algebra of the form

$$M_n(A)=\begin{bmatrix} A & M_{1\times(n-1)}(A)\\ M_{(n-1)\times 1}(A) & M_{n-1}(A)\end{bmatrix}.$$

Then according to Theorem 3.1 we have

Corollary 3.3 *If A is zero triple product determined algebra, then $M_n(A)(n\ge 1)$ is zero triple product determined algebra.*

In this corollary, it is not necessarily true that A is a zero triple product determined algebra.

3.2. Upper and lower matrix triangular algebras

Let $\mathcal{R}$ be a commutative ring with identity and A be a unital algebra over $\mathcal{R}$. We denote the set of all $p \times q$ matrices over $\mathcal{R}$ by $M_{p\times q}(A)$. Let us denote the set of all $n \times n$ upper triangular matrices over $\mathcal{R}$ and the set of all $n \times n$ lower triangular matrices over $\mathcal{R}$ by $\mathcal{T}_n(A)$ and $\mathcal{T}'_n(A)$, respectively. For $n \geq 2$ and each $1 \leq k \leq n-1$, the *upper triangular matrix algebra* $\mathcal{T}_n(A)$ and *lower triangular matrix algebra* $\mathcal{T}'_n(A)$ can be written as

$$\mathcal{T}_n(A) = \begin{bmatrix} \mathcal{T}_k(A) & M_{k\times(n-k)}(A) \\ & \mathcal{T}_{n-k}(A) \end{bmatrix} \text{ and } \mathcal{T}'_n(A) = \begin{bmatrix} \mathcal{T}'_k(A) & \\ M_{(n-k)\times k}(A) & \mathcal{T}'_{n-k}(A) \end{bmatrix},$$

respectively.

Then we have the following:

Corollary 3.4 *$\mathcal{T}_n(A)$ (resp. $\mathcal{T}'_n(A)$) is a zero triple product determined algebra if and only if A is a zero triple product determined algebra.*

3.3. Block upper and lower triangular matrix algebras

Let $\mathcal{R}$ be a commutative ring with identity and A be a unital algebra over $\mathcal{R}$. Let $\mathbb{N}$ be the set of all positive integers and let $n \in \mathbb{N}$. For any positive integer m with $m \leq n$, we denote by $\bar{d} = (d_1, \cdots, d_i, \cdots, d_m) \in \mathbb{N}^m$ an ordered m-vector of positive integers such that $n = d_1 + \cdots + d_i + \cdots + d_m$. The *block upper triangular matrix algebra* $B_n^{\bar{d}}(A)$ is a subalgebra of $M_n(A)$ with form

$$B_n^{\bar{d}}(A) = \begin{bmatrix} M_{d_1}(A) & \cdots & M_{d_1\times d_i}(A) & \cdots & M_{d_1\times d_m}(A) \\ & \ddots & \vdots & & \vdots \\ & & M_{d_i}(A) & \cdots & M_{d_i\times d_m}(A) \\ & O & & \ddots & \vdots \\ & & & & M_{d_m}(A) \end{bmatrix}.$$

Likewise, the *block lower triangular matrix algebra* $B_n'^{\bar{d}}(A)$ is a subalgebra of $M_n(A)$ with form

$$B_n'^{\bar{d}}(A) = \begin{bmatrix} M_{d_1}(A) & & & & \\ \vdots & \ddots & & & O \\ M_{d_i\times d_1}(A) & \cdots & M_{d_i}(A) & & \\ \vdots & & \vdots & \ddots & \\ M_{d_m\times d_1}(A) & \dots & M_{d_m\times d_i}(A) & \dots & M_{d_m}(A) \end{bmatrix}.$$

