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Existence of solutions for a first-order nonlocal boundary value problem with changing-sign nonlinearity

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Abstract: This work is concerned with the existence of positive solutions to a nonlinear nonlocal first-order multipoint problem. Here the nonlinearity is allowed to take on negative values, not only positive values.

Key words: Positive solution, nonlinear boundary condition, sign-changing problem

1. Introduction

In this paper, we are interested in the existence of positive solutions for the following first-order m-point nonlocal boundary value problem:

$$y'(t) + p(t)y(t) = \sum_{i=1}^{n} f_i(t, y(t)), \quad t \in [0,1], \tag{1.1}$$

$$y(0) = y(1) + \sum_{j=1}^{m} g_j(t_j, y(t_j)), \tag{1.2}$$

where $p : [0,1] \to [0,\infty)$ is continuous, the nonlocal points satisfy $0 \le t_1 < t_2 < ... < t_m \le 1$, and the nonlinear functions $f_i : [0,1] \times [0,\infty) \to (-\infty,\infty)$ and $g_j : [0,1] \times [0,\infty) \to [0,\infty)$ are continuous.

First-order equations with various boundary conditions, including multipoint and nonlocal conditions, are of recent interest; see [3–11,13] and the references therein.

In [12], Zhao applied a monotone iteration method to the problem (if $\mathbb{T} = \mathbb{R}$):

$$y'(t) + p(t)y(t) = f(t, y(t)), \quad t \in [0,1],$$
$$y(0) = g(x(1)),$$

where the functions f and g are positive-valued continuous functions.

In [2], using the Guo–Krasnosel'skiĭ fixed point theorem, Anderson was interested in the existence of at least one positive solution to the problem (if $\mathbb{T} = \mathbb{R}$):

$$y'(t) + p(t)y(t) = \lambda f(t, y(t)), \qquad t \in [0,1],$$

$$y(0) = y(1) + \sum_{j=2}^{n-1} \gamma_i y(t_j),$$

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where the function $f:(0,1)\times[0,\infty)\to(-\infty,\infty)$ is continuous.

In [3], using the Legget–Williams fixed point theorem, Anderson studied the existence of at least three positive solutions to the nonlinear first-order problem with a nonlinear nonlocal boundary condition given by

$$y'(t)-r(t)y(t)=\lambda\sum_{i=1}^{m}f_i(t,y(t)),\qquad t\in[0,1],$$

$$\lambda y(0)=y(1)+\sum_{j=1}^{n}\Lambda_j(\tau_j,y(\tau_j)),$$

where $r:[0,1]\to[0,\infty)$ is continuous, and the nonlinear functions $f_i:[0,1]\times[0,\infty)\to[0,\infty)$ and $\Lambda_j:[0,1]\times[0,\infty)\to[0,\infty)$ are also continuous.

In [7], Goodrich considered the existence of at least one positive solution to the first-order semipositone discrete fractional boundary value problem:

$$\triangle^{\nu}y(t)=\lambda f(t+\nu-1,y(t+\nu-1)),\qquad t\in[0,T]_{\mathbb{Z}},$$

$$y(\nu-1)=y(\nu+T)+\sum_{i=1}^{N}F(\tau_i,y(\tau_i)).$$

In a recent paper [8], using Krasnosel'skiĭ's fixed point theorem, Goodrich studied the existence of a positive solution to the first-order problem given by (if $\mathbb{T}=\mathbb{R}$)

$$y'(t)+p(t)y(t)=\lambda f(t,y(t)),\quad t\in(a,b),$$
$$y(a)=y(b)+\int_{\tau_1}^{\tau_2}F(s,y(s))ds,$$

where $\tau_1,\tau_2\in[a,b]_{\mathbb{T}}$ with $\tau_1<\tau_2$, p and F are nonnegative functions, and the nonlinearity f can be negative for some values of t and y.

