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A new approach to soft uniform spaces

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Abstract: The purpose of this paper is to introduce the concept of soft uniform spaces and the relationships between soft uniform spaces and uniform spaces. The notions of soft uniform structure, soft uniform continious function, and operations on soft uniform space are introduced and their basic properties are investigated.

Key words: Soft set, soft point, soft topological space, soft diagonal, soft uniform structure, soft uniform space, soft uniform continious function

1. Introduction

Many practical problems in economics, engineering, environment, social science, medical science, etc. cannot be dealt with by classical methods because classical methods have inherent difficulties. The reason for these difficulties may be the inadequacy of the theories of parameterization tools. Molodtsov [18] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Maji et al. [15,16] studied operations over a soft set. The algebraic structure of set theories dealing with uncertainties is an important problem. Many researchers heve contributed towards the algebraic structure of soft set theory. Aktaş and Çağman [2] defined soft groups and derived their basic properties. Acar et al. [1] introduced initial concepts of soft rings. Feng et al. [10] defined soft semirings and several related notions to establish a connection between soft sets and semirings. Shabir et al. [21] studied soft ideals over a semigroup. Sun et al. [24] defined soft modules and investigated their basic properties. Gunduz and Bayramov [11,12] introduced fuzzy soft modules and intuitionistic fuzzy soft modules and investigated some basic properties. Ozturk and Bayramov defined chain complexes of soft modules and their soft homology modules [19]. Ozturk et al. introduced the concept of inverse and direct systems in the category of soft modules [20].

Recently, Shabir and Naz [22] initiated the study of soft topological spaces. Theoretical studies of soft topological spaces have also been conducted by some authors in [6,14,17,23,25]. In [5], a soft point concept was given differently from the concepts that were given in other studies [3,6,14,17,23,25]. In this study, we use the soft point concept that was defined in [5].

Uniform spaces are like metric spaces; however, the application area of uniform spaces is more extensive than that of metric spaces. Since every uniform space can be transformed to a topological space, there exists a relation between uniform and topological spaces. Thus, it is important to carry the uniform spaces to soft sets.

In [7], the contraction of the product of soft sets to the diagonal was investigated when the soft uniform

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structure was given, and the definitions of soft point and soft Hausdorff space that were given in [23] were used. However, in [5], it was seen that these definitions of soft point and soft Hausdorff are not correct.

In this study, differently from [7], we give a new definition of soft uniform space. The soft uniform spaces defined in [7] are a special case of the soft uniform spaces defined by us. Furthermore, we investigate operations on soft uniform space and establish relationships between soft uniform space and uniform space.

2. Preliminaries

In this section we will introduce necessary definitions and theorems for soft sets. Molodtsov [18] defined the soft set in the following way:

Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the power set of X and $A, B \subseteq E$.

Definition 1 *[18] A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.*

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in A$, $F(e)$ may be considered as the set of e-elements of the soft set (F, A), or as the set of e-approximate elements of the soft set.

Definition 2 *[16] For two soft sets (F, A) and (G, B) over X, (F, A) is called a soft subset of (G, B) if:*

1. *$A \subset B$ and*
2. *$\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.*

This relationship is denoted by $(F, A) \tilde{\subset} (G, B)$. Similarly, (F, A) is called a soft superset of (G, B) if (G, B) is a soft subset of (F, A). This relationship is denoted by $(F, A) \tilde{\supset} (G, B)$. Two soft sets (F, A) and (G, B) over X are called soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 3 *[16] The intersection of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F, A) \tilde{\cap} (G, B) = (H, C)$.*

Definition 4 *[16] The union of two soft sets (F, A) and (G, B) over X is the soft set, where $C = A \cup B$ and $\forall e \in C$,*

$$H(\varepsilon) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e) & \text{if } e \in A \cup B. \end{cases}$$

This relationship is denoted by $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 5 *[16] A soft set (F, A) over X is said to be a NULL soft set denoted by Φ if for all $e \in A$, $F(e) = \varnothing$ (null set).*

Definition 6 *[16] A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e \in A$, $F(e) = X$.*

Definition 7 *[22] The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by $(F,E)\backslash(G,E)$, is defined as $H(e)=F(e)\backslash G(e)$ for all $e\in E$.*

Definition 8 *[22] Let Y be a nonempty subset of X, and then $\widetilde{Y}$ denotes the soft set (Y,E) over X for which $Y(e)=Y$, for all $e\in E$.*

In particular, (X,E) will be denoted by $\widetilde{X}$.

Definition 9 *[22] Let (F,E) be a soft set over X and Y be a nonempty subset of X. Then the sub-soft set of (F,E) over Y denoted by $\left({}^{Y}F,E\right)$ is defined as follows: ${}^{Y}F(e)=Y\cap F(e)$, for all $e\in E$. In other words, $\left({}^{Y}F,E\right)=\widetilde{Y}\cap(F,E)$.*

Definition 10 *[4] Let (F,A) and (G,B) be two soft sets over X_1 and X_2, respectively. The Cartesian product $(F,A)\times(G,B)$ is defined by $(F\times G)_{(A\times B)}$ where*

$$(F\times G)_{(A\times B)}(e,k)=F(e)\times G(k),\quad \forall(e,k)\in A\times B.$$

According to this definiton, the soft set $(F,A)\times(G,B)$ is the soft set over $X_1\times X_2$ and its parameter universe is $E_1\times E_2$.

