Turkish Journal of Mathematics

http://journals.tubitak.gov.tr/math/

Research Article

Turk J Math
(2016) 40: 90 – 97
© TÜBİTAK
doi:10.3906/mat-1505-84

Homothetic motions with dual octonions in dual $8-$space

Hesna KABADAYI*
Department of Mathematics, Ankara University, Ankara, Turkey

Received: 25.05.2015 • **Accepted/Published Online:** 21.07.2015 • **Final Version:** 01.01.2016

Abstract: In this study, for dual octonions in dual $8-$space D^8 over the domain of coefficients D, we give a matrix that is similar to a Hamilton operator. By means of this matrix a new motion is defined and this motion is proven to be homothetic. For this one parameter dual homothetic motion, we prove some theorems about dual velocities, dual pole points, and dual pole curves. Furthermore, after defining dual accelerations, we show that the motion defined by the regular order m dual curve, at every t-instant, has only one acceleration center of order $(m-1)$.

Key words: Dual octonions, homothetic motion, dual curve, dual space

1. Introduction

John T. Graves discovered octonions in 1843. Arthur Cayley also discovered them independently and, on account of this, they are sometimes referred to as Cayley numbers.

The octonions form a normed division algebra over real numbers. They are nonassociative and non-commutative but alternative, which is a weaker form of associativity. Octonions have applications in string theory, special relativity, and quantum logic. They have some interesting features and are related to a number of exceptional structures, such as exceptional Lie groups, in mathematics.

The set of dual numbers $D = \left\{\widehat{a} = a + \varepsilon a^* : \varepsilon \neq 0, \varepsilon^2 = 0, a, a^* \in \mathbb{R}\right\}$ is a commutative ring with a unit.

Dual numbers were introduced by Clifford [4] in the 19th century. They were applied to describe rigid body motions in three- dimensional space by Koltelnikov [7]. With the help of dual numbers, Yaglom [9] described geometrical objects in three- dimensional space. The notion of dual angle is defined by Study [8].

Recently, dual numbers have found applications in many areas such as in kinematics, dynamics, robotics, computer aided geometrical design, mechanism design and modeling of rigid bodies, group theory, and field theory.

In [6], a curve, which is represented by bicomplex numbers, in a hypersurface in E^4 is considered; then by the help of this curve a homothetic motion is defined. In this study, we considered a dual curve in dual $8-$space over the domain of coefficients D. We defined a homothetic motion by means of this dual curve. Under some conditions, this homothetic motion satisfies all of the properties in [10, 11]. For one parameter dual homothetic motion, we give some results about dual velocities, dual pole points, and dual pole curves. Moreover, after defining dual accelerations, we prove that the motion defined by the regular order m dual curve, at every t-instant, has only one acceleration center of order $(m-1)$. We hope that these results will contribute to the

*Correspondence: hesnakabadayi@gmail.com

2010 *AMS Mathematics Subject Classification:* Primary 53A17, 53B50, 53A25.

geometry of dual spaces and hence to the study of applications of physics and kinematics. Some recent papers on homothetic motions include [2, 5].

2. Dual 8−space, dual quaternions, and dual octonions

The set of all 8-tuples of dual numbers, i.e.

$$D^8 = \{\widetilde{\alpha} = (\widehat{\alpha_0}, \widehat{\alpha_1}, \widehat{\alpha_2}, \widehat{\alpha_3}, \widehat{\alpha_4}, \widehat{\alpha_5}, \widehat{\alpha_6}, \widehat{\alpha_7}) : \widehat{\alpha_i} \in D, 0 \leq i \leq 7\}$$

is called dual 8-space and denoted by D^8. It is a module over the ring D. Elements of D^8 are called dual vectors. A dual vector $\widetilde{\alpha}$ can be written as $\widetilde{\alpha} = \overrightarrow{\alpha} + \varepsilon \overrightarrow{\alpha}^* = (\overrightarrow{\alpha}, \overrightarrow{\alpha}^*)$, where $\overrightarrow{\alpha} = (\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7)$ and $\overrightarrow{\alpha}^* = (\alpha_0^*, \alpha_1^*, \alpha_2^*, \alpha_3^*, \alpha_4^*, \alpha_5^*, \alpha_6^*, \alpha_7^*)$ are vectors in $\mathbb{R}^8$. The norm of $\widetilde{\alpha} \in D^8$ is given by

$$\|\widetilde{\alpha}\| = \sqrt{\widehat{\alpha_0}^2 + \widehat{\alpha_1}^2 + \widehat{\alpha_2}^2 + \widehat{\alpha_3}^2 + \widehat{\alpha_4}^2 + \widehat{\alpha_5}^2 + \widehat{\alpha_6}^2 + \widehat{\alpha_7}^2}.$$

A dual vector with norm 1 is called a dual unit vector.

