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Super d-anti-magic labeling of subdivided kC_5

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Abstract: A graph $(G = (V, E, F))$ admits labeling of type $(1, 1, 1)$ if we assign labels from the set $\{1, 2, 3, ..., |V(G)| + |E(G)| + |F(G)|\}$ to the vertices, edges, and faces of a planar graph G in such a way that each vertex, edge, and face receives exactly one label and each number is used exactly once as a label and the weight of each face under the mapping is the same. Super d-antimagic labeling of type $(1, 1, 1)$ on snake kC_5, subdivided kC_5 as well as ismorphic copies of kC_5 for string $(1, 1, ..., 1)$ and string $(2, 2, ..., 2)$ is discussed in this paper.

Key words: Super d-anti-magic labeling, snake graph

1. Introduction

We consider a finite, connected, and planar graph $(G = (V, E, F))$ without loops and multiple edges, where $V(G)$, $E(G)$, and $F(G)$ are its vertex set, edge set, and face set, respectively.

Labeling of a graph is one-to-one mapping that carries a set of graph elements to a set of numbers (usually positive integers). Labeling of type (α, β, γ) (where $(\alpha, \beta, \gamma) \in \{0, 1\}$) assigns labels from the set $\{1, 2, 3, \ldots, | V(G) | + | E(G) | + | F(G) |\}$ to the set of vertices, edges, and faces of a plane graph G in such a way that each vertex, edge, and face receives exactly one label and each number is used exactly once as a label. The weight of a face under labeling of type $(1, 1, 1)$ is the sum of the labels carried by that face and the edges and vertices surrounding it.

Labeling of type (α, β, γ) is said to be face-magic if for every number $s \geq 3$ all s-sided faces have the same weight. We allow different weights for different s. The labeling is called super face-magic if the vertex set is assigned first. The labeling is called face antimagic if face weights form an arithmetic sequence with initial value a and common difference d.

The notion of magic labeling of planer graphs was defined by Ko-Wei Lih in [17], where magic labelings of type (1,1,0) for wheels, friendship graphs, and prisms are given. Bača gave magic labelings of type (1,1,1) for fans in [1, 5, 10]. Ali [16] gave magic labeling of type $(1, 1, 1)$ for wheels and subdivision of wheels. Bača determined magic labeling on different graphs in [2, 3, 4, 6, 8, 9]. Siddiqui [15] determined the existence of super d-antimagic labeling for a Jahangir graph for certain different d. Bača [7, 11, 12, 13] gave magic labelings of type (1,1,1) and type (1,1,0) for certain classes of convex polytopes. A general survey of graph labelings is given in [18].

The snake graph (kC_4) was first introduced by Barrientos [14], while Rosa [19] gave the generalization

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concept of the triangular snake. A kC_n-snake can be defined as a connected graph with k blocks; each of the blocks is isomorphic to the cycle C_n, such that the block-cut-vertex graph is a path.

In this paper we formulate super antimagic labeling of type $(1,1,1)$ for a kC_5 - snake graph, partition of a kC_5 - snake graph, and isomorphic copies of a kC_5 - snake graph; super antimagic labeling of type $(1,1,1)$ for kC_n is also discussed in this paper.

2. Main results I

In this section we formulate super antimagic labeling of a kC_5- snake graph as well as isomorphic copies of a kC_5 -snake graph.

Theorem 1 *For all* $k \geq 2$, $H \cong kC_5$ *- snake graph with string* $(2,2,...,2)$ *admits super 1-antimagic labeling of type (1,1,1).*

Proof Let $s = |V(H)|, e = |E(H)|$, and $f = |F(H)|$. Then $s = 4k+1, e = 5k$, and $f = k$.
Now we define labeling $\lambda : V(H) \cup E(H) \cup F(H) \rightarrow \{1,2,...,s+e+f\}$
as follows:

$$\lambda(u_i) = \{i, 1 \leq i \leq k+1\}$$
$$\lambda(v_i) = \{(k+1)+i, 1 \leq i \leq k\}$$
$$\lambda(w_i) = \{3k+2-i, 1 \leq i \leq k\}$$
$$\lambda(X_i) = \{4k+2-i, 1 \leq i \leq k\}$$

