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Existence of unique solution to switched fractional differential equations with p-Laplacian operator

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Abstract: In this paper, we study a class of nonlinear switched systems of fractional order with p-Laplacian operator. By applying a fixed point theorem for a concave operator on a cone, we obtain the existence and uniqueness of a positive solution for an integral boundary value problem with switched nonlinearity under some suitable assumptions. An illustrative example is included to show that the obtained results are effective.

Key words: Existence, positive solution, fractional-order switched system, integral boundary valued problems, p-Laplacian operator

1. Introduction

In this paper, we consider an integral boundary value problem (BVP for short) for fractional differential equations with switched nonlinearity and p-Laplacian operator:

$$\begin{cases} D_{0+}^{\beta}\phi_p(D_{0+}^{\alpha}u(t)) = f_{\sigma(t)}(t, u(t), D_{0+}^{\gamma}u(t)), \quad t \in J = [0,1], \\ u(0) = \mu\displaystyle\int_0^1 u(s)ds + \lambda u(\xi), \\ D_{0+}^{\alpha}u(0) = \kappa D_{0+}^{\alpha}u(\eta), \ \xi, \ \eta \in [0,1], \end{cases} \tag{1.1}$$

where ϕ_p is a p-Laplacian operator, $p > 1$, ϕ_p is invertible, and $(\phi_p)^{-1} = \phi_q$, $1/p + 1/q = 1$, $D_{0+}^{\alpha}, D_{0+}^{\beta}$ denote the Caputo fractional derivative of order α, β. $0 < \alpha, \beta \le 1 < \alpha + \beta \le 2$, $\mu, \lambda, \kappa \in (0,1), \mu + \lambda < 1$, ξ, $\eta \in [0,1]$, $\sigma(t) : [0,1] \to M = \{1, 2, \cdots, N\}$ is a finite switching signal that is a piecewise constant function depending on t, $\mathbb{R}^+ = (0, +\infty)$, $f_i \in C(J \times \mathbb{R}^+, \mathbb{R}^+)$, $i \in M$. Corresponding to the switching signal $\sigma(t)$, we have the following switching sequence:

$$\big\{(i_0, t_0), \cdots, (i_j, t_j), \cdots, (i_k, t_k)\big| i_j \in M, \ j = 0, 1, \cdots, k\big\}, \tag{1.2}$$

which means that the i_jth nonlinearity is activated when $t \in [t_j; t_{j+1})$ and the i_kth nonlinearity is activated when $t \in [t_k, 1]$.

Fractional differential equations play important roles in many research areas, such as physics, chemical technology, population dynamics, biotechnology, and economics (see [10, 16]). Since the p-Laplacian operator and fractional calculus arise from many applied fields, such as turbulent filtration in porous media, material

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science, blood flow problems, rheology, and modeling of viscoplasticity, it is worth studying the fractional p-Laplacian equations. In recent years, more and more researchers have been concerned with the BVP of fractional differential equations with p-Laplacian operator. For example, Chen and Liu [3] studied a class of BVPs for the fractional p-Laplacian equation:

$$\begin{cases} D_{0+}^{\beta}\phi_p(D_{0+}^{\alpha}u(t)) = f(t,u(t)), & t \in [0,1], \\ u(0) = -u(1),\ D_{0+}^{\alpha}u(0) = -D_{0+}^{\alpha}u(1), \end{cases}$$

where $0 < \alpha,\ \beta \leq 1,\ 1 < \alpha+\beta \leq 2,\ D_{0+}^{\alpha}$ is a Caputo fractional derivative, and $f : [0,1]\times\mathbb{R} \to \mathbb{R}$ is continuous. Under suitable nonlinear growth conditions, the existence result was presented by using Schaefer's fixed point theorem. Han et al. [9] considered a class of fractional BVP with p-Laplacian operator and boundary parameter, and they obtained several existence results for a positive solution in terms of the boundary parameter. In [17], Wang and Xiang investigated the following p-Laplacian fractional BVP:

$$\begin{cases} D_{0+}^{\gamma}\phi_p(D_{0+}^{\alpha}u(t)) = f(t,u(t)), & 0 < t < 1, \\ u(0) = 0,\ u(1) = au(\xi),\ D_{0+}^{\alpha}u(0) = 0,\ D_{0+}^{\alpha}u(1) = bD_{0+}^{\alpha}u(\eta)\ , \end{cases}$$

where $1 < \gamma,\ \alpha \leq 2,\ 0 \leq a,\ b \leq 1,\ 0 < \xi,\ \eta < 1,\ D_{0+}^{\alpha}$ is the standard Riemann–Liouville fractional differential operator of order α. By using the upper and lower solutions method, they got some existence results on the existence of a positive solution. For more work, the reader can refer to [2, 4, 12, 13, 14, 15] and the references therein.