Definition 11 *[22] Let τ be the collection of soft sets over X, and then τ is said to be a soft topology on X if:*

1) $\Phi,\widetilde{X}$ belongs to τ;

2) the union of any number of soft sets in τ belongs to τ;

3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

Definition 12 *[22] Let (X,τ,E) be a soft topological space over X, and then members of τ are said to be soft open sets in X.*

Definition 13 *[22] Let (X,τ,E) be a soft topological space over X. A soft set (F,E) over X is said to be a soft closed in X if its relative complement $(F,E)'$ belongs to τ.*

Proposition 1 *[22] Let (X,τ,E) be a soft topological space over X. Then the collection $\tau_e=\{F(e):(F,E)\in\tau\}$ for each $e\in E$ defines a topology on X.*

Definition 14 *[22] Let (X,τ,E) be a soft topological space over X and (F,E) be a soft set over X. Then the soft closure of (F,E), denoted by $(F,E)^c$, is the intersection of all soft closed super sets of (F,E). Clearly $(F,E)^c$ is the smallest soft closed set over X that contains (F,E).*

Definition 15 *[5] Let (F,E) be a soft set over X. The soft set (F,E) is called a soft point, denoted by (x_e,E), if for the element $e\in E$, $F(e)=\{x\}$ and $F(e')=\varnothing$ for all $e'\in E-\{e\}$ (briefly denoted by x_e).*

Definition 16 *[5] For two soft points (x_e, E) and $(y_{e'}, E)$ over a common universe X, we say that the points are different points if $x \neq y$ or $e \neq e'$.*

Definition 17 *[5] Let (X, τ, E) be a soft topological space over X. A soft set (F, E) in (X, τ, E) is called a soft neighborhood of the soft point $(x_e, E) \in (F, E)$ if there exists a soft open set (G, E) such that $(x_e, E) \in (G, E) \subset (F, E)$.*

Definition 18 *[13] Let (X, τ, E) and (Y, τ', E) be two soft topological spaces and $f : (X, \tau, E) \to (Y, \tau', E)$ be a mapping. For each soft neighborhood (H, E) of $(f(x)_e, E)$, if there exists a soft neighborhood (F, E) of (x_e, E) such that $f((F, E)) \subset (H, E)$, then f is said to be soft continuous mapping at (x_e, E).*

If f is a soft continuous mapping for all (x_e, E), then f is called a soft continuous mapping.

Definition 19 *[13] Let (X, τ, E) and $(Y, \tau^{'}, E)$ be two soft topological spaces and $f : X \to Y$ be a mapping. If f is a bijection, soft continuous and f^{-1} is a soft continuous mapping, then f is said to be a soft homeomorphism from X to Y. When a homeomorphism f exists between X and Y, we say that X is soft homeomorphic to Y.*

Definition 20 *[3] Let (X, τ, E) be a soft topological space over X. A subcollection β of τ is said to be a base for τ if every member of τ can be expressed as a union of members of β.*

Definition 21 *[3] Let (X, τ, E) be a soft topological space over X. A subcollection δ of τ is said to be a subbase for τ if the family of all finite intersections members of δ forms a base for τ.*

Definition 22 *[3] Let $\{(\varphi_s, \psi_s) : (X, \tau, E) \to (Y_s, \tau_s, E_s)\}_{s \in S}$ be a family of soft mappings and $\{(Y_s, \tau_s, E_s)\}_{s \in S}$ is a family of soft topological spaces. Then the topology τ generated from the subbase $\delta = \{(\varphi_s, \psi_s)^{-1}_{s \in S}(F, E) : (F, E) \in \tau_s, s \in S\}$ is called the soft topology (or initial soft topology) induced by the family of soft mappings $\{(\varphi_s, \psi_s)\}_{s \in S}$.*

Definition 23 *[3] Let $\{(X_s, \tau_s, E_s)\}_{s \in S}$ be a family of soft topological spaces. Then the initial soft topology on $X = \prod_{s \in S} X_s$ generated by the family $\{(p_s, q_s)\}_{s \in S}$ is called product soft topology on X. (Here, (p_s, q_s) is the soft projection mapping from X to X_s, $s \in S$.)*

The product soft topology is denoted by $\prod_{s \in S} \tau_s$.

Definition 24 *[6] Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. If there exist soft open sets (F, A) and (G, B) such that $x \in (F, A)$, $y \in (G, B)$ and $(F, A) \cap (G, B) = \Phi$, then (X, τ, E) is called a soft Hausdorff space.*

Definition 25 *[3] Let (X, τ, E) be a soft topological space. If there exists a soft finite subcovering of every soft open covering of the soft topological space (X, τ, E) then this soft space is called soft compact space.*

Definition 26 *[5] Let (X, τ, E) be a soft topological space over X. If for every soft point (x_e, E) in X, there exists a soft neighborhood (G, E) such that $\overline{(G, E)}$ is a soft compact subspace of X, then (X, τ, E) is said to be soft locally compact space.*

Definition 27 *[8] Let $\mathbb{R}$ be the set of real numbers and $B(\mathbb{R})$ the collection of all nonempty bounded subsets of $\mathbb{R}$ and E taken as a set of parameters. Then a mapping $F : E \to B(\mathbb{R})$ is called a soft real set. If a soft real set is a singleton soft set, it will be called a soft real number and denoted by $\widetilde{r}$, $\widetilde{s}$, $\widetilde{t}$, etc.*

$\overline{0}$, $\overline{1}$ are the soft real numbers where $\overline{0}(e) = 0$, $\overline{1}(e) = 1$ for all $e \in E$, respectively.