We symbolize the edges of H as follows:

$$\lambda(w_i x_i) = \{s+i, 1 \leq i \leq k\}$$
$$\lambda(x_i u_{i+1}) = \{s+2k+1-i, 1 \leq i \leq k\}$$
$$\lambda(u_{i+1} vi) = \{s+3k+1-i, 1 \leq i \leq k\}$$
$$\lambda(u_i v_i) = \{s+3k+i, 1 \leq i \leq k\}$$
$$\lambda(u_i w_i) = \{s+4k+i, 1 \leq i \leq k\}$$

We symbolize the faces of H as follows:

$$\lambda(f_i) = \{s+e+k+1-i, 1 \leq i \leq k\}$$

In this way the snake graph of string $(2,2,...,2)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1,1,1)$. □

Theorem 2 *For all* $k \geq 3$, $H \cong kC_5$ *- snake graph with string* $(1,1,...,1)$ *admits super 1-antimagic labeling of type (1,1,1).*

Proof Let $s = |V(H)|, e = |E(H)|$ and $f = |F(H)|$. Then $s = 4k+1, e = 5k$ and $f = k$.
Now we define labeling $\lambda : V(H) \cup E(H) \cup F(H) \rightarrow \{1,2,...,s+e+f\}$
as

$$\lambda(u_i) = \{i, 1 \leq i \leq k+1\}$$
$$\lambda(v_i) = \{(k+1)+i, 1 \leq i \leq k\}$$
$$\lambda(w_i) = \{3k+2-i, 1 \leq i \leq k\}$$
$$\lambda(X_i) = \{4k+2-i, 1 \leq i \leq k\}$$

We symbolize the edges of H as follows:

$$\lambda(u_iu_{i+1}) = \{s + k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(u_iw_i) = \{s + k + i, 1 \leq i \leq k\}$$
$$\lambda(w_ix_i) = \{s + 2k + i, 1 \leq i \leq k\}$$
$$\lambda(x_iv_i) = \{s + 4k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(v_iu_{i+1}) = \{s + 5k + 1 - i, 1 \leq i \leq k\}$$

We symbolize the faces of H as follows:

$$\lambda(f_i) = \{s + e + i, 1 \leq i \leq k\}$$

In this way the snake graph of string $(1, 1, ..., 1)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1, 1, 1)$. □

Theorem 3 *For all* $k \geq 2$, $H \cong mkC_5 - snake$ *graph with string* $(2, 2, ..., 2)$ *admits super 1-antimagic labeling of type* $(1, 1, 1)$.

Proof Let $s = |V(H)|$, $e = |E(H)|$, and $f = |F(H)|$. Then we have $s = n(4k + 1)$, $e = 5nk$, and $f = nk$, $1 \leq m \leq n$.
For vertices

$$\lambda(u_i^m) = \{i + (m - 1)(k + 1) \quad 1 \leq i \leq k + 1\}$$
$$\lambda(v_i^m) = \{n(k + 1) + nk + 1 - (m - 1)k - i \quad 1 \leq i \leq k\}$$
$$\lambda(x_i^m) = \{3nk + n + 1 - i - (m - 1)k \quad 1 \leq i \leq k\}$$
$$\lambda(w_i^m) = \{4nk + n + 1 - i - (m - 1)k \quad 1 \leq i \leq k\}$$

For edges

$$\lambda(u_i^m w_i^m) = \{s + (m - 1)k + i \quad 1 \leq i \leq k\}$$
$$\lambda(v_i^m u_{i+1}^m) = \{s + 2nk + 1 - i - (m - 1)k \quad 1 \leq i \leq k\}$$
$$\lambda(u_i^m v_i^m) = \{s + 3nk + 1 - (m - 1)k - i \quad 1 \leq i \leq k\}$$
$$\lambda(w_i^m x_i^m) = \{s + 3nk + (m - 1)k + i \quad 1 \leq i \leq k\}$$
$$\lambda(x_i^m u_{i+1}^m) = \{s + 4nk + (m - 1)k + i \quad 1 \leq i \leq k\}$$

For faces

$$\lambda(f_i^m) = \{\ s + e + n - (m - 1) + (i - 1)n \quad 1 \leq i \leq k$$

In this way the $H \cong mkC_5 - snake$ graph of string $(2, 2, ..., 2)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1, 1, 1)$. □

Theorem 4 *For all* $k \geq 3$, $H \cong mkC_5 - snake$ *graph with string* $(1, 1, ..., 1)$ *admits super 1-antimagic labeling of type* $(1, 1, 1)$.

