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On separating subadditive maps

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Abstract: Recall that a map $T\colon C(X,E)\to C(Y,F)$, where X,Y are Tychonoff spaces and E,F are normed spaces, is said to be separating, if for any 2 functions $f,g\in C(X,E)$ we have $c(T(f))\cap c(T(g))=\varnothing$ provided $c(f)\cap c(g)=\varnothing$. Here $c(f)$ is the co-zero set of f. A typical result generalizing the Banach–Stone theorem is of the following type (established by Araujo): if T is bijective and additive such that both T and T^{-1} are separating, then the realcompactification νX of X is homeomorphic to νY. In this paper we show that a similar result is true if additivity is replaced by subadditivity (a map T is called subadditive if $||T(f+g)(y)||\leq||T(f)(y)||+||T(g)(y)||$ for any $f,g\in C(X,E)$ and any $y\in Y$). Here is our main result (a stronger version is actually established): if $T\colon C(X,E)\to C(Y,F)$ is a separating subadditive map, then there exists a continuous map $S_Y\colon\beta Y\to\beta X$. Moreover, S_Y is surjective provided $T(f)=0$ iff $f=0$. In particular, when T is a bijection such that both T and T^{-1} are separating and subadditive, βX is homeomorphic to βY. We also provide an example of a biseparating subadditive map from $C(\mathbb{R})$ onto $C(\mathbb{R})$, which is not additive.

Key words: Function spaces, separating maps, supports, subadditive maps

1. Introduction

Recall that the Banach–Stone theorem [2,8] states that 2 compact spaces X and Y are homeomorphic provided there exists a linear isometry between the sup norm Banach spaces $C(X)$ and $C(Y)$ (everywhere below $C(X,Z)$ denotes the set of all continuous maps from X to Z; if Z is the real line we just write $C(X)$). In order to generalize this theorem, the so-called separating maps were introduced in [4]. Separating maps were explored in several other papers; see the survey in [5].

The definition of a separating map usually requires linearity or additivity of that map. To the best of this author's knowledge, subadditive separating maps were considered in only 2 papers [3,7]. The first was devoted to subadditive separating maps between function spaces $C(X)$ and $C(Y)$, where X and Y are compact spaces, while the second generalized the results of [3] to regular Banach algebras. Additive separating maps between vector-valued function spaces were considered in [1]. In this paper we show that some of the results established for separating additive maps remain true when additivity is weakened to subadditivity.

If E and F are normed linear spaces and X, Y 2 Tychonoff spaces, we consider maps $T\colon L(X,E)\to L(Y,F)$, where $L(X,E)$ and $L(Y,F)$ are linear subspaces of the function spaces $C(X,E)$ and $C(Y,F)$, respectively. Recall that such a map $T\colon L(X,E)\to L(Y,F)$ is said to be *separating* if $c(f)\cap c(g)=\varnothing$

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implies $c(T(f)) \cap c(T(g)) = \varnothing$ for every $f, g \in L(X,E)$, where, $c(f) = \{x \in X : f(x) \neq 0_E\}$ with 0_E being the zero element of E. When T is bijective and both T and T^{-1} are separating, T is called *biseparating*.

For any map $T\colon L(X,E) \to L(Y,F)$ we consider the maps $\mu_y : L(X,E) \to \beta F$ and $\varphi_y : L(X,E) \to [0,\infty]$, $y \in \beta Y$, defined by $\mu_y(f) = \beta(T(f))(y)$ and $\varphi_y(f) = |||\ \mu_y(f)\ |||$. Here, βF denotes the Čech–Stone compactification of F and $|||\ .\ ||| : \beta F \to [0,\infty]$ is the continuous extension of the norm $\|.\|$ of F considered as a function from F into $[0,\infty)$. Let us also explain that $\beta(T(f))$ is the Čech–Stone extension of the map $T(f)$, so $\beta(T(f))$ is a map from βY to βF (generally, if $h\colon Z_1 \to Z_2$, then $\beta h\colon \beta Z_1 \to \beta Z_2$). We say that T is *subadditive* if each φ_y is subadditive, i.e. $\varphi_y(f+g) \leq \varphi_y(f) + \varphi_y(g)$ for all $y \in Y$. According to Proposition 3.2 below, if $||T(f+g)(y)|| \leq ||T(f)(y)|| + ||T(g)(y)||$ for any $f, g \in L(X,E)$ and any y from a dense subset of Y, then all φ_y, $y \in \beta Y$, are subadditive.