Motivated greatly by the above-mentioned works, in this paper, we are interested in the existence and iteration of positive solutions for the nonlinear nonlocal first-order multipoint problem $(1.1)-(1.2)$. By applying the monotone iteration method, we not only obtain the existence of positive solutions, but we also establish iterative schemes for approximating the solutions. The following monotone iteration method [1] is fundamental and important to the proof of our main result.

Theorem 1.1 *Let K be a cone in a Banach space E and $v_0\leq w_0$. Suppose that:*

(i) $T:[v_0,w_0]\to E$ is completely continuous;

(ii) T is monotone increasing on $[v_0,w_0]$;

(iii) v_0 is a lower solution of T, that is $v_0\leq Tv_0$;

(iv) w_0 is an upper solution of T, that is $Tw_0\leq w_0$.

Then the iterative sequences

$$v_n=Tv_{n-1},\quad w_n=Tw_{n-1}(n=1,2,3,...)$$

satisfy

$$v_0 \le v_1 \le ... \le v_n \le ... \le w_n \le ... \le w_1 \le w_0,$$

and converge to, respectively, v *and* w *in* $[v_0, w_0]$*, which are fixed points of* T*.*

2. Main results

In this section we will state the sufficient conditions for the m-point nonlocal boundary value problem (1.1) − (1.2) to have positive solutions. For this purpose, first we will give some lemmas that will be used to give the main result.

Lemma 2.1 *The function* $y(t)$ *is a solution of the problem (1.1)-(1.2) if and only if*

$$y(t) = \sum_{i=1}^{n} \int_0^1 G(t,s) f_i(s, y(s)) ds + \frac{e^{-\int_0^t p(\xi)d\xi}}{1 + e^{-\int_0^1 p(\xi)d\xi}} \sum_{j=1}^{m} g_j(t_j, y(t_j))$$

where $G(t,s)$ *is defined by*

$$G(t,s) = \frac{e^{-\int_s^t p(\xi)d\xi}}{1 - e^{-\int_0^1 p(\xi)d\xi}} \begin{cases} 1, & s < t, \\ e^{-\int_0^1 p(\xi)d\xi} & t \le s. \end{cases}$$

If we take the derivation of $y(t)$, we can easily see that $y(t)$ is the solution of the problem (1.1)-(1.2). In the proof of Theorem 2.2 in [3], a similar result was given. Therefore, we do not restate the proof here.

In [8], Goodrich found the upper and lower bounds for Green's function on the general time scales in Lemma 2.4. Since the following lemma can be proven in a similar way, we give the lemma without the proof.

Lemma 2.2 *Green's function* $G(t,s)$ *satisfies*

$$e^{-\int_0^1 p(\xi)d\xi} G(s,s) \le G(t,s) \le e^{\int_0^1 p(\xi)d\xi} G(s,s).$$

The main result of this paper as follows.

Theorem 2.1 *Assume that conditions* $f_i : [0,1] \times [0,\infty) \to (-\infty,\infty), i = 1,2,...,n$ *and* $g_j : [0,1] \times [0,\infty) \to [0,\infty), j = 1,2,...,m$ *are continuous and there exists a constant* $M > 0$ *such that* $f_i(t,y) > -M$ *for all* $(t,y) \in [0,1] \times [0,\infty)$ *and* $\int_0^1 (f_i + M) ds > 0$*. If there exist positive constants* r *and* R *such that* $r > \frac{2M}{\gamma^2}$ *and the following conditions are satisfied:*

$$(A_1) f_i(t,u) \le f_i(t,v) \le \frac{1 - e^{-\int_0^1 p(\xi)d\xi}}{2n} R - M, \quad t \in [0,1], \quad \frac{r}{2} \le u \le v \le R,$$

$$(A_2) \frac{1 - e^{-\int_0^1 p(\xi)d\xi}}{e^{-\int_0^1 p(\xi)d\xi}} \frac{r}{m} \le g_j(t,u) \le g_j(t,v) \le \frac{1 - e^{-\int_0^1 p(\xi)d\xi}}{2m} R, \quad t \in [0,1], \quad \frac{r}{2} \le u \le v \le R,$$

where $\gamma = \dfrac{1 - e^{-\int_0^1 p(\xi)d\xi}}{1 + e^{\int_0^1 p(\xi)d\xi}} e^{-\int_0^1 p(\xi)d\xi}$*, then the boundary value problem* $(1.1) - (1.2)$ *has positive solutions.*