Proof Let $s = |V(H)|$, $e = |E(H)|$, and $f = |F(H)|$. Then we have $s = n(4k+1)$, $e = 5nk$, and $f = nk$, $1 \le m \le n$.
For vertices

$$\lambda(u_i^m) = \{i + (m-1)(k+1) \quad 1 \le i \le k+1\}$$
$$\lambda(v_i^m) = \{2nk + n + 1 - (m-1)k - i \quad 1 \le i \le k\}$$
$$\lambda(x_i^m) = \{3nk + n + 1 - i - (m-1)k \quad 1 \le i \le k\}$$
$$\lambda(w_i^m) = \{4nk + n + 1 - i - (m-1)k \quad 1 \le i \le k\}$$

For edges

$$\lambda(u_i^m w_i^m) = \{s + (m-1)k + i \quad 1 \le i \le k\}$$
$$\lambda(u_{i+1}^m v_i^m) = \{s + 2nk + 1 - i - (m-1)k \quad 1 \le i \le k\}$$
$$\lambda(u_i^m u_{i+1}^m) = \{s + 3nk + 1 - (m-1)k - i \quad 1 \le i \le k\}$$
$$\lambda(w_i^m x_i^m) = \{s + 3nk + (m-1)k + i \quad 1 \le i \le k\}$$
$$\lambda(x_i^m v_i^m) = \{s + 4nk + (m-1)k + i \quad 1 \le i \le k\}$$

For faces

$$\lambda(f_i^m) = \{\ s + e + n - (m-1) + (i-1)n \quad 1 \le i \le k\}$$

In this way the $H \cong mkC_5 - snake$ graph of string $(1,1,...,1)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1,1,1)$. □

3. Main results II

In this section we formulate super antimagic labeling of subdivision kC_5- snake graph as well as isomorphic copies of subdivision kC_5 -snake graph.

Theorem 5 *For all $k \ge 3$, $H \cong kC_5$ - snake graph of string $(1,1,...,1)$ with 1 subdivision admits super 1-antimagic labeling of type (1,1,1).*

Proof Let $s = |V(H)|$, $e = |E(H)|$, and $f = |F(H)|$. Then $s = 9k+1$, $e = 10k,$, and $f = k$.
Now we define labeling $\lambda : V(H) \cup E(H) \cup F(H) \to \{1, 2, ..., s+e+f\}$
as follows:

$$\lambda(u_i) = \{i, 1 \le i \le k+1\}$$
$$\lambda(v_i) = \{(k+1) + i, 1 \le i \le k\}$$
$$\lambda(w_i) = \{3k + 2 - i, 1 \le i \le k\}$$
$$\lambda(x_i) = \{4k + 2 - i, 1 \le i \le k\}$$

We symbolize the partitions of H as follows:

$$\lambda(a_i) = \{5k + 2 - i, 1 \le i \le k\}$$
$$\lambda(e_i) = \{6k + 2 - i, 1 \le i \le k\}$$

$$\lambda(b_i) = \{7k + 2 - i, 1 \leq i \leq k\}$$
$$\lambda(c_i) = \{7k + 1 + i, 1 \leq i \leq k\}$$
$$\lambda(d_i) = \{9k + 2 - i, 1 \leq i \leq k\}$$

We symbolize the edges of H as follows:

$$\lambda(u_i a_i) = \{s + i, 1 \leq i \leq k\}$$
$$\lambda(a_i u_{i+1}) = \{S + 2k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(v_i e_i) = \{s + 3k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(e_i u_{i+1}) = \{s + 3k + i, 1 \leq i \leq k\}$$
$$\lambda(x_i d_i) = \{s + 5k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(d_i v_i) = \{S + 5k + i, 1 \leq i \leq k\}$$
$$\lambda(w_i c_i) = \{s + 7k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(c_i x_i) = \{s + 7k + i, 1 \leq i \leq k\}$$
$$\lambda(w_i b_i) = \{s + 9k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(b_i u_i) = \{s + 9k + i, 1 \leq i \leq k\}$$