Note that the full matrix algebra $M_n(A)$ of all $n \times n$ matrices over A and the upper (resp. lower) triangular matrix algebra $\mathcal{T}_n(A)$ of all $n \times n$ upper triangular matrices over A are 2 special cases of block upper (resp. lower) triangular matrix algebras. If $n \geq 2$ and $B_n^{\bar{d}}(A) \neq M_n(A)$, then $B_n^{\bar{d}}(A)$ is an upper triangular algebra and can be written as

$$B_n^{\bar{d}}(A) = \begin{bmatrix} B_j^{\bar{d}_1}(A) & M_{j\times(n-j)}(A) \\ O_{(n-j)\times j} & B_{n-j}^{\bar{d}_2}(A) \end{bmatrix},$$

where $1 \leq j < m$ and $\bar{d}_1 \in \mathbb{N}^j, \bar{d}_2 \in \mathbb{N}^{m-j}$. Similarly, if $n \geq 2$ and $B_n'^{\bar{d}}(A) \neq M_n(A)$, then $B_n'^{\bar{d}}(A)$ is a lower triangular algebra and can be represented as

$$B_n'^{\bar{d}}(A) = \begin{bmatrix} B_j'^{\bar{d}_1}(A) & O_{j\times(n-j)} \\ M_{(n-j)\times j}(A) & B_{n-j}'^{\bar{d}_2}(A) \end{bmatrix},$$

where $1 \leq j < m$ and $\bar{d}_1 \in \mathbb{N}^j, \bar{d}_2 \in \mathbb{N}^{m-j}$.

Therefore, by Theorem 3.1, the next corollary is immediate.

Corollary 3.5 *Let $B_n^{\bar{d}}(A)$ (resp. $B_n'^{\bar{d}}(A)$) be a block upper triangular matrix algebra, where $\bar{d} = (d_1, \cdots, d_i, \cdots, d_m) \in \mathbb{N}^m$. Then we have*

(a) If A is a zero triple product determined algebra, then $B_n^{\bar{d}}(A)$ (resp. $B_n'^{\bar{d}}(A)$) is a zero triple product determined algebra.

(b) If $d_j = 1$ for some $1 \geq j \geq m$, then $B_n^{\bar{d}}(A)$ (resp. $B_n'^{\bar{d}}(A)$) is a zero triple product determined algebra if and only if A is a zero triple product determined algebra.

4. Zero Jordan triple product determined generalized matrix algebras

Now we assume that $\mathcal{R}$ contains the element $\frac{1}{2}$, i.e. 2 is invertible in $\mathcal{R}$.

Theorem 4.1 *Let $\mathcal{G}$ be a generalized matrix algebra consisting of algebras A, B and (A, B)-bimodules M, N with bimodule homomorphisms $\Phi_{MN} : M \underset{B}{\otimes} N \to A$ and $\Psi_{NM} : N \underset{A}{\otimes} M \to B$ be trivial. Then $\mathcal{G}$ is a zero Jordan triple product determined algebra if and only if A and B are zero Jordan triple product determined algebras.*

Proof Suppose that A and B are zero Jordan triple product determined algebras. Let $\mathcal{W}$ be a $\mathcal{R}$-module and $\phi : \mathcal{G}\times\mathcal{G}\times\mathcal{G} \to \mathcal{W}$ be a $\mathcal{R}$-trilinear map such that for all $X, Y, Z \in \mathcal{G}, X \circ Y \circ Z = 0$ implies $\phi(X, Y, Z) = 0$. □

It is easy to check that

$$(PXP) \circ (QYQ) \circ Z = (PXQ) \circ (PYQ) \circ Z = (QXP) \circ (QYP) \circ Z = 0$$

for all $X, Y, Z \in \mathcal{G}$. Thus we have

$$\begin{aligned} &\phi(PXP, QYQ, Z) = 0, \phi(QXQ, PYP, Z) = 0, \\ &\phi(PXQ, PYQ, Z) = 0, \\ &\phi(QXP, QYP, Z) = 0. \end{aligned} \tag{4.1}$$