As usual (see [3,9]), the support of μ_y (resp., φ_y), $y \in \beta Y$, is defined to be the set $\mathrm{supp}(\mu_y)$ (resp., $\mathrm{supp}(\varphi_y)$) of all $x \in \beta X$ such that for every neighborhood U of x in βX there is $f \in L(X,E)$ with $\beta f|(\beta X - U) = 0$ and $\mu_y(f) \neq 0$ (resp., $\varphi_y(f) \neq 0$). It follows from the definition that a point $x \in \beta X$ does not belong to $\mathrm{supp}(\mu_y)$ if there exists its neighborhood U in βX such that for every $f \in L(X,E)$ with $\beta f|(\beta X - U) = 0$ we have $\mu(f) = 0$. This implies that $\mathrm{supp}(\mu_y)$ are closed in βX. Similarly, $\mathrm{supp}(\varphi_y)$ are also closed in βX, and obviously $\mathrm{supp}(\varphi_y) \subset \mathrm{supp}(\mu_y)$ for every $y \in \beta Y$. Let us observe that $\mathrm{supp}(\varphi_y) = \mathrm{supp}(\mu_y)$ for all $y \in Y$.

We say that a family $\mathcal{A} \subset C(X,E)$ *separates the points of* βX if for every $x \in \beta X$ there exists $f \in \mathcal{A}$ with $|||\ (\beta f)(x)\ ||| \neq 0$ (for example, this is true if $\mathcal{A} = C(X,F)$ or $\mathcal{A} = C^*(X,F)$). Denote also by $\mathrm{Ker}(T)$ the set $\{f \in L(X,E) : T(f) = 0\}$. According to Corollary 2.3(i) below, $\mathrm{Ker}(T)$ contains the constant function 0 provided that T is subadditive.

Everywhere below we suppose that $L(X,E)$ and $L(Y,F)$ have the following properties: there exist subsets $A \subset C^*(X)$ and $B \subset C^*(Y)$ such that

- $L(X,E)$ is an A-module and $L(Y,F))$ is a B-module;
- for any finite open cover $\gamma = \{U_1,..,U_k\}$ of βX (resp., of βY) there exist functions $\{h_1,..,h_k\}$ from A (resp., from B) such that $\{h_1,..,h_k\}$ form a partition of unity subordinated to γ.

Our first result is the following theorem:

Theorem 1.1 *Let $T : L(X,E) \to L(Y,F)$ be a subadditive separating map such that $T(L(X,E))$ separate the points of βY. Then the support map $S_Y : \beta Y \to \beta X$, $S_Y(y) = \mathrm{supp}(\varphi_y)$, is single-valued and continuous. If, in addition, $L(X,E)$ separates the points of βX and $\mathrm{Ker}(T) = 0$, then $S_Y(\beta Y) = \beta X$.*

Corollary 1.2 *Let $L(X,E)$ and $L(Y,F)$ separate the points of βX and βY, respectively. If $T : L(X,E) \to L(Y,F)$ is a subadditive biseparating bijection such that T^{-1} is also subadditive, then the supporting map $S_Y : \beta Y \to \beta X$ is a homeomorphism.*

In our next results the requirement for T^{-1} to be subadditive is weakened. We show that any subadditive separating map $T : L(X,E) \to L(Y,F)$ is *strongly separating*, where T is strongly separating if for any $f, g \in L(X,E)$

$$\overline{c(f)}^{\beta X} \cap \overline{c(g)}^{\beta X} = \varnothing \text{ implies } \overline{c(T(f))}^{\beta Y} \cap \overline{c(T(g))}^{\beta Y} = \varnothing.$$

If T^{-1} in Corollary 1.2 is strongly separating (instead of being subadditive and separating), we still have that βX and βY are homeomorphic:

Theorem 1.3 *Let $L(X,E)$ and $L(Y,F)$ separate the points of βX and βY, respectively. If $T: L(X,E) \to L(Y,F)$ is a subadditive separating bijection such that T^{-1} is strongly separating, then the supporting map $S_Y : \beta Y \to \beta X$ is a homeomorphism.*

Question 1.4 *Is it true that the realcompactifications νX and νY are homeomorphic provided there exists a map $T: L(X,E) \to L(Y,F)$ satisfying the requirements from Corollary* 1.2 *?*

It follows from Corollary 1.2 that the above question has a positive answer provided both X and Y are first countable (then βX being homeomorphic to βY implies that νX and νY are also homeomorphic; see [6]). According to a result of Araujo [1, Theorem 3.1], the above question also has a positive answer if T is additive.

2. Proof of Theorem 1.1 and Corollary 1.2

Everywhere in this section, we assume that $T: L(X,E) \to L(Y,F)$ is a fixed subadditive map.

We extend the operations $a+b$ and $|a-b|$ on $[0,\infty]$ by defining $\infty + a = \infty$ for every $a \in [0,\infty]$, $|\infty - a| = |a - \infty| = \infty$ for $a \in [0,\infty)$ and $|\infty - \infty| = 0$.

Lemma 2.1 *For all $y \in \beta Y$ and $f, g \in L(X,E)$ we have $|\varphi_y(f) - \varphi_y(g)| \le \max\{\varphi_y(f-g), \varphi_y(g-f)\}$.*

Proof This inequality follows directly from subadditivity of the functions φ_y. □

Lemma 2.2 *Suppose $y \in \beta Y$ and U is a neighborhood of* $\operatorname{supp}(\varphi_y)$ *in βX. Then $\varphi_y(f) = 0$ for every $f \in L(X,E)$ with $\beta f = 0$ on U.*

Proof For every $x \notin \operatorname{supp}(\varphi_y)$ take a neighborhood $U(x)$ of x in βX such that $\varphi_y(g) = 0$ provided $g \in L(X,E)$ and $\beta g|(\beta X - U(x)) = 0$. We can suppose that all $U(x)$ coincide with the interior of their closures in βX and are disjoint from $\operatorname{supp}(\varphi_y)$. Take a finite cover $\gamma = \{U, U(x_i) : i = 1,2,..,k\}$ of βX and a real-valued function $\{h, h_i\}_{i \le k}$ from A forming a partition of unity subordinated to γ. Now, suppose $\beta f(U) = 0$ for some $f \in L(X,E)$. Set $g_0 = h \cdot f$ and $g_i = h_i \cdot f$. Since $L(X,E)$ is an A-module, $g_i \in L(X,E)$ for all $i = 0,1,..,k$. Obviously, $g_0 \equiv 0$. Moreover, $g_i|(X - U(x_i)) = 0$, $i = 1,,.,k$, and because $X - U(x_i)$ is dense in $\beta X - U(x_i)$, we have $\beta g_i|(\beta X - U(x_i)) = 0$. Hence, $\varphi_y(g_i) = 0$ for all $i = 1,..,k$. Finally, since $f = \sum\{g_i : i = 1,..,k\}$, the subadditivity of φ_y implies $\varphi_y(f) \le \sum\{\varphi_y(g_i) : i = 1..,k\}$. Therefore, $\varphi_y(f) = 0$. □

Corollary 2.3 *The following conditions are satisfied:*

(i) $T(0) = 0$*;*

(ii) if $\varphi_y(f) \ne 0$, where $y \in \beta Y$ and $f \in L(X,E)$, then $\operatorname{supp}(\varphi_y)$ *intersects the closure in βX of the set $c(\beta f) = \{z \in \beta X : (\beta f)(z) \ne 0\}$;*

(iii) if $U \subset \beta X$ *is open and* $f, g \in L(X,E)$ *are 2 functions with* $f(x) = g(x)$ *for all* $x \in U$ *, then* $\varphi_y(f) = \varphi_y(g)$ *for all* $y \in \beta Y$ *such that* $\mathrm{supp}(\varphi_y) \subset U$ *.*

Proof The first 2 items follow directly from Lemma 2.2. Since the functions $f - g$ and $g - f$ are 0 on U, the third item follows from Lemmas 2.1 and 2.2. □

Recall that a set-valued map $F : Y \to X$ is called lower semicontinuous (br., lsc) if $F^{-1}(V) = \{y \in Y : F(y) \cap V \neq \emptyset\}$ is open in Y for every open $V \subset X$.