We symbolize the faces of H as follows:

$$\lambda(f_i) = \{s + e + i, 1 \leq i \leq k\}$$

In this way the snake graph of string $(1, 1, ..., 1)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1, 1, 1)$, with 1 subdivision. □

Theorem 6 *For all* $k \geq 2$, $H \cong kC_5$ *- snake graph of string* $(2, 2, ..., 2)$ *with* 1 *subdivision admits super 1-antimagic labeling of type (1,1,1).*

Proof Let $s = |V(H)|$, $e = |E(H)|$, and $f = |F(H)|$. Then $s = 9k + 1$, $e = 10k,$, and $f = k$.
Now we define labeling $\lambda : V(H) \cup E(H) \cup F(H) \rightarrow \{1, 2, ..., s + e + f\}$
as follows:

$$\lambda(u_i) = \{i, 1 \leq i \leq k + 1\}$$
$$\lambda(v_i) = \{(k + 1) + i, 1 \leq i \leq k\}$$
$$\lambda(w_i) = \{3k + 2 - i, 1 \leq i \leq k\}$$
$$\lambda(x_i) = \{4k + 2 - i, 1 \leq i \leq k\}$$

We symbolize the partitions of H as follows:

$$\lambda(a_i) = \{5k + 2 - i, 1 \leq i \leq k\}$$
$$\lambda(e_i) = \{6k + 2 - i, 1 \leq i \leq k\}$$
$$\lambda(b_i) = \{7k + 2 - i, 1 \leq i \leq k\}$$
$$\lambda(c_i) = \{7k + 1 + i, 1 \leq i \leq k\}$$
$$\lambda(d_i) = \{9k + 2 - i, 1 \leq i \leq k\}$$

We symbolize the edges of H as follows:

$$\lambda(u_i a_i) = \{s + i, 1 \le i \le k\}$$
$$\lambda(a_i v_i) = \{S + 2k + 1 - i, 1 \le i \le k\}$$
$$\lambda(u_{i+1} e_i) = \{s + 3k + 1 - i, 1 \le i \le k\}$$
$$\lambda(e_i v_i) = \{s + 3k + i, 1 \le i \le k\}$$
$$\lambda(x_i d_i) = \{s + 5k + 1 - i, 1 \le i \le k\}$$
$$\lambda(d_i u_{i+1}) = \{S + 5k + i, 1 \le i \le k\}$$
$$\lambda(w_i c_i) = \{s + 7k + 1 - i, 1 \le i \le k\}$$
$$\lambda(c_i x_i) = \{s + 7k + i, 1 \le i \le k\}$$
$$\lambda(w_i b_i) = \{s + 9k + 1 - i, 1 \le i \le k\}$$
$$\lambda(b_i u_i) = \{s + 9k + i, 1 \le i \le k\}$$

We symbolize the faces of H as follows:

$$\lambda(f_i) = \{s + e + i, 1 \le i \le k\}$$

In this way the snake graph of string $(2, 2, ..., 2)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1, 1, 1)$, with 1 subdivision. □

Theorem 7 *For all* $k \ge 3$, $H \cong kC_5$ *- snake graph of string* $(1, 1, ..., 1)$ *with* 2 *subdivisions admits super 1-antimagic labeling of type (1,1,1).*

Proof Let $s = |V(H)|$, $e = |E(H)|$, and $f = |F(H)|$. Then $s = 14k + 1$, $e = 15k,$, and $f = k$.
Now we define labeling $\lambda : V(H) \cup E(H) \cup F(H) \to \{1, 2, ..., s + e + f\}$
as follows:

$$\lambda(u_i) = \{i, 1 \le i \le k + 1\}$$
$$\lambda(v_i) = \{(k + 1) + i, 1 \le i \le k\}$$
$$\lambda(w_i) = \{3k + 2 - i, 1 \le i \le k\}$$
$$\lambda(x_i) = \{4k + 2 - i, 1 \le i \le k\}$$