Definition 28 *[9] Let $\mathbb{R}(E)^*$ denote the set of all nonnegative soft real numbers and $SS(X,E)$ denote the set of all soft points on the set X. A mapping $\widetilde{d} : SS(X,E) \times SS(X,E) \to \mathbb{R}(E)^*$ is said to be a soft metric on the soft set (X,E) if $\widetilde{d}$ satisfies the following conditions:*

(M1) $\widetilde{d}(\widetilde{x}_{e_1}, \widetilde{y}_{e_2}) \widetilde{\geq} \widetilde{0}$ *for all* $\widetilde{x}_{e_1}, \widetilde{y}_{e_2} \in SS(X,E)$,

(M2) $\widetilde{d}(\widetilde{x}_{e_1}, \widetilde{y}_{e_2}) = \widetilde{0}$ *if and only if* $\widetilde{x}_{e_1} = \widetilde{y}_{e_2} \in SS(X,E)$,

(M3) $\widetilde{d}(\widetilde{x}_{e_1}, \widetilde{y}_{e_2}) = \widetilde{d}(\widetilde{y}_{e_2}, \widetilde{x}_{e_1})$ *for all* $\widetilde{x}_{e_1}, \widetilde{y}_{e_2} \in SS(X,E)$,

(M4) *For all* $\widetilde{x}_{e_1}, \widetilde{y}_{e_2}, \widetilde{z}_{e_3} \in SS(X,E)$, $\widetilde{d}(\widetilde{x}_{e_1}, \widetilde{z}_{e_3}) \widetilde{\leq} \widetilde{d}(\widetilde{x}_{e_1}, \widetilde{y}_{e_2}) + \widetilde{d}(\widetilde{y}_{e_2}, \widetilde{z}_{e_3})$.

The soft set (X,E) with a soft metric $\widetilde{d}$ is a called a soft metric space and denoted by $(X, \widetilde{d}, E)$.

3. Soft uniform spaces

Let X be an initial universe set and E be a set of parameters. The set of all soft points on the set X is denoted by $SS(X,E)$. It is obvious that each soft set can be expressed as a union of soft points. For this reason, to give the family of all soft sets on X it is sufficient to give only soft points on X.

Definition 29 ***a)*** *If (X,E) is a soft set, then the soft set*

$$\Delta_{(X,E)} = \{(x_e, x_e) : x_e \in (X,E)\}$$

is called the diagonal of $(X,E) \times (X,E)$. Here $\Delta_{(X,E)} = (\ \Delta_X,\ \Delta_E)$ is defined by $((x,x),(e,e)) = (x,x)_{(e,e)} = (x_e, x_e)$.

b) *If $(A, E \times E) \widetilde{\subset} (X,E) \times (X,E)$ is a soft subset and the soft subset $(A, E \times E)$ is denoted by $\widetilde{A}$, then*

$$\widetilde{A}^{-1} = \left\{ (y_{e'}, x_e) : (x_e, y_{e'}) \in \widetilde{A} \right\}.$$

If $\widetilde{A} = \widetilde{A}^{-1}$, then the soft set $\widetilde{A}$ is called soft symmetric.

c) *If $\widetilde{A}$, $\widetilde{B} \widetilde{\subset} (X,E) \times (X,E)$ are soft subsets, then*

$$\widetilde{A} \circ \widetilde{B} = \left\{ (x_e, y_{e'}) : \exists z_{e''} \in (X,E),\ (x_e, z_{e''}) \in \widetilde{A},\ (z_{e''}, y_{e'}) \in \widetilde{B} \right\}.$$

d) $\widetilde{A}^1 = \widetilde{A}$ *and* $\widetilde{A}^n = \widetilde{A}^{n-1} \circ \widetilde{A}$ *for* $n = 1, 2, 3, \ldots$.

Example 1 *Let* $X = \{x^1, x^2, x^3\}$ *be any set and* $E = \{e_1, e_2\}$ *be a set of parameters. In this case, the soft set* $SS(X, E)$ *is formed by the following soft points:*

$$\{x^1_{e_1}, x^1_{e_2}, x^2_{e_1}, x^2_{e_2}, x^3_{e_1}, x^3_{e_2}\}.$$

Thus, the soft diagonal $\Delta_{(X,E)}$ *of this soft set is*

$$\Delta_{(X,E)} = \{(x^1_{e_1}, x^1_{e_1}), (x^1_{e_2}, x^1_{e_2}), (x^2_{e_1}, x^2_{e_1}), (x^2_{e_2}, x^2_{e_2}), (x^3_{e_1}, x^3_{e_1}), (x^3_{e_2}, x^3_{e_2})\}.$$

Proposition 2 *If we fixed the parameter* $e_0 \in E$, *then we can obtain a soft subset* $\{x_{e_0} : x \in X\}$ *of the soft set* (X, E). *It is obvious that there exists a bijection mapping between the set* X *and the soft subset* $\{x_{e_0} : x \in X\}$. *Thus, we can consider the soft subset* $\{x_{e_0} : x \in X\}$ *as the set* X.

Proposition 3 *For every* $e \in E$, *we have*

$$\left(\Delta_{(X,E)}\right)_e = \{(x^i_e, x^i_e) : x^i \in X\} = \Delta_X.$$

Therefore, the diagonal of the soft set (X, E) *is the family of diagonals that is parameterized on the set* E *of* X. *That is,* $\Delta_{(X,E)} = \bigcup_{e \in E} (\Delta_X)_e$.

Let (X, E) be a soft set and $(V, E \times E)\widetilde{\subset}(X, E) \times (X, E)$ be a soft set that is defined as $V : E \times E \to P(X \times X)$, and the soft set $(V, E \times E)$ is denoted by $\widetilde{V}$.