A dual quaternion is given by

$$Q = \widehat{Q_0} + \widehat{Q_1} i + \widehat{Q_2} j + \widehat{Q_3} k,$$

where $\widehat{Q_0}, \widehat{Q_1}, \widehat{Q_2}, \widehat{Q_3} \in D$ and the products of $\{1, i, j, k\}$ are given as in the following table

	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

.

Note that

$$D_{\mathbb{C}} = \left\{\widehat{Q_0} + \widehat{Q_1} i : \widehat{Q_0}, \widehat{Q_1} \in D, i^2 = -1\right\}$$

is the set of dual complex numbers and

$$\widehat{Q_0} + \widehat{Q_1} i + \widehat{Q_2} j + \widehat{Q_3} k = (\widehat{Q_0} + \widehat{Q_1} i) + (\widehat{Q_2} + \widehat{Q_3} i) j = W + Zj,$$

where $W = \widehat{Q_0} + \widehat{Q_1} i$ and $Z = \widehat{Q_2} + \widehat{Q_3} i$ are dual complex numbers and $j^2 = -1$.

Thus, dual quaternions can be viewed as pairs of dual complex numbers. The set of dual quaternions is given as follows:

$$\begin{aligned} H_D &= \left\{Q = \widehat{Q_0} + \widehat{Q_1} i + \widehat{Q_2} j + \widehat{Q_3} k : \widehat{Q_0}, \widehat{Q_1}, \widehat{Q_2}, \widehat{Q_3} \in D\right\} \\ &= \left\{W + Zj : W, Z \in D_{\mathbb{C}}, j^2 = -1\right\} = D_{\mathbb{C}} \times D_{\mathbb{C}}. \end{aligned}$$

Now we define

$$(W_1 + Z_1 j)(W_2 + Z_2 j) = \left(W_1 W_2 - \overline{Z_2} Z_1\right) + \left(Z_1 \overline{W_2} + Z_2 W_1\right),$$

where $\overline{Z}$ denotes the conjugate of $Z \in D_{\mathbb{C}}$. This product in H_D is readily seen to be the product defined in terms of i, j, k.

Let

$$O_D = \{K = \Omega + \Psi e : \Omega, \Psi \in H_D\},$$

where e is an arbitrary unit with $e^2 = -1$. We define the product in O_D as follows:

$$K_1K_2 = (\Omega_1 + \Psi_1 e)(\Omega_2 + \Psi_2 e) = \left(\Omega_1\Omega_2 - \overline{\Psi_2}\Psi_1\right) + \left(\Psi_1\overline{\Omega_2} + \Psi_2\Omega_1\right)e, \tag{2.1}$$

where $\Omega_i, \Psi_i \in H_D, K_i \in O_D$ and the conjugate $\overline{\Omega}$ of the dual quaternion $\Omega = \widehat{P_0} + \widehat{P_1}i + \widehat{P_2}j + \widehat{P_3}k$ is $\widehat{P_0} - \widehat{P_1}i - \widehat{P_2}j - \widehat{P_3}k$. Note that O_D becomes a noncommutative and nonassociative, but alternative (i.e. $(K_1K_1)K_2 = K_1(K_1K_2), (K_1K_2)K_2 = K_1(K_2K_2)$, for $K_1, K_2 \in O_D$.) ring. The elements of O_D are called dual octonions. Unlike the real octonions, dual octonions do not form a division ring. It is known that the set of real octonions forms a noncommutative, nonassociative, but alternative division ring. [3, 12].

We can also give the product in O_D similar to Hamilton operators (see [1]), which has been given in (2.1).