We symbolize the partitions of H as follows:

$$\lambda(a_{i1}) = \{5k + 2 - i, 1 \le i \le k\}$$
$$\lambda(a_{i1}) = \{5k + 1 + i, 1 \le i \le k\}$$
$$\lambda(b_{i1}) = \{7k + 2 - i, 1 \le i \le k\}$$
$$\lambda(b_{i2}) = \{7k + 1 + i, 1 \le i \le k\}$$
$$\lambda(c_{i1}) = \{9k + 2 - i, 1 \le i \le k\}$$
$$\lambda(c_{i2}) = \{9k + 1 + i, 1 \le i \le k\}$$
$$\lambda(d_{i1}) = \{11k + 2 - i, 1 \le i \le k\}$$
$$\lambda(d_{i2}) = \{11k + 1 + i, 1 \le i \le k\}$$
$$\lambda(e_{i1}) = \{13k + 2 - i, 1 \le i \le k\}$$
$$\lambda(e_{i2}) = \{13k + 1 + i, 1 \le i \le k\}$$

We symbolize the edges of H as follows:

$$\lambda(u_i a_{i1}) = \{s + i, 1 \le i \le k\}$$
$$\lambda(a_{i1} a_{i2}) = \{s + k + i, 1 \le i \le k\}$$
$$\lambda(a_{i2} u_{i+1}) = \{s + 2k + i, 1 \le i \le k\}$$
$$\lambda(v_i e_{i1}) = \{s + 4k + 1 - i, 1 \le i \le k\}$$
$$\lambda(e_{i1} e_{i2}) = \{s + 5k + 1 - i, 1 \le i \le k\}$$
$$\lambda(e_{i2} u_{i+1}) = \{s + 6k + 1 - i, 1 \le i \le k\}$$
$$\lambda(x_i d_{i1}) = \{s + 6k + i, 1 \le i \le k\}$$
$$\lambda(d_{i1} d_{i2}) = \{s + 7k + i, 1 \le i \le k\}$$
$$\lambda(d_{i2} v_i) = \{s + 8k + i, 1 \le i \le k\}$$
$$\lambda(x_i c_{i1}) = \{s + 10k + 1 - i, 1 \le i \le k\}$$
$$\lambda(c_{i1} c_{i2}) = \{s + 11k + 1 - i, 1 \le i \le k\}$$
$$\lambda(c_{i2} w_i) = \{s + 12k + 1 - i, 1 \le i \le k\}$$
$$\lambda(w_i b_{i1}) = \{s + 13k + 1 - i, 1 \le i \le k\}$$
$$\lambda(b_{i1} b_{i2}) = \{s + 14k + 1 - i, 1 \le i \le k\}$$
$$\lambda(b_{i2} u_i) = \{s + 15k + 1 - i, 1 \le i \le k\}$$

We symbolize the faces of H as follows:

$$\lambda(f_i) = \{s + e + i, 1 \le i \le k\}$$

In this way the snake graph of string $(1, 1, ..., 1)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1, 1, 1)$, with 2 subdivisions. □

Theorem 8 *For all $k \ge 2$, $H \cong kC_5$ - snake graph of string $(2, 2, ..., 2)$ with 2 subdivisions admits super 1-antimagic labeling of type (1,1,1).*

Proof Let $s = |V(H)|$, $e = |E(H)|$, and $f = |F(H)|$. Then $s = 14k + 1$, $e = 15k,$, and $f = k$.
Now we define labeling $\lambda : V(H) \cup E(H) \cup F(H) \to \{1, 2, ..., s + e + f\}$
as follows:

$$\lambda(u_i) = \{i, 1 \le i \le k + 1\}$$
$$\lambda(v_i) = \{(k + 1) + i, 1 \le i \le k\}$$
$$\lambda(w_i) = \{3k + 2 - i, 1 \le i \le k\}$$
$$\lambda(x_i) = \{4k + 2 - i, 1 \le i \le k\}$$