Note that $(P - Q) \circ I \circ QXP = 0$ for any $X \in \mathcal{G}$, we get

$$\phi(P - Q, I, QXP) = \phi(QXP, I, P - Q) = 0,$$

and hence

$$\phi(P, I, QXP) = \phi(Q, I, QXP), \quad \phi(QXP, I, P) = \phi(QXP, I, Q).$$

On the other hand, the equation $(P-Q)\circ I\circ PXQ=0$ gives

$$\phi(P,I,PXQ)=\phi(Q,I,PXQ),\quad \phi(PXQ,I,P)=\phi(PXQ,I,Q).$$

In view of the above relations, we have

$$\begin{aligned}\phi(QXP,I,P)&=\phi(QXP,I,Q)=\frac{1}{2}\phi(QXP,I,I),\\ \phi(P,I,QXP)&=\phi(Q,I,QXP)=\frac{1}{2}\phi(I,I,QXP),\\ \phi(PXQ,I,P)&=\phi(PXQ,I,Q)=\frac{1}{2}\phi(PXQ,I,I),\\ \phi(P,I,PXQ)&=\phi(Q,I,PXQ)=\frac{1}{2}\phi(I,I,PXQ).\end{aligned}\tag{4.2}$$

From $(PXP+QYPXP)\circ I\circ(Q-QYP)=0$, we obtain

$$\phi(PXP,I,QYP)=\phi(QYPXP,I,Q),\ \phi(QYP,I,PXP)=\phi(Q,I,QYPXP).$$

Letting $X=P$ in the above equations, we arrive at

$$\begin{aligned}\phi(P,I,QYP)&=\phi(QYP,I,Q),\\ \phi(QYP,I,P)&=\phi(Q,I,QYP).\end{aligned}\tag{4.3}$$

Likewise, $(PXP+PXPYQ)\circ I\circ(Q-PYQ)=0$ yields

$$\phi(PXP,I,PYQ)=\phi(PXPYQ,I,Q),\ \phi(PYQ,I,PXP)=\phi(Q,I,PXPYQ).$$

Next, setting $X=P$ in the above equations, we have

$$\begin{aligned}\phi(P,I,PYQ)&=\phi(PYQ,I,Q),\\ \phi(PYQ,I,P)&=\phi(Q,I,PYQ).\end{aligned}\tag{4.4}$$

Next we will complete the proof of this part via the following 10 steps.

Step 1. Define $\psi:A\times A\times A\to\mathcal{W}$ by

$$\psi(a_1,a_2,a_3)=\phi\left(\begin{bmatrix}a_1&0\\0&0\end{bmatrix},\begin{bmatrix}a_2&0\\0&0\end{bmatrix},\begin{bmatrix}a_3&0\\0&0\end{bmatrix}\right)$$

for all $a_1,a_2,a_3\in A$. Therefore, ψ is a $\mathcal{R}$-trilinear map such that for all $a_1,a_2,a_3\in A$, $a_1\circ a_2\circ a_3=0$ implies $\psi(a_1,a_2,a_3)=0$. Since A is a zero Jordan triple product determined algebra, there is an $\mathcal{R}$-linear map $T_1:A\to\mathcal{W}$ such that $\psi(a_1,a_2,a_3)=T_1(a_1\circ a_2\circ a_3)$ for all $a_1,a_2,a_3\in A$. Therefore, $T_1(a)=\frac{1}{2}\psi(a,1_A,1_A)$ for any $a\in A$ and hence $\psi(a_1,a_2,a_3)=\frac{1}{2}\psi(a_1a_2a_3,1_A,1_A)+\frac{1}{2}\psi(a_3a_2a_1,1_A,1_A)$ for all $a_1,a_2,a_3\in A$, that is,

$$\phi(PXP,PYP,PZP)=\frac{1}{2}\phi(PXPYPZP,P,P)+\frac{1}{2}\phi(PZPYPXP,P,P),$$

or we can write it as

$$\phi(PXP, PYP, PZP) = \frac{1}{2}\phi(PXPYPZP, I, I) + \frac{1}{2}\phi(PZPYPXP, I, I).$$

Note that $PXP \circ (PYP + PYPZQ) \circ (Q - PZQ) = 0$ and $(PXP + PXPYPZQ) \circ (Q - PYPZQ) \circ Q = 0$; therefore,

$$\phi(PXP, PYP, PZQ) = \phi(PXP, PYPZQ, Q) = \phi(PXPYPZQ, Q, Q).$$

Thus, we get from (4.2) that

$$\phi(PXP, PYP, PZQ) = \phi(PXPYPZQ, I, Q) = \frac{1}{2}\phi(PXPYPZQ, I, I).$$

Taking account of $PXP \circ (PYP + QZPYP) \circ (Q - QZP) = 0$ and $(PXP + QZPYPXP) \circ (Q - QZPYP) \circ Q = 0$, we have