Proof First we consider the following boundary value problem:

$$u'(t) + p(t)u(t) = \sum_{i=1}^{n} F_i(t, u_x(t)), \qquad t \in [0,1], \tag{2.1}$$

$$u(0) = u(1) + \sum_{j=1}^{m} g_j(t_j, u_x(t_j)), \tag{2.2}$$

where $F_i(t, u_x(t)) = f_i(t, u_x(t)) + M$ and $u_x(t) = \max\{(u-x)(t), 0\}$ such that $x(t) = M\omega(t)$ and $\omega(t)$ is the solution of

$$y'(t) + p(t)y(t) = 1, \qquad t \in [0,1],$$
$$y(0) = y(1).$$

Using Lemma 2.1 and Lemma 2.2 we can easily see that the unique solution $\omega(t)$ of the above problem satisfies

$$\omega(t) = \int_0^1 G(t,s)ds \le \int_0^1 e^{\int_0^1 p(\xi)d\xi} G(s,s)ds = \frac{1}{1 - e^{-\int_0^1 p(\xi)d\xi}} < \frac{1 + e^{\int_0^1 p(\xi)d\xi}}{1 - e^{-\int_0^1 p(\xi)d\xi}} = \frac{e^{-\int_0^1 p(\xi)d\xi}}{\gamma} < \frac{1}{\gamma}.$$

Let $E := \{y | \ y : [0,1] \to \mathrm{R}$ continuous $\}$ with the norm $\|y\| = \max_{t \in [0,1]} |y(t)|$. Denote $K := \{y \in E : y(t) \ge \gamma \|y\|, t \in [0,1]\}$. Then K is a normal cone of E. Now we define an operator $T : K \to K$ by

$$Tu(t) := \int_0^1 G(t,s) \sum_{i=1}^{n} F_i(s, u_x(s))ds + \frac{e^{-\int_0^t p(\xi)d\xi}}{1 + e^{-\int_0^1 p(\xi)d\xi}} \sum_{j=1}^{m} g_j(t_j, u_x(t_j)),$$

and then it is easy to see that fixed points of T are nonnegative solutions of the BVP (2.3)-(2.4).

Let $v_0(t) = r$ and $w_0(t) = R$ for $t \in [0,1]$.

Now we will verify that $T : [v_0, w_0] \to K$ is completely continuous.

First, T is continuous. Let $u_n (n = 1, 2, ...), u \in [v_0, w_0]$ and $\lim_{n\to\infty} u_n = u$. Then,

$$r \le u_n \le R, \quad r \le u \le R, \qquad t \in [0,1].$$

For any given $\epsilon > 0$, since f_i are uniformly continuous on $[0,1] \times [\frac{r}{2}, R]$, there exists $\delta_1 > 0$ such that for any $u_1, u_2 \in [\frac{r}{2}, R]$ with $|u_1 - u_2| < \delta_1$

$$|f_i(s, u_1) - f_i(s, u_2)| < \frac{\epsilon}{2n}\left(1 - e^{-\int_0^1 p(\xi)d\xi}\right), \quad s \in [0,1].$$

On the other hand, since g_j are uniformly continuous on $[0,1] \times [\frac{r}{2}, R]$, there exists $\delta_2 > 0$ such that for any $u_1, u_2 \in [\frac{r}{2}, R]$ with $|u_1 - u_2| < \delta_2$

$$|g_j(s, u_1) - g_j(s, u_2)| < \frac{\epsilon}{2m}, \quad s \in [0,1].$$

Let $\delta = \min\{\delta_1, \delta_2\}$. Then it follows from $\lim_{n\to\infty} u_n = u$ that there exists a positive N such that for any $n > N$,

$$|u_n(s) - u(s)| < \delta, \quad s \in [0,1].$$

We can easily see that $|u_n - u| < \delta$ implies $|u_{n_x} - u_x| < \delta$.