We symbolize the partitions of H as follows:

$$\lambda(a_{i1}) = \{5k + 2 - i, 1 \le i \le k\}$$
$$\lambda(a_{i1}) = \{5k + 1 + i, 1 \le i \le k\}$$
$$\lambda(b_{i1}) = \{7k + 2 - i, 1 \le i \le k\}$$
$$\lambda(b_{i2}) = \{7k + 1 + i, 1 \le i \le k\}$$
$$\lambda(c_{i1}) = \{9k + 2 - i, 1 \le i \le k\}$$
$$\lambda(c_{i2}) = \{9k + 1 + i, 1 \le i \le k\}$$
$$\lambda(d_{i1}) = \{11k + 2 - i, 1 \le i \le k\}$$

$$\lambda(d_{i2}) = \{11k + 1 + i, 1 \leq i \leq k\}$$
$$\lambda(e_{i1}) = \{13k + 2 - i, 1 \leq i \leq k\}$$
$$\lambda(e_{i2}) = \{13k + 1 + i, 1 \leq i \leq k\}$$

We symbolize the edges of H as follows:

$$\lambda(u_i a_{i1}) = \{s + i, 1 \leq i \leq k\}$$
$$\lambda(a_{i1} a_{i2}) = \{s + k + i, 1 \leq i \leq k\}$$
$$\lambda(a_{i2} v_i) = \{s + 2k + i, 1 \leq i \leq k\}$$
$$\lambda(u_{i+1} e_{i1}) = \{s + 4k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(e_{i1} e_{i2}) = \{s + 5k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(e_{i2} v_i) = \{s + 6k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(x_i d_{i1}) = \{s + 6k + i, 1 \leq i \leq k\}$$
$$\lambda(d_{i1} d_{i2}) = \{s + 7k + i, 1 \leq i \leq k\}$$
$$\lambda(d_{i2} u_{i+1}) = \{s + 8k + i, 1 \leq i \leq k\}$$
$$\lambda(x_i c_{i1}) = \{s + 10k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(c_{i1} c_{i2}) = \{s + 11k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(c_{i2} w_i) = \{s + 12k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(w_i b_{i1}) = \{s + 13k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(b_{i1} b_{i2}) = \{s + 14k + 1 - i, 1 \leq i \leq k\}$$
$$\lambda(b_{i2} u_i) = \{s + 15k + 1 - i, 1 \leq i \leq k\}$$

We symbolize the faces of H as follows:

$$\lambda(f_i) = \{s + e + i, 1 \leq i \leq k\}$$

In this way the snake graph of string $(2, 2, ..., 2)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1, 1, 1)$, with 2 subdivisions. □

Theorem 9 *For all* $k \geq 3$, $H \cong mkC_5$ *-* m *copies of snake graph of string* $(1, 1, ..., 1)$ *with* 1 *subdivision admit super 1-antimagic labeling of type (1,1,1).*

Proof Let $s = |V(H)|$, $e = |E(H)|$, and $f = |F(H)|$. Then we have $s = n(9k + 1)$, $e = 10nk$, and $f = nk$, $1 \leq m \leq n$.
For vertices

$$\lambda(u_i^m) = \{i + (m-1)(k+1) \quad 1 \leq i \leq k+1\}$$
$$\lambda(v_i^m) = \{n(k+1) + nk + 1 - (m-1)k - i \quad 1 \leq i \leq k\}$$
$$\lambda(x_i^m) = \{3nk + n + 1 - i - (m-1)k \quad 1 \leq i \leq k\}$$
$$\lambda(w_i^m) = \{4nk + n + 1 - i - (m-1)k \quad 1 \leq i \leq k\}$$

We symbolize the partitions of H as follows:

$$\lambda(a_i^m) = \{4nk + n + i + (m-1)k, 1 \leq i \leq k\}$$
$$\lambda(d_i^m) = \{6nk + n + 1 - i - (m-1)k, 1 \leq i \leq k\}$$

$$\lambda(e_i^m) = \{7nk + n + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(b_i^m) = \{7nk + n + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(c_i^m) = \{8nk + n + i + (m-1)k, 1 \le i \le k\}$$