Definition 30 *If* $\Delta_{(X,E)}\widetilde{\subset}\ \widetilde{V}$ *and* $\widetilde{V} = \widetilde{V}^{-1}$ *are satisfied, then the soft set* $\widetilde{V}$ *is called the soft neighborhood of the soft diagonal.*

Let us denote the all soft neighborhoods of the soft diagonal $\Delta_{(X,E)}$ *by* $D_{(X,E)}$.

Example 2 *According to Example 1, we can give a few examples of the soft neighborhood* $\widetilde{V}$.

$$\begin{aligned}
\widetilde{V}_1 &= \{\Delta_{(X,E)}, (x^1_{e_1}, x^2_{e_2}), (x^2_{e_2}, x^1_{e_1})\} \\
\widetilde{V}_2 &= \{\Delta_{(X,E)}, (x^1_{e_1}, x^2_{e_1}), (x^2_{e_1}, x^1_{e_1})\} \\
\widetilde{V}_3 &= \{\Delta_{(X,E)}, (x^1_{e_1}, x^2_{e_1}), (x^2_{e_1}, x^1_{e_1}), (x^2_{e_2}, x^3_{e_2}), (x^3_{e_2}, x^2_{e_2})\} \\
&\dots
\end{aligned}$$

Proposition 4 *For every* $(e, e) \in E \times E$, *the set* $\left(\widetilde{V}_i\right)_{(e,e)}$ *is a neighborhood of the diagonal of the order set* X.

Example 3 *Here, by the soft contraction of the soft neighborhoods* $\widetilde{V}_i$ *to the parameter* (e_i, e_i), *according to* *the above example we obtain the following neighborhoods of the ordinary set X for every i.*

$$\begin{aligned}
\left(\widetilde{V}_1\right)_{(e_1,e_1)} &= \left(\widetilde{V}_1\right)_{(e_2,e_2)} = \{\Delta_X\} \\
\left(\widetilde{V}_2\right)_{(e_1,e_1)} &= \left\{\Delta_X,(x^1,x^2),(x^2,x^1)\right\}, \left(\widetilde{V}_2\right)_{(e_2,e_2)} = \{\Delta_X\} \\
\left(\widetilde{V}_3\right)_{(e_1,e_1)} &= \left\{\Delta_X,(x^1,x^2),(x^2,x^1)\right\}, \left(\widetilde{V}_3\right)_{(e_2,e_2)} = \left\{\Delta_X,(x^2,x^3),(x^3,x^2)\right\} \\
&\dots
\end{aligned}$$

Definition 31 *Let (X,E) be a soft set, $(F,E)\widetilde{\subset}(X,E)$ be a soft subset, $\widetilde{V}\in D_{(X,E)}$ be a soft neighborhood, and x_e, $y_{e'}\in(X,E)$ be soft points.*

a) *If $(x_e,y_{e'})\in\widetilde{V}$, then it is said that the distance between the soft point x_e and the soft point $y_{e'}$ is smaller than $\widetilde{V}$ and denoted by $|x_e-y_{e'}|\,\widetilde{<}\widetilde{V}$. Otherwise, $|x_e-y_{e'}|\,\widetilde{>}\widetilde{V}$.*

b) *If $(F,E)\times(F,E)\widetilde{\subset}\ \widetilde{V}$, then it is said that the diameter of the soft set (F,E) is smaller than $\widetilde{V}$ and denoted by $\delta((F,E))\,\widetilde{<}\widetilde{V}$.*

For all soft points $x_e,y_{e'},z_{e''}\in(X,E)$ and $\widetilde{V},\widetilde{V}_1,\widetilde{V}_2\in D_{(X,E)}$ the following properties are satisfied:

1. $|x_e-x_e|\,\widetilde{<}\widetilde{V}$.
2. $|x_e-y_{e'}|\,\widetilde{<}\widetilde{V}\Longleftrightarrow|y_{e'}-x_e|\,\widetilde{<}\widetilde{V}$.
3. If $|x_e-y_{e'}|\,\widetilde{<}\widetilde{V}_1$ and $|y_{e'}-z_{e''}|\,\widetilde{<}\widetilde{V}_2$, then $|x_e-z_{e''}|\,\widetilde{<}\widetilde{V}_1\circ\widetilde{V}_2$.

Definition 32 *Let (X,E) be a soft set, $x_e^0\in(X,E)$ be a soft point, and $\widetilde{V}\in D_{(X,E)}$. The set*

$$B\left(x_e^0,\widetilde{V}\right)=\left\{y_{e'}\in(X,E):|x_e^0-y_{e'}|\,\widetilde{<}\widetilde{V}\right\}$$

is called the soft sphere whose center is the soft point x_e^0 with diameter $\widetilde{V}$. $B\left((F,E),\widetilde{V}\right)=\underset{x_e\in(F,E)}{\cup}B\left(x_e,\widetilde{V}\right)$ is defined for every soft set $(F,E)\widetilde{\subset}(X,E)$.