However, we note that the above works just considered a single-mode nonlinearity. Fractional differential equations often have switched nonlinearity in practice, which is called 'switched systems'. Switched systems arise as models for phenomena that cannot be described as exclusively continuous or exclusively discrete processes [1]. A class of dynamic systems for hybrid systems are the switched systems [18]. Due to their applications in chemical processing, traffic control, switching power converters, etc., switched systems have been studied by many researchers and a lot of excellent results have been obtained in the last decades (see [6, 5, 8, 11] and the references therein).

Motivated by the above-mentioned works, we study a class of fractional differential equations' integral BVPs with switched nonlinearity and p-Laplacian operator. To the best of our knowledge, there are relatively few results on BVPs for fractional p-Laplacian equations, and no paper is concerned with the existence of a unique solution for the fractional p-Laplacian BVP (1.1). By applying a fixed point theorem for a concave operator on a cone, we will obtain the existence and uniqueness of a positive solution for an integral BVP with switched nonlinearity under some suitable assumptions.

This paper is organized as follows. In Section 2, we present some material to prove our main results. In Section 3, we prove the existence of a uniqueness solution for nonlinear fractional differential equations' BVP (1.1). Finally, an example is given to illustrate the main results in Section 4.

2. Preliminaries and lemmas

In this section, we recall the following known definitions and some preliminary facts.

Definition 2.1 ([16, 10]) *The fractional order integral of a function $f : (0,\infty) \to \mathbb{R}$ of order $\alpha > 0$ is defined by*

$$I_{0+}^{\alpha}f(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}f(s)ds.$$

Definition 2.2 **([16, 10])** *The Riemann–Liouville derivative of order $\alpha > 0$ for a function $f : [0, +\infty) \to \mathbb{R}$ can be written as*

$$^{L}\boldsymbol{D}^{\alpha}_{0+} f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_0^x \frac{f(s)}{(x-s)^{\alpha-n+1}} ds,$$

where n is the smallest integer greater than α.

Definition 2.3 **([16, 10])** *The Caputo fractional derivative of order $\alpha > 0$ for a function $f : [0, +\infty) \to \mathbb{R}$ can be written as*

$$D^{\alpha}_{0+} f(x) = \ ^{L}\boldsymbol{D}^{\alpha}_{0+} \left[f(x) - \sum_{k=0}^{n-1} \frac{x^k}{k!} f^{(k)}(0) \right],$$

where n is the smallest integer greater than α.

Remark 1 *If $f \in AC^n[0, +\infty)$, then it is the standard Caputo fractional derivative*

$$^{C}D^{\alpha}_{0+} f(x) = D^{\alpha}_{0+} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x \frac{f^{(n)}(s)}{(x-s)^{\alpha+1-n}} ds,$$

where n is the smallest integer greater than α.

Furthermore, the Caputo derivative of a constant is equal to zero.

Lemma 2.4 **([10], Lemma 2.22)** *Let $\alpha > 0$ and $u \in AC^n[0,1]$. Then the following equality holds:*

$$I^{\alpha}_{0+} D^{\alpha}_{0+} u(t) = u(t) + c_0 + c_1 t + \cdots + c_{n-1} t^{n-1},$$

where $c_i \in \mathbb{R},\ i = 0, 1, ..., n-1;\ n$ is the smallest integer greater than α.

Lemma 2.5 **([14], Lemma 3.2)** *Let $\varphi(t) \in C[0,1],\ \alpha,\ \beta \in (0,1],\ \mu,\ \lambda,\ \kappa \in \mathbb{R}$ such that $k \neq 1,\ \mu + \lambda \neq 1$, and then a function $u \in \{u \mid u \in AC[0,1]$ and $D^{\alpha}_{0+} u \in AC[0,1]\}$ is a solution of the following fractional differential equation:*

$$\begin{cases} D^{\beta}_{0+} \phi_p(D^{\alpha}_{0+} u(t)) = \varphi(t), \quad t \in [0,1], \\ u(0) = \mu \displaystyle\int_0^1 u(s) ds + \lambda u(\xi), \\ D^{\alpha}_{0+} u(0) = \kappa D^{\alpha}_{0+} u(\eta),\ \xi,\ \eta \in [0,1], \end{cases} \tag{2.1}$$