If $K = \Omega + \Psi e \in O_D, \Omega = \widehat{P_0} + \widehat{P_1}i + \widehat{P_2}j + \widehat{P_3}k \in H_D, \Psi = \widehat{P_4} + \widehat{P_5}i + \widehat{P_6}j + \widehat{P_7}k \in H_D, \widehat{P_i} \in D$, then N^+ is defined as

$$N^+(K) = \begin{bmatrix}
\widehat{P_0} & -\widehat{P_1} & -\widehat{P_2} & -\widehat{P_3} & -\widehat{P_4} & -\widehat{P_5} & -\widehat{P_6} & -\widehat{P_7} \\
\widehat{P_1} & \widehat{P_0} & -\widehat{P_3} & \widehat{P_2} & -\widehat{P_5} & \widehat{P_4} & \widehat{P_7} & -\widehat{P_6} \\
\widehat{P_2} & \widehat{P_3} & \widehat{P_0} & -\widehat{P_1} & -\widehat{P_6} & -\widehat{P_7} & \widehat{P_4} & \widehat{P_5} \\
\widehat{P_3} & -\widehat{P_2} & \widehat{P_1} & \widehat{P_0} & -\widehat{P_7} & \widehat{P_6} & -\widehat{P_5} & \widehat{P_4} \\
\widehat{P_4} & \widehat{P_5} & \widehat{P_6} & \widehat{P_7} & \widehat{P_0} & -\widehat{P_1} & -\widehat{P_2} & -\widehat{P_3} \\
\widehat{P_5} & -\widehat{P_4} & \widehat{P_7} & -\widehat{P_6} & \widehat{P_1} & \widehat{P_0} & \widehat{P_3} & -\widehat{P_2} \\
\widehat{P_6} & -\widehat{P_7} & -\widehat{P_4} & \widehat{P_5} & \widehat{P_2} & -\widehat{P_3} & \widehat{P_0} & \widehat{P_1} \\
\widehat{P_7} & \widehat{P_6} & -\widehat{P_5} & -\widehat{P_4} & \widehat{P_3} & \widehat{P_2} & -\widehat{P_1} & \widehat{P_0}
\end{bmatrix}$$

$$N^+(K) = \begin{bmatrix} H^+(\Omega) & -M^T \\ M & H^-(\Omega) \end{bmatrix},$$

where M is a 4×4 dual matrix and H^+ and H^- are dual Hamilton operators (i.e. quaternion replaced by dual quaternion). By means of the definition of N^+, the multiplication of $K_1, K_2 \in O_D$ is given by

$$K_1K_2 = N^+(K_1)K_2.$$

3. Homothetic motion in dual $8-$space

Let us consider the following parametrized dual curve:

$$\begin{aligned}
\widetilde{\alpha} &: I \subset \mathbb{R} \longrightarrow D^8 \text{ given by,} \\
\widetilde{\alpha}(t) &= \left(\widehat{\alpha_0}(t), \widehat{\alpha_1}(t), \widehat{\alpha_2}(t), \widehat{\alpha_3}(t), \widehat{\alpha_4}(t), \widehat{\alpha_5}(t), \widehat{\alpha_6}(t), \widehat{\alpha_7}(t)\right), \text{ for } \forall t \in I
\end{aligned}$$

$\widetilde{\alpha}(t) = \overrightarrow{\alpha}(t) + \varepsilon\overrightarrow{\alpha}^*(t)$,

where $\overrightarrow{\alpha}(t) = (\alpha_0(t), \alpha_1(t), \alpha_2(t), \alpha_3(t), \alpha_4(t), \alpha_5(t), \alpha_6(t), \alpha_7(t))$, (the real part of $\widetilde{\alpha}(t)$ is called indicatrix),

$$\overrightarrow{\alpha}^*(t) = (\alpha_0^*(t), \alpha_1^*(t), \alpha_2^*(t), \alpha_3^*(t), \alpha_4^*(t), \alpha_5^*(t), \alpha_6^*(t), \alpha_7^*(t))$$

are curves in $\mathbb{R}^8$. If the real valued functions $\alpha_i(t)$ and $\alpha_i^*(t)$, $0 \leq i \leq 7$ are differentiable then the dual space curve $\widetilde{\alpha}(t)$ is differentiable in D^8. We suppose that the curve $\widetilde{\alpha}(t)$ is a differentiable regular order m dual curve. (i.e. $\left\|\widetilde{\alpha}^{(m)}(t)\right\| \neq 0$). Corresponding to $\widetilde{\alpha}(t)$ the operator $\widetilde{B} = N^+(\widetilde{\alpha}(t))$ is given by the following dual matrix:

$$\widetilde{B} = \begin{bmatrix} \widehat{\alpha_0}(t) & -\widehat{\alpha_1}(t) & -\widehat{\alpha_2}(t) & -\widehat{\alpha_3}(t) & -\widehat{\alpha_4}(t) & -\widehat{\alpha_5}(t) & -\widehat{\alpha_6}(t) & -\widehat{\alpha_7}(t) \\ \widehat{\alpha_1}(t) & \widehat{\alpha_0}(t) & -\widehat{\alpha_3}(t) & \widehat{\alpha_2}(t) & -\widehat{\alpha_5}(t) & \widehat{\alpha_4}(t) & \widehat{\alpha_7}(t) & -\widehat{\alpha_6}(t) \\ \widehat{\alpha_2}(t) & \widehat{\alpha_3}(t) & \widehat{\alpha_0}(t) & -\widehat{\alpha_1}(t) & -\widehat{\alpha_6}(t) & -\widehat{\alpha_7}(t) & \widehat{\alpha_4}(t) & \widehat{\alpha_5}(t) \\ \widehat{\alpha_3}(t) & -\widehat{\alpha_2}(t) & \widehat{\alpha_1}(t) & \widehat{\alpha_0}(t) & -\widehat{\alpha_7}(t) & \widehat{\alpha_6}(t) & -\widehat{\alpha_5}(t) & \widehat{\alpha_4}(t) \\ \widehat{\alpha_4}(t) & \widehat{\alpha_5}(t) & \widehat{\alpha_6}(t) & \widehat{\alpha_7}(t) & \widehat{\alpha_0}(t) & -\widehat{\alpha_1}(t) & -\widehat{\alpha_2}(t) & -\widehat{\alpha_3}(t) \\ \widehat{\alpha_5}(t) & -\widehat{\alpha_4}(t) & \widehat{\alpha_7}(t) & -\widehat{\alpha_6}(t) & \widehat{\alpha_1}(t) & \widehat{\alpha_0}(t) & \widehat{\alpha_3}(t) & -\widehat{\alpha_2}(t) \\ \widehat{\alpha_6}(t) & -\widehat{\alpha_7}(t) & -\widehat{\alpha_4}(t) & \widehat{\alpha_5}(t) & \widehat{\alpha_2}(t) & -\widehat{\alpha_3}(t) & \widehat{\alpha_0}(t) & \widehat{\alpha_1}(t) \\ \widehat{\alpha_7}(t) & \widehat{\alpha_6}(t) & -\widehat{\alpha_5}(t) & -\widehat{\alpha_4}(t) & \widehat{\alpha_3}(t) & \widehat{\alpha_2}(t) & -\widehat{\alpha_1}(t) & \widehat{\alpha_0}(t) \end{bmatrix}. \tag{3.1}$$

Let $\left\|\dot{\widetilde{\alpha}}(t)\right\| = 1$, i.e. $\widetilde{\alpha}(t)$ be a unit velocity dual curve. If $\widetilde{\alpha}(t)$ does not pass through the origin, and $\overrightarrow{\alpha}(t) \neq 0$, the matrix in (3.1) can be written as

$$\widetilde{B} = \widetilde{h}\widetilde{A}, \tag{3.2}$$

where $\widetilde{A} = \frac{\widetilde{B}}{\widetilde{h}}$ and

$$\begin{aligned} \widetilde{h} \quad &: \quad I \subset \mathbb{R} \longrightarrow D \\ s \quad &\longrightarrow \quad \widetilde{h}(s) = \|\widetilde{\alpha}(s)\| = \sqrt{\widehat{\alpha_0}^2 + \widehat{\alpha_1}^2 + \widehat{\alpha_2}^2 + \widehat{\alpha_3}^2 + \widehat{\alpha_4}^2 + \widehat{\alpha_5}^2 + \widehat{\alpha_6}^2 + \widehat{\alpha_7}^2}. \end{aligned}$$

Theorem 1 *The matrix* $\widetilde{A}$ *in the equation* (3.2) *is a dual ortogonal matrix.*

Proof By the definition of the matrix $\widetilde{A}$, it is easy to see that $\widetilde{A}\widetilde{A}^T = \widetilde{A}^T\widetilde{A} = I_8$ and $\det \widetilde{A} = 1$. □

4. One parameter dual motion

Let the dual motional space and the dual fixed space be respectively $\widetilde{R}$ and $\widetilde{R_0}$. Now the one-parameter dual motion of $\widetilde{R_0}$ with respect to $\widetilde{R}$ will be denoted by $\widetilde{R_0}/\widetilde{R}$.