Definition 33 *Let (X,E) be a soft set and $\widetilde{\vartheta}\widetilde{\subset}D_{(X,E)}$ be a soft subfamily. If the conditions*

a) *If $\widetilde{V}\in\widetilde{\vartheta}$ and $\widetilde{V}\widetilde{\subset}\widetilde{W}\in D_{(X,E)}$, then $\widetilde{W}\in\widetilde{\vartheta}$*

b) *If $\widetilde{V}_1,\widetilde{V}_2\in\widetilde{\vartheta}$ then $\widetilde{V}_1\widetilde{\cap}\widetilde{V}_2\in\widetilde{\vartheta}$*

c) *For every $\widetilde{V}\in\widetilde{\vartheta}$, there exists $\widetilde{W}\in\widetilde{\vartheta}$ such that $\widetilde{W}^2\widetilde{\subset}\widetilde{V}$*

d) $\underset{\widetilde{V}\in\widetilde{\vartheta}}{\widetilde{\cap}}\widetilde{V}=\Delta_{(X,E)}$

are satisfied for the soft family $\widetilde{\vartheta}$, then the soft family $\widetilde{\vartheta}$ is called a soft uniform structure and the triple $(X,\widetilde{\vartheta},E)$ is called a soft uniform space.

Proposition 5 *For every $(e,e) \in E \times E$, the family $\widetilde{\vartheta}_{(e,e)}$ is the ordinary uniform structure of the set X. Thus, the soft uniform spaces are the parameterized family of ordinary uniform spaces.*

Definition 34 *Let $(X,\widetilde{\vartheta},E)$ be a soft uniform space and $\widetilde{\Gamma}\widetilde{\subset}\widetilde{\vartheta}$ be a soft subfamily. If for every $\widetilde{V} \in \widetilde{\vartheta}$ there exists $\widetilde{W}\widetilde{\subset}\widetilde{\Gamma}$ such that $\widetilde{W}\widetilde{\subset}\widetilde{V}$ is satisfied, then the family of $\widetilde{\Gamma}$ is called a soft base of the soft uniform structure $\widetilde{\vartheta}$.*

Every soft base $\widetilde{\Gamma}$ of the soft uniform structure $\widetilde{\vartheta}$ in (X,E) has the following properties:

a) For every $\widetilde{V}_1,\widetilde{V}_2 \in \widetilde{\Gamma}$, there exists $\widetilde{V} \in \widetilde{\Gamma}$ such that $\widetilde{V}\widetilde{\subset}\widetilde{V}_1 \cap \widetilde{V}_2$;

b) For every $\widetilde{V} \in \widetilde{\Gamma}$, there exists $\widetilde{W} \in \widetilde{\Gamma}$ such that $\widetilde{W}^2\widetilde{\subset}\widetilde{V}$;

c) $\widetilde{\cap}\widetilde{\Gamma} = \Delta_{(X,E)}$.

Theorem 1 *Let (X,E) be a soft set and $\widetilde{\Gamma}$ be the family of soft subsets of $(X,E) \times (X,E)$. If the conditions*

a) $\widetilde{B}\widetilde{\subset}\ \widetilde{\Gamma} \Rightarrow \Delta_{(X,E)}\widetilde{\subset}\ \widetilde{B}$,

b) $\widetilde{B}_1,\widetilde{B}_2 \in\ \widetilde{\Gamma} \Rightarrow \exists\widetilde{B}_3 \in\ \widetilde{\Gamma} : \widetilde{B}_3\widetilde{\subset}\widetilde{B}_1 \cap \widetilde{B}_2$,

c) $\widetilde{B}\widetilde{\subset}\ \widetilde{\Gamma} \Rightarrow \exists\widetilde{C} \in \widetilde{\Gamma} : \widetilde{C} \circ \widetilde{C}\widetilde{\subset}\widetilde{B}$,

d) $\widetilde{B}\widetilde{\subset}\ \widetilde{\Gamma} \Rightarrow \exists\widetilde{C} \in \widetilde{\Gamma} : \widetilde{C}^{-1}\widetilde{\subset}\widetilde{B}$,

e) $\widetilde{\cap}\widetilde{\Gamma} = \Delta_{(X,E)}$,

are satisfied for the soft family $\widetilde{\Gamma}$, then the soft family $\widetilde{\Gamma}$ is a soft base of a soft uniform structure on the soft set (X,E).

Proof The soft family $\widetilde{\vartheta} = \left\{\widetilde{V}\widetilde{\subset}(X,E) \times (X,E) : \exists\widetilde{B} \in \widetilde{\Gamma} \text{ such that } \widetilde{B}\widetilde{\subset}\widetilde{V}\right\}$ is a soft uniform structure on (X,E) and the soft family $\widetilde{\Gamma}$ is a soft base of this soft uniform structure. □

Theorem 2 *If $(X,\widetilde{\vartheta},E)$ is a soft uniform space, then the soft family*

$$\widetilde{\tau} = \left\{(G,E)\widetilde{\subset}(X,E) : \textit{for every } x_e \in (G,E), \exists\widetilde{V} \in \widetilde{\vartheta} \textit{ such that } B\left(x_e,\widetilde{V}\right)\widetilde{\subset}(G,E)\right\}$$

is a soft topology on (X,E). The soft set (X,E) is a T_1-space together with this topology. The soft topology $\widetilde{\tau}$ is called the generated soft uniform topology from the soft uniform structure $\widetilde{\vartheta}$.

Proof Let us show that the family $\widetilde{\tau}$ is a soft topology. Let $(G_\alpha,E) \in \widetilde{\tau}$ and $x_e \in \widetilde{\bigcup_\alpha}(G_\alpha,E)$ be any soft point. In this case, $x_e \in (G_{\alpha_0},E)$. From the definition of $\widetilde{\tau}$, there exists $\widetilde{V} \in \widetilde{\vartheta}$ such that

$$B\left(x_e,\widetilde{V}\right)\widetilde{\subset}(G_{\alpha_0},E)\widetilde{\subset}\widetilde{\bigcup_\alpha}(G_\alpha,E),$$

i.e. $\underset{\alpha}{\widetilde{\cup}}(G_\alpha, E) \in \widetilde{\tau}$.