Lemma 2.4 *The set-valued map* $\mathrm{supp}(\varphi_y) : \beta Y \to \beta X$ *is lsc.*

Proof Suppose $x \in \mathrm{supp}(\varphi_y) \cap U$ for some $y \in \beta Y$ and an open $U \subset \beta X$. Take an open set $W \subset \beta X$ such that $x \in W$ and $\overline{W} \subset U$. Since $x \in \mathrm{supp}(\varphi_y)$, there exists $f \in L(X,E)$ with $\beta f(\beta X - W) = 0$ and $\varphi_y(f) \neq 0$. Let $c_\varphi(f) = \{z \in \beta Y : \varphi_z(f) \neq 0\}$. Obviously, $c_\varphi(f)$ is open in βY and contains y. If there is $z \in c_\varphi(f)$ such that $\mathrm{supp}(\varphi_z) \cap U = \varnothing$, then $\mathrm{supp}(\varphi_z) \subset \beta X - \overline{W}$. Thus, by Lemma 2.2, $\varphi_z(f) = 0$, which contradicts $z \in c_\varphi(f)$. Therefore, $\mathrm{supp}(\varphi_z) \cap U \neq \emptyset$ for all $z \in c_\varphi(f)$. □

Lemma 2.5 *If* $L(X,E)$ *separates the points of* βX *and* $\mathrm{Ker}(T) = 0$*, then* $\bigcup\{\mathrm{supp}(\varphi_y) : y \in \beta Y\}$ *is dense in* βX*.*

Proof Suppose $P = \overline{\cup\{\mathrm{supp}(\varphi_y) : y \in \beta Y\}} \neq \beta X$ and take an open set $U \subset \beta X$ such that $\overline{U} \cap P = \varnothing$ and $X - U$ is dense in $\beta X - U$. According to the properties of A, there exists $h \in A$ and $x \in U \cap X$ with $h(x) \neq 0$ and $h(\beta X - U) = 0$. On the other hand, since $L(X,E)$ separates the points of βX, there is $g \in L(X,E)$ with $\beta g(x) \neq 0$. Then $f = g \cdot h \in L(X,E)$ and $f \neq 0$. Moreover, βf is 0 on the set $\beta X - \overline{U}$. Hence, according to Lemma 2.2, $\varphi_y(f) = 0$ for every $y \in \beta Y$. This implies $T(f) = 0$, which contradicts $\mathrm{Ker}(T) = 0$. □

Lemma 2.6 *If* $T(L(X,E))$ *separates the points of* βY*, then* $\mathrm{supp}(\varphi_y) \neq \varnothing$ *for all* $y \in \beta Y$*.*

Proof Suppose $\mathrm{supp}(\varphi_y) = \varnothing$ for some $y \in \beta Y$. As in the proof of Lemma 2.2, we can choose a finite open cover $\gamma = \{U_i; i = 1,..,k\}$ of βX such that each U_i has the following property: $\varphi_y(g) = 0$ provided βg is 0 on the set $\beta X - U_i$. If $\{h_i : i = 1,..,k\} \subset A$ is a partition of unity subordinated to γ, then $\varphi_y(f \cdot h_i) = 0$ for all $f \in L(X,E)$ and $i \leq k$. Consequently, $\varphi_y(f) = 0$ for all $f \in L(X,E)$ (see the proof of Lemma 2.2). On the other hand, because $T(L(X,E))$ separates the points of βY, $\varphi_y(f_0) \neq 0$ for some $f_0 \in L(X,E)$, a contradiction. □

The next lemma is the first one in this section using that T is separating (observe that the subadditivity of T is not used).