This dual motion can be expressed as follows:

$$\begin{bmatrix} \widetilde{X} \\ 1 \end{bmatrix} = \begin{bmatrix} \widetilde{h}\widetilde{A} & \widetilde{C} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \widetilde{X_0} \\ 1 \end{bmatrix} \tag{4.1}$$

or equivalently

$$\widetilde{X} = \widetilde{h}\widetilde{A}\widetilde{X_0} + \widetilde{C}. \tag{4.2}$$

Here dual position vectors of any point respectively in $\widetilde{R}$ and $\widetilde{R_0}$ are represented by $\widetilde{X}$ and $\widetilde{X_0}$, and $\widetilde{C}$ represents any dual translation vector.

Definition 1 *In dual* 8*-space, the one-parameter dual homothetic motion of a body is generated by the transformation given in* (4.2)*. Here* $\widetilde{h}$ *is called the homothetic scale, which is a dual scalar matrix,* $\widetilde{A}$ *is an* 8×8 *dual orthogonal matrix,* $\widetilde{X_0}$ *and* $\widetilde{C}$ *are* 8×1 *dual matrices, and* $\widetilde{A},\widetilde{C},$ *and* $\widetilde{h}$ *are of class* C^m*.*

In order not to encounter the case of affine transformation we suppose that

$$\widetilde{h}\,(t) = h\,(t) + \varepsilon h^*\,(t) \neq Cons\tan t, h\,(t) \neq 0,$$

and to prevent the cases of pure rotation and pure translation we also suppose that

$$\dot{\widetilde{h}}\widetilde{A} + \widetilde{h}\dot{\widetilde{A}} \neq 0, \dot{\widetilde{C}} \neq 0.$$

Theorem 2 *In dual* 8*-space, the motion determined by the equation* (4.1) *is a dual homothetic motion.*

Proof The matrix $\widetilde{A}$ is a dual orthogonal matrix by Theorem 1; therefore the motion in (4.1) is a dual homothetic motion. □

Now the matrix $\widetilde{B} = \widetilde{h}\widetilde{A}$ is said to be a dual homothetic matrix.

Theorem 3 *The derivation operator* $\dot{\widetilde{B}}$ *of the dual homothetic matrix* $\widetilde{B} = \widetilde{h}\widetilde{A}$ *is a dual ortogonal matrix.*

Proof Using (3.1), we have that $\dot{\widetilde{B}}\dot{\widetilde{B}}^T = \dot{\widetilde{B}}^T\dot{\widetilde{B}} = I_8$ and $\det\dot{\widetilde{B}} = 1.$ □

Theorem 4 *In dual* 8*-space, the dual motion is a regular motion and it is independent of* $\widetilde{h}$*.*

Proof From Theorem 3, $\det\dot{\widetilde{B}} = 1$ and hence the value of $\det\dot{\widetilde{B}}$ is independent of $\widetilde{h}$. □

5. Dual velocities, dual pole points, and dual pole curves

From (4.2) we have

$$\widetilde{X} = \widetilde{B}\widetilde{X_0} + \widetilde{C} \tag{5.1}$$

then

$$\widetilde{X_0} = \widetilde{B}^{-1}(\widetilde{X} - \widetilde{C}). \tag{5.2}$$

If we let $\widetilde{C}' = -\widetilde{B}^{-1}\widetilde{C}$, then (5.2) leads to

$$\widetilde{X_0} = \widetilde{B}^{-1}\widetilde{X} + \widetilde{C}'. \tag{5.3}$$

The equations (5.1) and (5.3) are coordinate transformations between the fixed and moving dual spaces.

Differentiating the equation (5.1) with respect to t we get

$$\dot{\widetilde{X}} = \dot{\widetilde{B}}\widetilde{X_0} + \dot{\widetilde{C}} + \widetilde{B}\dot{\widetilde{X_0}}, \tag{5.4}$$

where $\widetilde{B}\dot{\widetilde{X_0}}$ is the dual relative velocity, $\dot{\widetilde{B}}\widetilde{X_0} + \dot{\widetilde{C}}$ is the dual sliding velocity, and $\dot{\widetilde{X}}$ is the dual absolute velocity of the dual point $\widetilde{X_0}$. Hence we give the following:

Theorem 5 *In dual* 8*-space, for a dual homothetic motion with one parameter, the dual absolute velocity vector of a moving dual system of a dual point* $\widetilde{X_0}$ *at time* t *is the sum of the dual sliding velocity and dual relative velocity of* $\widetilde{X_0}$.