Let $(G_1, E), (G_2, E) \in \widetilde{\tau}$ and $x_e \in (G_1, E) \cap (G_2, E)$. From the definition of $\widetilde{\tau}$, there exists $\widetilde{V}_1, \widetilde{V}_2 \in \widetilde{\vartheta}$ such that $B\left(x_e, \widetilde{V}_1\right) \widetilde{\subset} (G_1, E)$ and $B\left(x_e, \widetilde{V}_2\right) \widetilde{\subset} (G_2, E)$. Consequently, since $\widetilde{V} = \widetilde{V}_1 \widetilde{\cap} \widetilde{V}_2 \in \widetilde{\vartheta}$ and $B\left(x_e, \widetilde{V}\right) = B\left(x_e, \widetilde{V}_1\right) \cap B\left(x_e, \widetilde{V}_2\right) \widetilde{\subset} (G_1, E) \cap (G_2, E)$, we obtain $(G_1, E) \cap (G_2, E) \in \widetilde{\tau}$.

Now let us show that the space (X, τ, E) is a soft T_1-space. For this, it is sufficient to show that the soft set $(G, E) = (X, E) \backslash \{x_e\}$ is soft open for evey soft point $x_e \in (X, E)$. Since $x_e \neq y_{e'}$ for every soft point $y_{e'} \in (G, E)$, we have

$$\exists \widetilde{V} \in \widetilde{\vartheta} : |x_e - y_{e'}| \widetilde{>} \widetilde{V}.$$

As a result, since $B\left(y_{e'}, \widetilde{V}\right) \widetilde{\subset} (G, E)$, we have $(G, E) \in \widetilde{\tau}$. □

Example 4 *Let* $X = \mathbb{R}$ *and* $E = \{1, 2\}$ *be a set of parameters. In this case, for every* $x_e, y_{e'} \in (X, E)$, *the soft function*

$$\widetilde{d}(x_e, y_{e'}) = \left|e - e^{'}\right| + |x - y|$$

is a soft metric on the soft set (X, E). *Indeed,*

d$_1$) $\widetilde{d}(x_e, y_{e'}) = \left|e - e^{'}\right| + |x - y| \geq 0$;

d$_2$) $\widetilde{d}(x_e, y_{e'}) = 0 \Longleftrightarrow \left|e - e^{'}\right| + |x - y| = 0 \Longleftrightarrow \left|e - e^{'}\right| = 0$ *and* $|x - y| = 0 \Longleftrightarrow e = e^{'}$ *and* $x = y$;

d$_3$) $\widetilde{d}(x_e, y_{e'}) = \left|e - e^{'}\right| + |x - y| = \left|e^{'} - e\right| + |y - x| = \widetilde{d}(y_{e'}, x_e)$;

d$_4$) $\widetilde{d}(x_e, y_{e'}) \leq \widetilde{d}\left(x_e, z_{e''}\right) + \widetilde{d}\left(z_{e''}, y_{e'}\right)$. *Here, there exist two cases according to the parameters;*

- **a)** *For* $e = e^{'} \neq e^{''}$, *the triangle inequalitiy*

  $$|x - y| \leq 1 + |x - z| + |z - y| + 1$$

  is satisfied.

- **b)** *For* $e \neq e^{'} = e^{''}$, *the triangle inequalitiy*

  $$1 + |x - y| \leq 1 + |x - z| + |z - y|$$

  is satisfied.

Therefore, the soft function $\widetilde{d}(x_e, y_{e'})$ *is a soft metric. In this case, let*

$$\widetilde{V}_\alpha = \left\{(x_e, y_{e'}) \in (\mathbb{R}, E) \times (\mathbb{R}, E) : \widetilde{d}(x_e, y_{e'}) \widetilde{<} \alpha\right\}$$

for every $\alpha > 0$. *Thus, the soft family* $\widetilde{\Gamma} = \left\{\widetilde{V}_\alpha\right\}_{\alpha > 0}$ *is a soft base of a soft uniform structure.*

Here, each of the families $\left\{\left(\widetilde{V}_\alpha\right)_{e_1}\right\}_{\alpha>0}$ *and* $\left\{\left(\widetilde{V}_\alpha\right)_{e_2}\right\}_{\alpha>0}$ *is a base of ordinary uniform structure of the set* X.

Proposition 6 *Let* $(X,\widetilde{\vartheta},E)$ *be a soft uniform space and* $\widetilde{\tau}$ *be a soft topology that is generated from this uniform space.* $\widetilde{\tau}_e$ *is the topology that is generated from the soft structure* $\widetilde{\vartheta}_e$ *for every* $e\in E$.

Theorem 3 *If* $(X,\widetilde{\vartheta},E)$ *is a soft uniform space,* $(F,E)\times(F,E)$ *is a soft subset, and* $\widetilde{\tau}$ *is a soft uniform topology generated from* $\widetilde{\vartheta}$ *on the soft set* (X,E), *then the soft set*

$$\widetilde{B}=\left\{x_e\in(X,E):\exists\widetilde{V}\in\widetilde{\vartheta}\text{ such that }B\left(x_e,\widetilde{V}\right)\widetilde{\subset}(F,E)\right\}$$

is equal to $Int(F,E)$.