Thus, using Lemma 2.2 and the continuity of the functions $f_i (i = 1, 2, ..., n)$ and $g_j (j = 1, 2, ..., m)$, we can easily get

$$
\begin{aligned}
|Tu_n(t) - Tu(t)| &\leq \int_0^1 G(t,s) \sum_{i=1}^{n} |f_i(s, u_{n_x}(s)) - f_i(s, u_x(s))| ds + \sum_{j=1}^{m} |g_j(t_j, u_{n_x}(t_j)) - g_j(t_j, u_x(t_j))| \\
&\leq \int_0^1 e^{\int_0^1 p(\xi)d\xi} G(s,s) \sum_{i=1}^{n} \frac{\epsilon}{2n} \left(1 - e^{\int_0^1 p(\xi)d\xi}\right) ds + \sum_{j=1}^{m} \frac{\epsilon}{2m} \\
&= \frac{1}{1 - e^{-\int_0^1 p(\xi)d\xi}} \frac{\epsilon}{2n} \left(1 - e^{\int_0^1 p(\xi)d\xi}\right) n + \frac{\epsilon}{2m} m = \epsilon,
\end{aligned}
$$

which indicates that $\lim_{n\to\infty} Tu_n = Tu$. So $T : [v_0, w_0] \to K$ is continuous.

Next, we will show that $T : [v_0, w_0] \to K$ is compact. Let $A \subset [v_0, w_0]$ be a bounded set. Define $Q := \max_{[0,1]\times[\frac{r}{2},R]} f_i(t, u(t))$ and $S := \max_{[0,1]\times[\frac{r}{2},R]} g_j(t_j, u(t_j))$. If $u \in [r, R]$ we can see easily that $u_x = \max\{(u - x)(t), 0\} \geq u(t) - x(t) = u(t) - Mw(t) \geq u(t) - \frac{M}{\gamma} \geq r - \frac{r}{2} = \frac{r}{2}$ and $u_x \leq u \leq R$. It shows that $u_x \in [\frac{r}{2}, R]$.

For $u \in A$ and $t \in [0, 1]$, using Lemma 2.2,

$$
\begin{aligned}
Tu(t) &\leq \frac{1}{1 - e^{-\int_0^1 p(\xi)d\xi}} \int_0^1 \sum_{i=1}^{n} (f_i(s, u_x(s)) + M) ds + \frac{1}{1 - e^{-\int_0^1 p(\xi)d\xi}} \sum_{j=1}^{m} g_j(t_j, u_x(t_j)) \\
&\leq \frac{1}{1 - e^{-\int_0^1 p(\xi)d\xi}} \{(Q + M)n + Sm\},
\end{aligned}
$$

which shows that $T(A)$ is uniformly bounded.

On the other hand, for any $u \in A$ and $t_1, t_2 \in [0, 1]$ with $t_1 \geq t_2$ and $c \in (t_2, t_1)$, we have

$$
\begin{aligned}
|Tu(t_1) - Tu(t_2)| &\leq \left| \int_0^1 (G(t_1, s) - G(t_2, s)) \sum_{i=1}^{n} (f_i(s, u_x(s)) + M) ds \right| \\
&\quad + \left| \frac{e^{-\int_0^{t_1} p(\xi)d\xi} - e^{-\int_0^{t_2} p(\xi)d\xi}}{1 - e^{-\int_0^1 p(\xi)d\xi}} \sum_{j=1}^{m} g_j(t_j, u_x(t_j)) \right| \\
&\leq \int_0^1 \left| \frac{e^{-\int_s^{t_1} p(\xi)d\xi} - e^{-\int_s^{t_2} p(\xi)d\xi}}{1 - e^{-\int_0^1 p(\xi)d\xi}} \right| \sum_{i=1}^{n} (f_i(s, u_x(s)) + M) ds \\
&\quad + \left| \frac{e^{-\int_0^{t_1} p(\xi)d\xi} - e^{-\int_0^{t_2} p(\xi)d\xi}}{1 - e^{-\int_0^1 p(\xi)d\xi}} \right| \sum_{j=1}^{m} g_j(t_j, u_x(t_j)) \\
&\leq \frac{1}{1 - e^{-\int_0^1 p(\xi)d\xi}} |p(c)||t_1 - t_2| \left\{ \int_0^1 \sum_{i=1}^{n} (f_i(s, u_x(s)) + M) ds + \sum_{j=1}^{m} g_j(t_j, u_x(t_j)) \right\} \\
&\leq \frac{1}{1 - e^{-\int_0^1 p(\xi)d\xi}} |p(c)||t_1 - t_2| \{(Q + M)n + Sm\},
\end{aligned}
$$