To obtain the pole points, the equation

$$\dot{\widetilde{B}}\widetilde{X}+\dot{\widetilde{C}}=0 \tag{5.5}$$

has to be solved.

Any solution of this equation is a dual pole point of the dual motion at that instant t in $\widetilde{R_0}$. From Theorem 3, $\det\dot{\widetilde{B}}=1$. Thus the equation in (5.5) has the solution $\widetilde{X}=-\dot{\widetilde{B}}^{-1}\dot{\widetilde{C}}$ at every t-instant, which is the only solution and therefore we have the following:

Theorem 6 *In* $\widetilde{R_0}$*, the dual pole point corresponding to each* t*-instant is the rotation by* $\dot{\widetilde{B}}^T$ *of the dual speed vector* $\dot{\widetilde{C}}$ *at that instant.*

Proof Since the matrix $\dot{\widetilde{B}}$ is dual orthogonal, then the matrix $\dot{\widetilde{B}}^T$ is dual orthogonal. Thus, it makes a rotation. □

We know that $\widetilde{C}'=-\widetilde{B}^{-1}\widetilde{C}$; then

$$\widetilde{X}_0=\widetilde{p}=-\dot{\widetilde{B}}^{-1}\dot{\widetilde{C}}=\widetilde{C}'+\dot{\widetilde{B}}^{-1}\widetilde{B}\widetilde{C}', \tag{5.6}$$

where $\widetilde{p}$ is a dual pole point of the fixed space. In the moving system, this dual pole point can be written as follows:

$$\widetilde{X}=\widetilde{q}=\widetilde{B}\widetilde{X_0}+\widetilde{C}=\widetilde{B}\widetilde{p}+\widetilde{C}. \tag{5.7}$$

The equations (5.6) and (5.7) give the equations of fixed and moving dual pole curves.

If we differentiate the equation (5.7) with respect to t, we get $\dot{\widetilde{q}}=\dot{\widetilde{B}}\widetilde{p}+\dot{\widetilde{C}}+\widetilde{B}\dot{\widetilde{p}}$. Because $\dot{\widetilde{B}}\widetilde{p}+\dot{\widetilde{C}}=0$, then $\dot{\widetilde{q}}=\widetilde{B}\dot{\widetilde{p}}$, which gives the dual sliding velocity of $\widetilde{q}$ at time t. Now we have the following:

Theorem 7 *In dual* 8*-space, in a dual homothetic motion, after the dual rotation* $\widetilde{A}$ *and dual translation* $\widetilde{h}$*, the dual tangent vectors of dual pole curves coincide.*

The equation $\dot{\widetilde{q}}=\widetilde{B}\dot{\widetilde{p}}$ leads to $\left\|\dot{\widetilde{q}}\right\|=\left\|\widetilde{B}\dot{\widetilde{p}}\right\|$; hence $\left\|\dot{\widetilde{q}}\right\|dt=\left\|\widetilde{h}\widetilde{A}\dot{\widetilde{p}}\right\|dt$, and thus $\left\|\dot{\widetilde{q}}\right\|dt=\widetilde{h}^8\left\|\dot{\widetilde{p}}\right\|dt$. Therefore, $\widetilde{s}=\int\widetilde{h}^8ds$, which yields:

Theorem 8 *In dual* 8*-space, during the dual homothetic motion, dual pole curves roll by sliding on top of each other. The rolling by sliding scale is* $\widetilde{h}^8$.

6. Dual accelerations and dual acceleration centers

Differentiating (5.4) implies that $\ddot{\widetilde{X}} = \ddot{\widetilde{B}}\widetilde{X_0} + \ddot{\widetilde{C}} + 2\dot{\widetilde{B}}\dot{\widetilde{X_0}} + \widetilde{B}\ddot{\widetilde{X_0}}$. Then the dual absolute acceleration is expressed as $\widetilde{\beta}_a = \ddot{\widetilde{X}}$, the dual sliding acceleration is given by

$$\widetilde{\beta}_s = \ddot{\widetilde{B}}\widetilde{X_0} + \ddot{\widetilde{C}},$$

the dual Coriolis acceleration by

$$\widetilde{\beta}_c = 2\dot{\widetilde{B}}\dot{\widetilde{X_0}},$$

and the dual relative acceleration by $\widetilde{\beta}_r = \widetilde{B}\ddot{\widetilde{X_0}}$. Then we have the following:

Theorem 9 *In dual* 8*-space, the relation between dual absolute, dual sliding, dual relative, and dual Coriolis accelerations of a point* $\widetilde{X_0}$ *of a moving system under the dual homothetic motion is stated as:*

$$\widetilde{\beta}_a = \widetilde{\beta}_s + \widetilde{\beta}_c + \widetilde{\beta}_r.$$

Definition 2 *The set of zeros of the dual sliding acceleration of order* m *is said to be the dual acceleration center of order* $(m-1)$.

By the above definition we have to find the solutions of the equation

$$\widetilde{B}^{(m)}\widetilde{X} + \widetilde{C}^{(m)} = 0, \tag{6.1}$$

where

$$\widetilde{B}^{(m)} = \frac{d^m\widetilde{B}}{dt^m} \text{ and } \widetilde{C}^{(m)} = \frac{d^m\widetilde{C}}{dt^m}.$$

We know that the order m dual curve $\widetilde{\alpha}(t)$ is a regular dual curve; then

$$\sum_{i=0}^{7}\left[\widehat{\alpha}_i{}^{(m)}\right]^2 \neq 0, \widehat{\alpha}_i{}^{(m)} = \frac{d^m\widehat{\alpha}_i}{dt^m}.$$

Furthermore,

$$\det \widetilde{B}^{(m)} = \left\{\sum_{i=0}^{7}\left[\widehat{\alpha}_i{}^{(m)}\right]^2\right\}^4;$$

hence $\det \widetilde{B}^{(m)} \neq 0$, and thus the dual matrix $\widetilde{B}^{(m)}$ has an inverse. Therefore, from the equation (6.1), at every t-instant, the dual acceleration center of order $(m-1)$ is $\widetilde{X} = \left[\widetilde{B}^{(m)}\right]^{-1}\left[-\widetilde{C}^{(m)}\right]$.

Special cases:

(1) $K = \Omega + \Psi e,\ \Omega, \Psi \in H_D$, if $\Psi = 0$, then K is a dual quaternion. Hence $N^+ = H^+$ and everything in [1] and [10] could easily be verified.

(2) $K = \Omega + \Psi e,\ \Omega, \Psi \in H_D$, if $\Psi = 0$ and $\Omega = \widehat{x} + i\widehat{y},\ \widehat{x}, \widehat{y} \in D$, then $K \in D_{\mathbb{C}}$ and

$$N^+ = \begin{bmatrix} \widehat{x} & -\widehat{y} \\ \widehat{y} & \widehat{x} \end{bmatrix}.$$

References

[1] Agrawal OP. Hamilton operators and dual number quaternions in spatial kinematics. Mech Mach Theory 1987; 22: 569–575.

[2] Aksoyak FK, Yayli Y. Homothetic motions and surfaces in E^4. Bull Malays Math Sci Soc 2015; 38: 259–269.

[3] Avital S, Kleiner I. Themes in the evolution of number systems. Int J Math Educ Sci Technol 1992; 23: 445–461.

[4] Clifford WK. Preliminary sketch of bi-quaternions. Proc of London Math Soc 1873; 4: 361–395.

[5] Jafari M. Homothetic motions in Euclidean 3-space. Kuwait J Sci 2014; 41: 65–73.

[6] Kabadayi H, Yayli Y. Homothetic motions at E^4 with bicomplex numbers. Adv Appl Clifford Algebras 2011; 21: 541–546.

[7] Koltelnikov AP. Screw Calculus and Some of Its Applications in Geometry and Mechanics. Kazan, 1895 (in Russian).

[8] Study E. Geometrie der Dynamen. Leipzig, 1903 (in German).

[9] Yaglom IM. A Simple Non-Euclidean Geometry and Its Physical Basis. New York, NY, USA: Springer-Verlag, 1979.

[10] Yayli Y. Homothetic motions at E^4. Mech Mach Theory 1992; 27: 303–305.

[11] Yayli Y, Bukcu B. Homothetic motions at E^8 with Cayley numbers. Mech Mach Theory 1995; 30: 417–420.

[12] Zvengrowski P. A 3-fold vector product in R^8. Comment Math Helv 1965/1966; 40: 149–152.