Proof It is obvious that every soft open set $(G,E)\widetilde{\subset}(F,E)$ belongs to $\widetilde{B}$. Let us show that $\widetilde{B}$ is a soft open to complete the proof. There exists $\exists\widetilde{V}\in\widetilde{\vartheta}$ such that $B\left(x_e,\widetilde{V}\right)\widetilde{\subset}(F,E)$ for every $x_e\in\widetilde{B}$. We take $\widetilde{W}\in\widetilde{\vartheta}$ under the condition $\widetilde{W}^2\in\widetilde{V}$. In this case, we have

$$B\left(y_{e'},\widetilde{W}\right)\widetilde{\subset}B\left(x_e,\widetilde{V}\right)\widetilde{\subset}(F,E)$$

for every $y_{e'}\in B\left(x_e,\widetilde{W}\right)$. Thus, $B\left(x_e,\widetilde{V}\right)\widetilde{\subset}\widetilde{B}$ and $\widetilde{B}$ is a soft open. □

Conclusion 1 *Let the topology of the soft topological space* $(X,\widetilde{\tau},E)$ *be generated from the soft uniform structure* $\widetilde{\vartheta}$. *Then, for every* $x_e\in(X,E)$ *and every* $(F,E)\widetilde{\subset}(X,E)$,

$$x_e\in(F,E)^c\Longleftrightarrow(F,E)\widetilde{\cap}B\left(x_e,\widetilde{V}\right)\neq\Phi\text{ for every }\widetilde{V}\in\widetilde{\vartheta}.$$

Proof The proof is obvious. □

Conclusion 2 *If the topology of the soft topological space* $(X,\widetilde{\tau},E)$ *is generated from the soft uniform structure* $\widetilde{\vartheta}$, *then* $\delta\left((F,E)^c\right)\widetilde{<}\widetilde{V}^3$ *for every* $\widetilde{V}\in\widetilde{\vartheta}$ *and every* $(F,E)\widetilde{\subset}(X,E)$ *where the condition* $\delta(F,E)\widetilde{<}\widetilde{V}$ *is satisfied.*

Proof From Conclusion 1, for every $x_{e_1},\ y_{e_2}\in(F,E)^c$, we can find $x'_{e'_1},\ y'_{e'_2}\in(F,E)$ such that $x'_{e'_1}\in B\left(x_{e_1},\widetilde{V}\right)$ and $y'_{e'_2}\in B\left(y_{e_2},\widetilde{V}\right)$. From here, $|x_{e_1}-y_{e_2}|\widetilde{<}\widetilde{V}\circ\widetilde{V}\circ\widetilde{V}=\widetilde{V}^3$. □

Definition 35 *Let* $(X,\widetilde{\vartheta},E)$, $(Y,\widetilde{\vartheta}',E')$ *be two soft uniform spaces and* $(f,g):(X,E)\to(Y,E')$ *be a pair of function.*

a) *If for every* $\widetilde{V}'\in\widetilde{\vartheta}'$

$$\exists\widetilde{V}\in\widetilde{\vartheta}:\text{for every }\left|x^1_{e_1}-x^2_{e_2}\right|\widetilde{<}\widetilde{V},\ \left|f\left(x^1_{g(e_1)}\right)-f\left(x^2_{g(e_2)}\right)\right|\widetilde{<}\widetilde{V}'$$

is satisfied, then the soft function (f,g) *is called soft uniform continuous.*

b) *If (f,g) is a soft bijection mapping and each of the soft functions (f,g) and $(f,g)^{-1}$ are soft uniform continuous, then the soft function (f,g) is called a soft uniform isomorphism and these two soft uniform spaces are called soft isomorphic spaces.*

Theorem 4 *Every soft uniform continuous function is soft continuous.*

Proof Let $(X,\widetilde{\vartheta},E)$, $(Y,\widetilde{\vartheta}^{'},E^{'})$ be two soft uniform space, $(f,g):(X,\widetilde{\vartheta},E)\to(Y,\widetilde{\vartheta}^{'},E^{'})$ be a soft uniform continuous function, and $x^1_{e_1}\in(X,E)$ be any soft point. If $B\left(f\left(x^1_{g(e_1)}\right),\widetilde{V}^{'}\right)$ is any soft neighborhood of the soft uniform topology of $f\left(x^1_{g(e_1)}\right)$ since (f,g) is soft uniform continuous, then

$$\exists\widetilde{V}\in\widetilde{\vartheta}:\text{for every }\left|x^1_{e_1}-x^2_{e_2}\right|\widetilde{<}\widetilde{V},\ \left|f\left(x^1_{g(e_1)}\right)-f\left(x^2_{g(e_2)}\right)\right|\widetilde{<}\widetilde{V}^{'}$$

is satisfied for $\widetilde{V}^{'}\in\widetilde{\vartheta}^{'}$. From here, $(f,g)\left(B\left(x^1_{e_1},\widetilde{V}\right)\right)\widetilde{\subset}B\left(f\left(x^1_{g(e_1)}\right),\widetilde{V}^{'}\right)$ is obtained easily. That is, the soft function (f,g) is soft continuous at the soft point $x^1_{e_1}$. □

Proposition 7 *Let $(f,g):(X,\widetilde{\vartheta},E)\to(Y,\widetilde{\vartheta}^{'},E^{'})$ be a soft uniform continuous function of soft uniform spaces. The function*

$$f:\left(X,\left(\widetilde{\vartheta}\right)_e\right)\to\left(Y,\left(\widetilde{\vartheta}^{'}\right)_{g(e)}\right)$$

is uniform continuous for every $e\in E$.