if and only if $u \in C[0,1]$ is a solution of the fractional integral equation

$$\begin{aligned} u(t) &= I^{\alpha}_{0+} \phi_q(I^{\beta}_{0+} \varphi(t) + F_1 \varphi(t)) + F_2 \varphi(t) \\ &= \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \phi_q \left(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)} \varphi(\tau) d\tau + F_1 \varphi(s) \right) ds + F_2 \varphi(t) \end{aligned} \tag{2.2}$$

where for any $t \in [0,1]$

$$F_1 \varphi(t) = \frac{\phi_p(\kappa)}{1 - \phi_p(\kappa)} \int_0^\eta \frac{(\eta - s)^{\beta-1}}{\Gamma(\beta)} \varphi(s) ds, \tag{2.3}$$

$$F_2\varphi(t) = \int_0^1 \frac{\mu(1-s)^{\alpha}}{(1-\mu-\lambda)\Gamma(\alpha+1)}\phi_q\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\varphi(\tau)d\tau + F_1\varphi(s)\Big)ds + \int_0^{\xi} \frac{\lambda(\xi-s)^{\alpha-1}}{(1-\mu-\lambda)\Gamma(\alpha)}\phi_q\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\varphi(\tau)d\tau + F_1\varphi(s)\Big)ds. \quad (2.4)$$

3. Main results

In this section, we consider (1.1) in the real Banach space $E = C[0,1]$ with the norm $\|x\| = \max_{t\in[0,1]} |x(t)|$. Let $P = \{x \in E : x(t) \geq 0,\ t \in [0,1]\}$. Then P is a normal solid cone of E with $P^{\circ} = \{x \in E : x(t) > 0,\ t \in [0,1]\}$. A function $x \in E$ is said to be a positive solution to BVP (1.1), if $x \in P$ and $x(t) \not\equiv 0$. We consider the existence of positive solutions to the fractional differential equation BVP (1.1) in P.

For $x \in E$, let us define an operator $K : E \to E$ as follows:

$$Ku(t) = I_{0+}^{\alpha}\phi_q\big(I_{0+}^{\beta}u(t) + F_1u(t)\big) + F_2u(t),$$

$$F^{\sigma}(t,u) = \begin{cases} f_{i_0}(t,u), & t \in [0,t_1], \\ \vdots & \\ f_{i_j}(t,u), & t \in [t_j, t_{j+1}), \\ \vdots & \\ f_{i_k}(t,u), & t \in [t_k, 1], \end{cases}$$

where the operators F_1 and F_2 are giver by (2.3) and (2.4).

Remark 2 *Define an operator $T : E \to E$ as $Tu(t) = K\big(F^{\sigma}(t,u(t))\big)$. Then Lemma 2.5 implies that a function $u \in E$ is a solution to BVP (1.1) if and only if u is a fixed point of T, i.e. $u(t) = Tu(t)$.*

In the following, we study the existence of a unique positive solution to BVP (1.1). To this end, we need the following definition and fixed point theorem.

Definition 3.1 ([7]) *Let P be a normal solid cone in a real Banach space E and P° be the interior of P. Suppose that $T : P^{\circ} \to P^{\circ}$ is an operator and $0 \leq \theta < 1$. Then T is called a θ-concave operator if*

$$T(ku) \geq k^{\theta}Tu, \quad \forall\ k \in (0,1),\ u \in P^{\circ}$$

.

Lemma 3.2 ([7]) *Assume that P is a normal solid cone in a real Banach space E, $0 \leq \theta < 1$, and $T : P^{\circ} \to P^{\circ}$ is a θ-concave increasing operator. Then T has a unique fixed point in P°.*

Now we list some assumptions on the nonlinearity of BVP (1.1).

(A_1) For any $i \in M$, $f_i : J \times \mathbb{R}^+ \to \mathbb{R}^+$, and $f_i(t,x)$ is increasing in x for $x \in \mathbb{R}^+$.

(A_2) For any $i \in M$, there exists a $\theta_i \in [0,1)$ such that

$$f_i(t,kx) \geq k^{(p-1)\theta_i} f_i(t,x),\ k \in (0,1),\ t \in J,\ x \in \mathbb{R}^+$$

.