which implies that $T(A)$ is equi-continuous.

Consequently, $T:[v_0,w_0]\to K$ is compact.

Now we will show that T is monotone increasing on $[v_0,w_0]$.

Suppose that $u,v\in[v_0,w_0]$ and $u\le v$. Then $r\le u(t)\le v(t)\le R$ for $t\in[0,1]$. As we have shown previously for u_x, we easily have $v_x\in[\frac{r}{2},R]$, and so we get $\frac{r}{2}\le u_x(t)\le v_x(t)\le R$. Thus, by (A_1) and (A_2), for $t\in[0,1]$, we have

$$\begin{aligned} Tu(t) &= \int_0^1 G(t,s)\sum_{i=1}^n (f_i(s,u_x(s))+M)ds+\frac{e^{-\int_0^t p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^m g_j(t_j,u_x(t_j)) \\ &\le \int_0^1 G(t,s)\sum_{i=1}^n (f_i(s,v_x(s))+M)ds+\frac{e^{-\int_0^t p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^m g_j(t_j,v_x(t_j))=Tv(t), \end{aligned}$$

which shows that $Tu\le Tv$.

Now we will prove that $v_0=r$ is a lower solution of T. For any $t\in[0,1]$, since $v_{0_x}\in[\frac{r}{2},R]$, it is obvious that

$$\begin{aligned} Tv_0(t) &= \int_0^1 G(t,s)\sum_{i=1}^n (f_i(s,v_{0_x}(s))+M)ds+\frac{e^{-\int_0^t p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^m g_j(t_j,v_{0_x}(t_j)) \\ &\ge \frac{e^{-\int_0^t p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^m g_j(t_j,v_{0_x}(t_j)) \\ &\ge \frac{e^{-\int_0^1 p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^m \frac{1-e^{-\int_0^1 p(\xi)d\xi}}{e^{-\int_0^1 p(\xi)d\xi}}\frac{r}{m}=r=v_0(t), \end{aligned}$$

which implies that $v_0\le Tv_0$.

We show that $w_0=R$ is an upper solution of T. In view of (A_1) and (A_2) and using $w_{0_x}=\max\{w_0-x,0\}\le w_0=R$, we have

$$\begin{aligned} Tw_0(t) &= \int_0^1 G(t,s)\sum_{i=1}^n (f_i(s,w_{0_x}(s))+M)ds+\frac{e^{-\int_0^t p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^m g_j(t_j,w_{0_x}(t_j)) \\ &\le \int_0^1 G(t,s)\sum_{i=1}^n (f_i(s,R)+M)ds+\frac{e^{-\int_0^t p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^m g_j(t_j,R) \\ &\le \int_0^1 \frac{1}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{i=1}^n\left(\frac{R}{2n}\right)\left(1-e^{-\int_0^1 p(\xi)d\xi}\right)ds+\frac{1}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^m\left(\frac{R}{2m}\right)\left(1-e^{-\int_0^1 p(\xi)d\xi}\right) \\ &= \frac{1}{1-e^{-\int_0^1 p(\xi)d\xi}}\left(\frac{R}{2}+\frac{R}{2}\right)\left(1-e^{-\int_0^1 p(\xi)d\xi}\right)=R=w_0(t), \end{aligned}$$

which implies that $Tw_0\le w_0$.