Lemma 2.7 *If* T *is separating, then each* $\mathrm{supp}(\varphi_y)$*,* $y \in \beta Y$*, contains at most one point.*

Proof Since $\mathrm{supp}(\varphi_y) \subset \mathrm{supp}(\mu_y)$, it suffices to show that $\mathrm{supp}(\mu_y)$ consists of no more than one point. Suppose $\mathrm{supp}(\mu_y)$ contains 2 different points x_1 and x_2 for some $y \in \beta Y$. Let U_1, U_2 be disjoint open subsets of βX with $x_i \in U_i$, $i = 1, 2$. Then, according to the definition of $\mathrm{supp}(\mu_y)$, there exist $f_1, f_2 \in L(X,E)$ such that $\beta f_i|(\beta X - U_i) = 0$ and $\mu_y(f_i) \neq 0$, $i = 1, 2$. Consider the sets $V_i = \{z \in \beta Y : \beta(T(f_i))(z) \neq 0\}$. Obviously, V_1 and V_2 are open in βY and both contain y. Therefore, $V_1 \cap V_2$ meets Y. On the other hand,

$V_i \cap Y = c(T(f_i))$, $i = 1, 2$. Hence, $c(T(f_1)) \cap c(T(f_2)) \neq \varnothing$, which contradicts the fact that $c(f_1) \cap c(f_2) = \varnothing$ and T is separating. □

Proof of Theorem 1.1.

Suppose T is a subadditive separating map such that $T(L(X,E))$ separate the points of βY. It follows from Lemma 2.6 and Lemma 2.7 that $\mathrm{supp}(\varphi_y)$ consists of exactly one point for every $y \in \beta Y$, and we define $S_Y(y) = \mathrm{supp}(\varphi_y)$. By Lemma 2.4, S_Y is continuous (recall that every single-valued lsc map is continuous). If, in addition, $L(X,E)$ separates the points of βX and $\mathrm{Ker}(T) = 0$, then $S_Y(\beta Y)$ is dense in βX (see Lemma 2.5). Hence, $S_Y(\beta Y) = \beta X$. □

Proof of Corollary 1.2.

For any $x \in \beta X$ we define the map $\psi_x : L(Y,F) \to [0,\infty]$, $\psi_x(g) = |||\beta(T^{-1}(g))(x)|||$, where $|||\,.\,||| : \beta E \to [0,\infty]$ is the continuous extension of the norm $\|.\|$ of E considered as a function from E into $[0,\infty)$. Because T and T^{-1} are subadditive and separating, by Theorem 1.1, both $S_Y\colon \beta Y \to \beta X$ and $S_X\colon \beta X \to \beta Y$, $S_X(x) = \mathrm{supp}(\psi_x)$, are single-valued and continuous surjections.

We claim that $S_X(S_Y(y)) = y$ for all $y \in \beta Y$. Indeed, if $y_0 \neq S_X(S_Y(y_0)) = y_1$ for some $y_0 \in \beta Y$, we take disjoint open sets U and V in βY with $y_0 \in U$ and $y_1 \in V$. Since $y_1 \in \mathrm{supp}(\psi_{x_0})$, where $x_0 = S_Y(y_0)$, there exists a function $g \in L(Y,F)$ such that $c(\beta g) \subset V$ and $\psi_{x_0}(g) \neq 0$. Thus, $\beta(T^{-1}(g))(x_0) \neq 0$. We choose a function $f \in L(Y,F)$ with $|||\,\beta f(y_0)\,||| \neq 0$. Because $x_0 = \mathrm{supp}(\varphi_{y_0})$, by Lemma 2.3(ii), $x_0 \in \overline{c(\beta(T^{-1}(f))}$. This implies $c(\beta(T^{-1}(g))) \cap c(\beta(T^{-1}(f))) \neq \varnothing$. Consequently, $c(T^{-1}(g)) \cap c(T^{-1}(f))) \neq \varnothing$, which contradicts that T^{-1} is separating and $c(f) \cap c(g) = \varnothing$. Therefore, S_Y is a homeomorphism. □

3. Proof of Theorem 1.3

Proposition 3.1 *Any subadditive separating surjection $T : L(X,E) \to L(Y,F)$, where $L(Y,F)$ separates the points of βY, is strongly separating.*