We symbolize the edges of H as follows:

$$\lambda(u_i^m e_i^m) = \{s + nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(e_i^m u_{i+1}^m) = \{s + nk + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(v_i^m d_i^m) = \{s + 3nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(d_i^m u_{i+1}^m) = \{s + 4nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(w_i^m a_i^m) = \{s + 4nk + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(a_i^m u_i^m) = \{s + 6nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(w_i^m b_i^m) = \{s + 7nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(b_i^m x_i^m) = \{s + 7nk + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(x_i^m c_i^m) = \{s + 8nk + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(c_i^m v_i^m) = \{s + 9nk + i + (m-1)k, 1 \le i \le k\}$$

We symbolize the faces of H as follows:

$$\lambda(f_i^m) = \{s + e + nk - n(i-1) - (m-1), 1 \le i \le k\}$$

In this way the m copies of snake graph of string $(1, 1, ..., 1)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1, 1, 1)$, with 1 subdivision. □

Theorem 10 *For all $k \ge 2$, $H \cong mkC_5$ - m copies of snake graph of string $(2, 2, ..., 2)$ with 1 subdivision admit super 1-antimagic labeling of type (1,1,1).*

Proof Let $s = |V(H)|$, $e = |E(H)|$, and $f = |F(H)|$. Then we have $s = n(9k+1)$, $e = 10nk$, and $f = nk$, $1 \le m \le n$.
For vertices

$$\lambda(u_i^m) = \{i + (m-1)(k+1) \quad 1 \le i \le k+1\}$$
$$\lambda(v_i^m) = \{n(k+1) + nk + 1 - (m-1)k - i \quad 1 \le i \le k\}$$
$$\lambda(x_i^m) = \{3nk + n + 1 - i - (m-1)k \quad 1 \le i \le k\}$$
$$\lambda(w_i^m) = \{4nk + n + 1 - i - (m-1)k \quad 1 \le i \le k\}$$

We symbolize the partitions of H as follows:

$$\lambda(a_i^m) = \{4nk + n + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(d_i^m) = \{6nk + n + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(e_i^m) = \{7nk + n + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(b_i^m) = \{7nk + n + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(c_i^m) = \{8nk + n + i + (m-1)k, 1 \le i \le k\}$$

We symbolize the edges of H as follows:

$$\lambda(u_i^m e_i^m) = \{s + nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(e_i^m v_i^m) = \{s + nk + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(u_{i+1}^m d_i^m) = \{s + 3nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(d_i^m v_i^m) = \{s + 4nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(w_i^m a_i^m) = \{s + 4nk + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(a_i^m u_i^m) = \{s + 6nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(w_i^m b_i^m) = \{s + 7nk + 1 - i - (m-1)k, 1 \le i \le k\}$$
$$\lambda(b_i^m x_i^m) = \{s + 7nk + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(x_i^m c_i^m) = \{s + 8nk + i + (m-1)k, 1 \le i \le k\}$$
$$\lambda(c_i^m u_{i+1}^m) = \{s + 9nk + i + (m-1)k, 1 \le i \le k\}$$

We symbolize the faces of H as follows:

$$\lambda(f_i^m) = \{s + e + nk - n(i-1) - (m-1), 1 \le i \le k\}$$

□

In this way the m copies of snake graph of string $(2, 2, ..., 2)$ can be labeled in the best way to show super 1-antimagic labeling of type $(1, 1, 1)$, with 1 subdivision.

4. Open problems

Open Problem 1 *For all* $k \ge 2$, $H \cong kC_5 - snake$ *graph with string* $(2, 2, ..., 2)$ *with* p *subdivisions admits super 1-antimagic labeling of type* $(1, 1, 1)$.

Open Problem 2 *For all* $k \ge 3$, $H \cong kC_5 - snake$ *graph with string* $(1, 1, ..., 1)$ *with* p *subdivisions admits super 1-antimagic labeling of type* $(1, 1, 1)$.

Open Problem 3 *For all* $k \ge 2$, $H \cong mkC_5 - snake$ *graph with string* $(2, 2, ..., 2)$ *with* p *subdivisions admits super 1-antimagic labeling of type* $(1, 1, 1)$.

Open Problem 4 *For all* $k \ge 3$, $H \cong mkC_5 - snake$ *graph with string* $(1, 1, ..., 1)$ *with* p *subdivisions admits super 1-antimagic labeling of type* $(1, 1, 1)$.

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