Remark 3 *Conditions (A_1) and (A_2) imply the following conditions of $F^\sigma(t,x)$:*
(A'_1) For $t \in J, x \in \mathbb{R}^+$, $F^\sigma(t,x) > 0$, and $F^\sigma(t,x)$ is increasing in x for $x \in \mathbb{R}^+$.
(A'_2) $F^\sigma(t,x) \geq k^{(p-1)\theta} F^\sigma(t,x)$, $k \in (0,1)$, $t \in J, x \in \mathbb{R}^+$, where $\theta = \max_{i\in M} \theta_i$.

Theorem 3.3 *Suppose that $0 < \kappa < 1, 0 < \mu + \lambda < 1$, and assumptions ($A_1$)–($A_2$) hold. Then for any finite switching signal $\sigma(t) : J \to M$, BVP (1.1) has an unique positive solution.*

Proof We first prove that $T : P^\circ \to P^\circ$. For any $u \in P^\circ$, we have $u(t) > 0,\ t \in [0,1]$. Then (A'_1) implies

$$
\begin{aligned}
Tu(t) =& I^{\alpha}_{0+}\phi_q\big(I^{\beta}_{0+}F^\sigma(t,u(t)) + F_1(F^\sigma(t,u(t)))\big) + F_2(F^\sigma(t,u(t)))\\
=& \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^\sigma(\tau,u(\tau))d\tau\\
&+ \frac{\phi_p(\kappa)}{1-\phi_p(\kappa)}\int_0^\eta \frac{(\eta-\tau)^{\beta-1}}{\Gamma(\beta)}F^\sigma(\tau,u(\tau))d\tau\bigg)ds\\
&+ \int_0^1 \frac{\mu(1-s)^{\alpha}}{(1-\mu-\lambda)\Gamma(\alpha+1)}\phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^\sigma(\tau,u(\tau))d\tau\\
&+ \frac{\phi_p(\kappa)}{1-\phi_p(\kappa)}\int_0^\eta \frac{(\eta-\tau)^{\beta-1}}{\Gamma(\beta)}F^\sigma(\tau,u(\tau))d\tau\bigg)ds\\
&+ \int_0^\xi \frac{\lambda(\xi-s)^{\alpha-1}}{(1-\mu-\lambda)\Gamma(\alpha)}\phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^\sigma(\tau,u(\tau))d\tau\\
&+ \frac{\phi_p(\kappa)}{1-\phi_p(\kappa)}\int_0^\eta \frac{(\eta-\tau)^{\beta-1}}{\Gamma(\beta)}F^\sigma(\tau,u(\tau))d\tau\bigg)ds > 0, \quad \forall t \in [0,1],
\end{aligned}
$$

and thus $Tu \in P^\circ$.

Next we prove that T is increasing in P°. For any $x_1, x_2 \in P^\circ$ with $x_1 \leq x_2$, from the monotonicity of F^σ and x^{q-1}, we have $Tx_2(t) - Tx_1(t) \geq 0,\ t \in [0,1]$, which implies that T is increasing in P°.

Finally, we prove that T is a θ-concave operator. In fact, from (A'_2), for any $0 < k < 1,\ u \in P^\circ$, it is easy to see that

$$
\begin{aligned}
F_1(F^\sigma(t,ku(t))) =& \frac{\phi_p(\kappa)}{1-\phi_p(\kappa)}\int_0^\eta \frac{(\eta-\tau)^{\beta-1}}{\Gamma(\beta)}F^\sigma(\tau,ku(\tau))d\tau\\
\geq& k^{(p-1)\theta}\frac{\phi_p(\kappa)}{1-\phi_p(\kappa)}\int_0^\eta \frac{(\eta-\tau)^{\beta-1}}{\Gamma(\beta)}F^\sigma(\tau,u(\tau))d\tau\\
=& k^{(p-1)\theta}F_1(F^\sigma(t,u(t))),
\end{aligned}
$$