$$\phi(PXP, PYP, QZP) = \phi(PXP, QZPYP, Q) = \phi(QZPYPXP, Q, Q)$$

$$= \phi(QZPYPXP, I, Q) = \frac{1}{2}\phi(QZPYPXP, I, I),$$

where we used Eq. (4.2) in the last step. One can easily check that

$$\phi(PXP, PYP, QZQ) = 0.$$

Now we can claim that

$$\phi(PXP, PYP, Z) = \frac{1}{2}\phi(PXPYPZ, I, I) + \frac{1}{2}\phi(ZPYPXP, I, I). \tag{4.5}$$

Step 2. In a similar way, by the hypothesis that B is a zero Jordan triple determined algebra, one sees that

$$\phi(QXQ, QYQ, QZQ) = \frac{1}{2}\phi(QXQYQZQ, I, I) + \frac{1}{2}\phi(QZQYQXQ, I, I).$$

Note that $QXQ \circ (PZQYQ - QYQ) \circ (P + PZQ) = 0$ and $(PZQYQXQ - QXQ) \circ (P + PZQYQ) \circ P = 0$, by (4.2), we find that

$$\phi(QXQ, QYQ, PZQ) = \phi(QXQ, PZQYQ, P)$$

$$= \phi(PZQYQXQ, P, P) = \phi(PZQYQXQ, I, P) = \frac{1}{2}\phi(PZQYQXQ, I, I).$$

Similarly, by $QXQ \circ (QYQZP - QYQ) \circ (P + QZP) = 0$ and $(QXQYQZP - QXQ) \circ (P + QYQZP) \circ P = 0$ we can get

$$\phi(QXQ, QYQ, QZP) = \phi(QXQ, QYQZP, P)$$

$$= \phi(QXQYQZP, P, P) = \phi(QXQYQZP, I, P) = \frac{1}{2}\phi(QXQYQZP, I, I).$$

By $QXQ \circ QYQ \circ PZP = 0$ we have

$$\phi(QXQ, QYQ, PZP) = 0.$$

Therefore,

$$\phi(QXQ, QYQ, Z) = \frac{1}{2}\phi(ZQYQXQ, I, I) + \frac{1}{2}\phi(QXQYQZ, I, I). \tag{4.6}$$

Step 3. Considering $PXP \circ (P + QYP) \circ (QZQYP - QZQ) = 0$ and $(PXP + QZQYPXP) \circ I \circ (Q - QZQYP) = 0$, and applying (4.2) we obtain

$$\phi(PXP, QYP, QZQ) = \phi(PXP, P, QZQYP)$$
$$= \phi(PXP, I, QZQYP) = \phi(QZQYPXP, I, Q) = \frac{1}{2}\phi(QZQYPXP, I, I).$$

It is easy to check that

$$\phi(PXP, QYP, QZP) = 0,$$
$$\phi(PXP, QYP, PZQ) = 0,$$
$$\phi(PXP, QYP, PZP) = 0.$$

Now we claim that

$$\phi(PXP, QYP, Z) = \frac{1}{2}\phi(QZQYPXP, I, I). \tag{4.7}$$

Step 4. From $(PXP+PXPYQ)\circ(Q-PYQ)\circ QZQ = 0$ and $(P+PXPYQ)\circ I\circ(PXPYQZQ-QZQ) = 0$, and applying (4.4) we get

$$\phi(PXP, PYQ, QZQ) = \phi(PXPYQ, Q, QZQ) = \phi(PXPYQ, I, QZQ)$$
$$= \phi(P, I, PXPYQZQ) = \phi(PXPYQZQ, I, Q).$$

Taking into account (4.2), we obtain

$$\phi(PXP, PYQ, QZQ) = \phi(PXPYQZQ, I, Q) = \frac{1}{2}\phi(PXPYQZQ, I, I).$$

We can also get

$$\phi(PXP, PYQ, QZP) = 0,$$
$$\phi(PXP, PYQ, PZQ) = 0,$$
$$\phi(PXP, PYQ, PZP) = 0.$$