If we construct sequences $\{v_n\}_{n=1}^{\infty}$ and $\{w_n\}_{n=1}^{\infty}$ as follows:

$$v_n=Tv_{n-1},\quad w_n=Tw_{n-1},\quad n=1,2,3,...,$$

then it follows from Theorem 1 that

$$v_0 \leq v_1 \leq ... \leq v_n \leq ... \leq w_n \leq ... \leq w_1 \leq w_0,$$

and $\{v_n\}$ and $\{w_n\}$ converge to, respectively, v and w, which are the solutions of the problem (2.1)-(2.2).

It follows from

$$\begin{aligned} Tv_0(1) &= \int_0^1 G(1,s)\sum_{i=1}^{n}(f_i(s,v_{0_x}(s))+M)ds + \frac{e^{-\int_0^1 p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\sum_{j=1}^{m} g_j(t_j, v_{0_x}(t_j)) \\ &\geq \frac{e^{-\int_0^1 p(\xi)d\xi}}{1-e^{-\int_0^1 p(\xi)d\xi}}\left\{\sum_{i=1}^{n}\int_0^1 (f_i(s,v_{0_x}(s))+M)ds + \sum_{j=1}^{m} g_j(t_j, v_{0_x}(t_j))\right\} > 0 \end{aligned}$$

that

$$0 < (Tv_0)(1) \leq (Tv)(1) = v(1) \leq w(1),$$

which shows that v and w are positive solutions of the problem (2.1)-(2.2).

Moreover, we get

$$v(t) \geq \gamma\|v\| \geq \gamma r \geq 2M\gamma^{-1} \text{ and } w(t) \geq \gamma\|w\| \geq 2M\gamma^{-1}.$$

Hence, for $t \in [0,1]$

$$y(t) = v(t) - x(t) \geq \frac{2M}{\gamma} - \frac{M}{\gamma} = \frac{M}{\gamma} > 0 \text{ and } \tilde{y}(t) = w(t) - x(t) > 0,$$

and it can be easily seen that y and $\tilde{y}$ are the positive solutions of the problem (1.1)-(1.2). □

Example 2.1 *We consider the following first-order m-point nonlocal boundary value problem:*

$$y'(t) + \sqrt[3]{t}y(t) = f_1(t,y) + f_2(t,y) + f_3(t,y), \qquad t \in [0,1], \tag{2.3}$$

$$y(0) = y(1) + g_1(t,y), \tag{2.4}$$

where $f_1(t,y) = \frac{1}{20}t\sqrt[4]{y(t)+3}$, $f_2(t,y) = \frac{1}{4}((t-2)^3 + \sqrt{y(t)})$, $f_3(t,y) = \frac{1}{200}(y(t)-2)$ *and*

$$g_1(t,y) = \begin{cases} \frac{1}{2}\left(145 + e^{-\frac{1}{y(\frac{1}{2})-32}}\right), & y > 32; \\ \frac{145}{2}, & y \leq 32. \end{cases}$$

Since $f_1 \leq 0, f_2 \leq -2$ *and* $f_3 \leq -\frac{1}{100}$, *then* $M = 2$ *and we can calculate* $\gamma \cong 0,08$. *If we choose* $r = 64$ *and* $R = 280$, *then all the conditions of Theorem 2.1 are fulfilled. Thus, the problem (2.5)-(2.6) has positive solutions* y *and* $\tilde{y}$. *Furthermore, if we construct sequences* $\{v_n\}$ *and* $\{w_n\}$ *such that* $v_n = Tv_{n-1}$ *and* $w_n = Tw_{n-1}$, $n = 1,2,...$ *where* $v_0(t) = 32$ *and* $w_0(t) = 280$, *then* $\lim_{n\to\infty} v_n - x = v - x = y$ *and* $\lim_{n\to\infty} w_n - x = w - x = \tilde{y}$.

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