$$
\begin{aligned}
T(ku)(t) =& I_{0^+}^{\alpha}\phi_q\big(I_{0^+}^{\beta}F^{\sigma}(t,u(t)) + F_1(F^{\sigma}(t,u(t)))\big) + F_2\big(F^{\sigma}(t,u(t))\big)\\
\geq& \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^{\sigma}(\tau,ku(\tau))d\tau + k^{(p-1)\theta}F_1(F^{\sigma}(s,u(s)))\bigg)ds\\
&+\int_0^1 \frac{\mu(1-s)^{\alpha}}{(1-\mu-\lambda)\Gamma(\alpha+1)}\\
&\times \phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^{\sigma}(\tau,ku(\tau))d\tau + k^{(p-1)\theta}F_1(F^{\sigma}(s,u(s)))\bigg)ds\\
&+\int_0^{\xi} \frac{\lambda(\xi-s)^{\alpha-1}}{(1-\mu-\lambda)\Gamma(\alpha)}\phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^{\sigma}(\tau,ku(\tau))d\tau\\
&+k^{(p-1)\theta}F_1(F^{\sigma}(s,u(s)))\bigg)ds\\
\geq& k^{\theta}\int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^{\sigma}(\tau,u(\tau))d\tau + F_1(F^{\sigma}(s,u(s)))\bigg)ds\\
&+k^{\theta}\int_0^1 \frac{\mu(1-s)^{\alpha}}{(1-\mu-\lambda)\Gamma(\alpha+1)}\\
&\times \phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^{\sigma}(\tau,u(\tau))d\tau + F_1(F^{\sigma}(s,u(s)))\bigg)ds\\
&+k^{\theta}\int_0^{\xi} \frac{\lambda(\xi-s)^{\alpha-1}}{(1-\mu-\lambda)\Gamma(\alpha)}\phi_q\bigg(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}F^{\sigma}(\tau,u(\tau))d\tau + F_1(F^{\sigma}(s,u(s)))\bigg)ds\\
=& k^{\theta}Tu(t),
\end{aligned}
$$

which implies that T is a θ-concave operator. By Lemma 3.2, BVP (1.1) has a unique positive solution. □

4. An illustrative example

In this section, we give an example to illustrate the usefulness of our main results.

Example 1 *Consider the following boundary value problem consisting of the equation*

$$
\begin{cases}
D_{0^+}^{\frac{2}{3}}\phi_3\big(D_{0^+}^{\frac{1}{2}}u(t)\big) = f_{\sigma(t)}(t,u(t)),\ t\in J=[0,1],\\
u(0) = \dfrac{1}{4}\displaystyle\int_0^1 u(t)dt + \dfrac{1}{4}u(1),\quad D_{0^+}^{\frac{1}{2}}u(0) = \dfrac{1}{4}D_{0^+}^{\frac{1}{2}}u(1),
\end{cases} \tag{4.1}
$$

where $\sigma(t): J\to M=\{1,2,3\}$ *is a finite switching signal,*

$$
f_1(t,u) = (1+t)\sqrt{u},\quad f_2(t,u) = (2+\sin t)\sqrt[3]{u},\quad f_3(t,u) = (\frac{1}{3}+t^2)u^{\frac{3}{4}}.
$$

It is not difficult to verify that problem (4.1) is of the form of (1.1). For the particular case $p=3,\ q=\frac{3}{2},\ \alpha=\frac{1}{2},\ \beta=\frac{2}{3},\ \mu=\lambda=\kappa=\frac{1}{4},\ \xi=\eta=1$, and

$$
f_i(t,u) > 0,\quad \forall t\in J, u\in(0,+\infty).
$$

Through some calculation, for $\forall t \in J, u \in (0,+\infty)$, we have

$$\frac{\partial f_1(t,u)}{\partial u} = \frac{1+t}{2\sqrt{u}} > 0, \quad \frac{\partial f_2(t,u)}{\partial u} = \frac{2+\sin t}{2u^{\frac{2}{3}}} > 0, \quad \frac{\partial f_3(t,u)}{\partial u} = \frac{3t^2+1}{4\sqrt[4]{u}} > 0.$$

Hence, (A_1) holds.

Moreover, for $\forall t \in J, u \in (0,+\infty)$, we have

$$f_1(t,ku) = k^{\frac{1}{2}}(1+t)\sqrt{u} \geq k(1+t)\sqrt{u} = k^{\frac{p-1}{2}} f_1(t,u), \qquad \forall k \in (0,1),$$

$$f_2(t,ku) = k^{\frac{1}{3}}(2+\sin t)\sqrt[3]{u} \geq= k^{\frac{2}{3}}(2+\sin t)\sqrt[3]{u} = k^{\frac{p-1}{3}} f_2(t,u), \qquad \forall k \in (0,1),$$

$$f_3(t,ku) = k^{\frac{3}{4}}(\frac{1}{3}+t^2)u^{\frac{3}{4}} \geq k^{\frac{3}{4}}(\frac{1}{3}+t^2)u^{\frac{3}{4}} = k^{\frac{(p-1)3}{4}} f_3(t,u), \qquad \forall k \in (0,1),$$

and therefore (A_2) holds.

Hence, the problem (4.1) satisfies all assumptions of Theorem 3.3. Then for any finite switching signal $\sigma(t): J \to M$, BVP (4.1) has a unique positive solution.

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