Thus we have

$$\phi(PXP, PYQ, Z) = \frac{1}{2}\phi(PXPYQZQ, I, I). \tag{4.8}$$

Step 5. Similarly, from $(Q - PXQ) \circ (PYP + PYPXQ) \circ PZP = 0$ and $Q \circ (Q - PYPXQ) \circ (PZP + PZPYPXQ) = 0$ we obtain

$$\phi(PXQ, PYP, PZP) = \phi(Q, PYPXQ, PZP) = \phi(Q, Q, PZPYPXQ)$$
$$= \phi(Q, I, PZPYPXQ) = \phi(PZPYPXQ, I, P) = \frac{1}{2}\phi(PZPYPXQ, I, I).$$

It is not difficult to verify

$$\phi(PXQ, PYP, QZP) = 0,$$

$$\phi(PXQ, PYP, PZQ) = 0,$$

$$\phi(PXQ, PYP, QZQ) = 0.$$

Therefore,

$$\phi(PXQ, PYP, Z) = \frac{1}{2}\phi(PZPYPXQ, I, I). \tag{4.9}$$

Step 6. We can routinely compute that $(P + PXQ) \circ (PXQYQ - QYQ) \circ QZQ = 0$ and $P \circ (P + PXQYQ) \circ (PXQYQZQ - QZQ) = 0$. This implies that

$$\phi(PXQ, QYQ, QZQ) = \phi(P, PXQYQ, QZQ) = \phi(P, I, PXQYQZQ),$$

and hence by (4.4) and (4.2) we get

$$\phi(PXQ, QYQ, QZQ) = \phi(PXQYQZQ, I, Q) = \frac{1}{2}\phi(PXQYQZQ, I, I).$$

Obviously,

$$\phi(PXQ, QYQ, QZP) = 0,$$

$$\phi(PXQ, QYQ, PZP) = 0,$$

$$\phi(PXQ, QYQ, PZQ) = 0.$$

As a consequence,

$$\phi(PXQ, QYQ, Z) = \frac{1}{2}\phi(PXQYQZQ, I, I). \tag{4.10}$$

Step 7. Likewise, using $(PYQXQ - QXQ) \circ (P + PYQ) \circ PZP = 0$ and $(Q - PYQXQ) \circ I \circ (PZP + PZPYQXQ) = 0$, we can also get

$$\phi(QXQ, PYQ, PZP) = \phi(PYQXQ, P, PZP) = \phi(PYQXQ, I, PZP)$$

$$= \phi(Q, I, PZPYQXQ) = (PZPYQXQ, I, P) = \frac{1}{2}\phi(PZPYQXQ, I, I).$$

Moreover,

$$\phi(QXQ, PYQ, QZP) = 0,$$

$$\phi(QXQ, PYQ, QZQ) = 0,$$

$$\phi(QXQ, PYQ, PZQ) = 0.$$

Therefore,

$$\phi(QXQ, PYQ, Z) = \frac{1}{2}\phi(PZPYQXQ, I, I). \tag{4.11}$$

Step 8. Since $(P+QXP)\circ(QYQXP-QYQ)\circ QZQ = 0$ and $P\circ(P+QYQXP)\circ(QZQYQXP-QZQ) = 0$, we have

$$\phi(QXP, QYQ, QZQ) = \phi(P, QYQXP, QZQ) = \phi(P, P, QZQYQXP)$$

$$= \phi(P, I, QZQYQXP).$$

Considering (4.2) and (4.3), we obtain

$$\phi(QXP, QYQ, QZQ) = \phi(P, I, QZQYQXP) = \phi(QZQYQXP, I, Q)$$
$$= \frac{1}{2}\phi(QZQYQXP, I, I).$$

Obviously,

$$\phi(QXP, QYQ, QZP) = 0,$$
$$\phi(QXP, QYQ, PZP) = 0,$$
$$\phi(QXP, QYQ, PZQ) = 0.$$