Proof Suppose $\overline{c(f)}^{\beta X} \cap \overline{c(g)}^{\beta X} = \varnothing$ for some $f, g \in L(X,E)$, but there exists $y_0 \in \overline{c(T(f))}^{\beta Y} \cap \overline{c(T(g))}^{\beta Y}$. According to Lemmas 2.4, 2.6, and 2.7, the map $\mathrm{supp}(\varphi_y)\colon \beta Y \to \beta X$ is well-defined and continuous. We are going to show that $\mathrm{supp}(\varphi_{y_0}) \in \overline{c(f)}^{\beta X}$. Indeed, otherwise there would be a neighborhood U of y_0 in βY such that $\mathrm{supp}(\varphi_y) \notin \overline{c(f)}^{\beta X}$ for all $y \in U$. Take a point $y_1 \in U \cap c(T(f))$. Then $\mathrm{supp}(\varphi_{y_1}) \notin \overline{c(f)}^{\beta X} = \overline{c(\beta f)}^{\beta X}$. On the other hand, by Corollary 2.3(ii), $\mathrm{supp}(\varphi_{y_1}) \in \overline{c(\beta f)}^{\beta X}$. Hence, $\mathrm{supp}(\varphi_{y_0}) \in \overline{c(f)}^{\beta X}$. Similarly, $\mathrm{supp}(\varphi_{y_0}) \in \overline{c(g)}^{\beta X}$, which completes the proof. □

Proof of Theorem 1.3

Proof By Theorem 1.1, the supporting map S_Y is a single-valued continuous surjection. Therefore, we need only to prove that S_Y is one-to-one. Suppose $S_Y^{-1}(x_0)$ contains 2 different points y_1 and y_2 for some $x_0 \in \beta X$. Then there exist 2 functions $g_1, g_2 \in L(Y,F)$ such that $|||(\beta g_i)(y_i)||| \neq 0$, $i = 1, 2$, and $\overline{c(\beta g_1)}^{\beta Y} \cap \overline{c(\beta g_2)}^{\beta Y} = \varnothing$.

Since T^{-1} is strongly separating, we have $\overline{c(\beta f_1)}^{\beta X} \cap \overline{c(\beta f_2)}^{\beta X} = \varnothing$, where $f_i = T^{-1}(g_i)$. Obviously, $\varphi_{y_i}(f_i) = (\beta g_i)(y_i)$. Thus, by Corollary 2.3(ii), x_0 belongs to $\overline{c(\beta f_i)}^{\beta X}$, $i = 1, 2$, a contradiction. Therefore, S_Y is bijective. □

The next proposition establishes a sufficient condition for T to be subadditive.

Proposition 3.2 *If $M \subset Y$ is dense and $||T(f+g)(y)|| \leq ||T(f)(y)|| + ||T(g)(y)||$ for any $f, g \in L(X, E)$ and any $y \in M$, then T is subadditive.*

Proof Fix $y \in \beta Y$ and $f, g \in L(X, E)$, and take a net $\{y_\alpha\}$ in M converging to y. Then for each α we have $||T(f + g)(y_\alpha)|| \leq ||T(f)(y_\alpha)|| + ||T(g)(y_\alpha)||$. This implies $\varphi_y(f + g) \leq \varphi_y(f) + \varphi_y(g)$ because the net $\{T(h)(y_\alpha)\}$ converges to $\beta T(h)(y)$ for any $h \in L(X, E)$ and the map $|||.|||$ is continuous on βF. □

Finally, we provide an example of a subadditive biseparating map between 2 function spaces, which is not additive.

Example 3.3 *There exists a subadditive biseparating map $T\colon C(\mathbb{R}) \to C(\mathbb{R})$, which is not additive.*

Proof Define the map $\phi\colon \mathbb{R} \to \mathbb{R}$ by $\phi(x) = \sqrt{x}$ if $x \geq 0$ and $\phi(x) = -\sqrt{-x}$ if $x \leq 0$. It is easily seen that ϕ is subadditive and surjective, but not additive. Then the map $T\colon C(\mathbb{R}) \to C(\mathbb{R})$, $T(f)(x) = \phi(f(x))$, is subadditive and injective. Since $T(f)(x) = 0$ if and only if $f(x) = 0$, T is biseparating. □

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