Hence,

$$\phi(QXP, QYQ, Z) = \frac{1}{2}\phi(QZQYQXP, I, I). \tag{4.12}$$

Step 9. Similarly, from $(QXQYP - QXQ) \circ (P + QYP) \circ PZP = 0$ and $(Q - QXQYP) \circ I \circ (PZP + QXQYPZP) = 0$ we get

$$\phi(QXQ, QYP, PZP) = \phi(QXQYP, P, PZP) = \phi(Q, I, QXQYPZP)$$
$$= \phi(QXQYPZP, I, P) = \frac{1}{2}\phi(QXQYPZP, I, I).$$

It is easy to check that

$$\phi(QXQ, QYP, QZP) = 0,$$
$$\phi(QXQ, QYP, PZQ) = 0,$$
$$\phi(QXQ, QYP, QZQ) = 0.$$

Now we have

$$\phi(QXQ, QYP, Z) = \frac{1}{2}\phi(QXQYPZP, I, I). \tag{4.13}$$

Step 10. Likewise, by $(Q - QXP) \circ (PYP + QXPYP) \circ PZP = 0$ and $Q \circ (Q - QXPYP) \circ (PZP + QXPYPZP) = 0$ we obtain

$$\phi(QXP, PYP, PZP) = \phi(Q, QXPYP, PZP) = \phi(Q, Q, QXPYPZP)$$
$$= \phi(Q, I, QXPYPZP) = \phi(QXPYPZP, I, P) = \frac{1}{2}\phi(QXPYPZP, I, I).$$

Particularly,

$$\phi(QXP, PYP, QZP) = 0,$$
$$\phi(QXP, PYP, PZQ) = 0,$$
$$\phi(QXP, PYP, QZQ) = 0.$$

Consequently,

$$\phi(QXP, PYP, Z) = \frac{1}{2}\phi(QXPYPZP, I, I). \tag{4.14}$$

In view of Eqs. (4.1) and (4.5)–(4.14), we have

$$\begin{aligned}\phi(X,Y,Z) =&\phi(PXP,PYP,Z)+\phi(QXQ,QYQ,Z)+\phi(PXP,PYQ,Z)+\phi(PXP,QYP,Z)\\&+\phi(PXQ,QYQ,Z)+\phi(PXQ,PYP,Z)+\phi(PXQ,QYP,Z)+\phi(QXQ,PYQ,Z)\\&+\phi(QXQ,QYP,Z)+\phi(QXP,PYQ,Z)+\phi(QXP,QYQ,Z)+\phi(QXP,PYP,Z)\end{aligned}$$

$$\begin{aligned}=&\frac{1}{2}\phi(PXPYPZ,I,I)+\frac{1}{2}\phi(ZPYPXP,I,I)\\&+\frac{1}{2}\phi(QXQYQZ,I,I)+\frac{1}{2}\phi(ZQYQXQ,I,I)\\&+\frac{1}{2}\phi(PXPYQZQ,I,I)+\frac{1}{2}\phi(QZQYPXP,I,I)\\&+\frac{1}{2}\phi(PXQYQZQ,I,I)+\frac{1}{2}\phi(PZPYPXQ,I,I)\\&+\frac{1}{2}\phi(QXQYPZP,I,I)+\frac{1}{2}\phi(PZPYQXQ,I,I)\\&+\frac{1}{2}\phi(QXPYPZP,I,I)+\frac{1}{2}\phi(QZQYQXP,I,I)\end{aligned}$$

$$\begin{aligned}=&\frac{1}{2}\phi(PXPYPZ,I,I)+\frac{1}{2}\phi(ZPYPXP,I,I)\\&+\frac{1}{2}\phi(QXQYQZ,I,I)+\frac{1}{2}\phi(ZQYQXQ,I,I)\\&+\frac{1}{2}\phi(PXPYQZ,I,I)+\frac{1}{2}\phi(ZQYPXP,I,I)\\&+\frac{1}{2}\phi(PXQYQZ,I,I)+\frac{1}{2}\phi(ZPYPXQ,I,I)\\&+\frac{1}{2}\phi(QXQYPZ,I,I)+\frac{1}{2}\phi(ZPYQXQ,I,I)\\&+\frac{1}{2}\phi(QXPYPZ,I,I)+\frac{1}{2}\phi(ZQYQXP,I,I),\end{aligned}$$

and so $\phi(X,Y,Z)=\frac{1}{2}\phi(XYZ,I,I)+\frac{1}{2}\phi(ZYX,I,I)$. If we define the $\mathcal{R}$-linear mapping $T:\mathcal{G}\to\mathcal{W}$ by $T(Z)=\frac{1}{2}\phi(Z,I,I)$ for all $Z\in\mathcal{G}$, then T satisfies all the requirements in the definition of zero Jordan triple product determined algebras. Thus, $\mathcal{G}$ is a zero Jordan triple product determined algebra.

Conversely, assume $\mathcal{G}$ is a zero Jordan triple product determined algebra. Let $\mathcal{W}$ be an $\mathcal{R}$-module and $\psi: A\times A\times A\to\mathcal{W}$ be an $\mathcal{R}$-trilinear map such that for all $a_1,a_2,a_3\in\mathcal{A}, a_1\circ a_2\circ a_3=0$ implies $\psi(a_1,a_2,a_3)=0$. Define an $\mathcal{R}$-trilinear map $\phi:\mathcal{G}\times\mathcal{G}\times\mathcal{G}\to\mathcal{W}$ by $\phi(X',Y',Z')=\psi(a_1,a_2,a_3)$ where $X'=\begin{pmatrix}a_1&m_1\\n_1&b_1\end{pmatrix}, Y'=\begin{pmatrix}a_2&m_2\\n_2&b_2\end{pmatrix}, Z'=\begin{pmatrix}a_3&m_3\\n_3&b_3\end{pmatrix}\in\mathcal{G}$. If $X\circ Y\circ Z=0$, then $\phi(X,Y,Z)=0$. Since $\mathcal{G}$ is a zero Jordan triple product determined algebra, there is an $\mathcal{R}$-linear map $T:\mathcal{G}\to\mathcal{W}$ such that $\phi(X',Y',Z')=T(X'\circ Y'\circ Z')$ for all $X',Y',Z'\in\mathcal{G}$. Define an $\mathcal{R}$-linear map $T_1: A\to\mathcal{W}$ by $T_1(a)=T\begin{pmatrix}a&0\\0&0\end{pmatrix}$ for all $a\in A$. Therefore, $\psi(a_1,a_2,a_3)=T_1(a_1\circ a_2\circ a_3)$ for all $a_1,a_2,a_3\in A$. Thus, A is a zero Jordan triple product determined algebra. Similarly, we can show that B is a zero Jordan triple product determined algebra. The proof is completed.

A direct consequence of this theorem is the following:

Corollary 4.2 *$\mathcal{T}^{\mathcal{U}}$ (resp. $\mathcal{T}_{\mathcal{L}}$) is a zero Jordan triple product determined algebra if and only if A and B are zero Jordan triple product determined algebras.*

By Theorem 4.1, we can also get the following corollary.

Corollary 4.3 *$\mathcal{T}_n(A)$ (resp. $\mathcal{T}'_n(A)$) is a zero Jordan triple product determined algebra if and only if A is a zero triple product determined algebra.*

Corollary 4.4 *Let $B_n^{\bar{d}}(A)$ (resp. $B_n'^{\bar{d}}(A)$) be a block upper triangular matrix algebra, where $\bar{d} = (d_1, \cdots, d_i, \cdots, d_m) \in \mathbb{N}^m$. Then we have*

(a) If A is zero Jordan triple product determined algebra, then $B_n^{\bar{d}}(A)$ (resp. $B_n'^{\bar{d}}(A)$) is a zero Jordan triple product determined algebra.

(b) If $d_j = 1$ for some $1 \geq j \geq m$, then $B_n^{\bar{d}}(A)$ (resp. $B_n'^{\bar{d}}(A)$) is a zero Jordan triple product determined algebra if and only if A is a zero Jordan triple product determined algebra.

Acknowledgments

I would like to thank the referee for his/her valuable suggestions and comments, which help to improve the presentation of this paper. This work is supported by the Doctor Foundation of Henan Polytechnic University (B2010-21), the National Natural Science Foundation of China (No.11301155), and the Research Project for Young Teachers of Higher School of Education Department of Henan Province (No. 2013GGJS-054).

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