Conclusion 3 *Soft uniform spaces form a category and this category is a generalization of a category of the ordinary uniform spaces.*

4. Operations on soft uniform spaces

Let $(X,\widetilde{\vartheta},E)$ be a soft uniform space and $(M,E)\widetilde{\subset}(X,E)$ be a soft subset. The conditions of soft uniform structure are satisfied for the soft family

$$\widetilde{\vartheta}_{(M,E)}=\left\{(M\times M,E\times E)\widetilde{\cap}(V,E\times E):(V,E\times E)\in\widetilde{\vartheta}\right\}.$$

The soft family $\widetilde{\vartheta}_{(M,E)}$ will be denoted by $\widetilde{\vartheta}_M$.

Definition 36 *The soft uniform space $(M,\widetilde{\vartheta}_M,E)$ is called a soft subspace of the soft uniform space $(X,\widetilde{\vartheta},E)$.*

If $\widetilde{\tau}$ is a soft uniform topology generated by the soft uniform structure $\widetilde{\vartheta}$ and $\widetilde{\tau}_M$ is a soft topology generated by the soft uniform substructure $\widetilde{\vartheta}_M$, then $\widetilde{\tau}_M$ is a topology of soft subspace of $\widetilde{\tau}$. The soft embedding mapping, which is defined by

$$(i_M,1_E):(M,\widetilde{\vartheta}_M,E)\to(X,\widetilde{\vartheta},E),\ \ (i_M,1_E)(x_e)=x_e,$$

is soft uniform continuous in soft uniform subspaces.

Let (X,E) be a soft set, $\left\{(X_s,\widetilde{\vartheta}_s,E_s)\right\}_{s\in S}$ be a family of soft uniform spaces, and $(f_s,g_s):(X,E)\rightarrow (X_s,E_s)$ be a soft function for every $s\in S$. The soft family, which is formed by the intersections of all the finite subfamilies of the following family,

$$\left\{(f_s\times f_s,g_s\times g_s)^{-1}(\widetilde{V}_s)\widetilde{\subset}(X\times X,E\times E):\widetilde{V}_s\in\widetilde{\vartheta}_s,\ s\in S\right\},$$

$$(f_s\times f_s,g_s\times g_s)^{-1}(x_e,y_{e'})=\left\{\begin{array}{c}(a_\alpha,b_\beta):(a,b)\in(f_s\times f_s)^{-1}(x,y),\\ (\alpha,\beta)\in(g_s\times g_s)^{-1}(e,e^{'})\end{array}\right\}$$

is a soft soft base of a soft uniform structure on the soft set (X,E). Every soft function $(f_s,g_s):(X,E)\rightarrow (X_s,E_s)$ is soft uniform continuous in this soft uniform structure. This soft uniform structure is called a soft uniform structure generated by the family of soft functions $\{(f_s,g_s):(X,E)\rightarrow(X_s,E_s)\}_{s\in S}$.

Let $\left\{(X_s,\widetilde{\vartheta}_s,E_s)\right\}_{s\in S}$ be a family of soft uniform spaces. We choose any finite subset $\{s_1,...,s_k\}$ of the set S and define the set

$$\left\{\left(\left\{x^{s_i}_{e_{s_i}}\right\},\left\{y^{s}_{e_s}\right\}\right):\left|x^{s_i}_{e_{s_i}}-y^{s}_{e_s}\right|\widetilde{<}\widetilde{V}_{s_i},\ i=\overline{1,k}\right\}$$

containing the soft diagonal of the set $\left(\prod\limits_{i=1}^{k}X_{s_i},\prod\limits_{i=1}^{k}E_{s_i}\right)\times\left(\prod\limits_{s\in S}X_s,\prod\limits_{s\in S}E_s\right)$. The family of the sets defined above is satisfied by the conditions of the base of soft uniform structure. The soft uniform structure generated by this base is called the product of the soft uniform structures $\left\{\widetilde{\vartheta}_s\right\}_{s\in S}$ and denoted by $\prod\limits_{s\in S}\widetilde{\vartheta}_s$. In addition, the soft uniform space $\left(\prod\limits_{s\in S}X_s,\prod\limits_{s\in S}\widetilde{\vartheta}_s,\prod\limits_{s\in S}E_s\right)$ is called the product of the family of the soft uniform spaces $\left\{(X_s,\widetilde{\vartheta}_s,E_s)\right\}_{s\in S}$.

Proposition 8 *If the soft topology $\widetilde{\tau}_s$ is generated from the soft uniform structure $\widetilde{\vartheta}_s$ for every $s\in S$, then the topology generated from soft uniform structure $\prod\limits_{s\in S}\widetilde{\vartheta}_s$ in the soft set $\left(\prod\limits_{s\in S}X_s,\prod\limits_{s\in S}E_s\right)$ is equal to the product of the soft topologies $\{\widetilde{\tau}_s\}_{s\in S}$.*

It is obvious that the soft projection mapping $(p_s,q_s):\left(\prod\limits_{s\in S}X_s,\prod\limits_{s\in S}\widetilde{\vartheta}_s,\prod\limits_{s\in S}E_s\right)\rightarrow(X_s,\widetilde{\vartheta}_s,E_s)$ is a soft uniform continuous mapping for every $s\in S$.

Theorem 5 *Let $(X,\widetilde{\vartheta},E)$ be a soft uniform space, $\left\{(Y_s,\widetilde{\vartheta}_s,E_s)\right\}_{s\in S}$ be a family of soft uniform spaces, and $(f,g):(X,\widetilde{\vartheta},E)\rightarrow\left(\prod\limits_{s\in S}Y_s,\prod\limits_{s\in S}\widetilde{\vartheta}_s,\prod\limits_{s\in S}E_s\right)$ be a soft function.*

(f,g) is a soft uniform continuous function $\Longleftrightarrow$ $(p_s\circ f,q_s\circ g):(X,\widetilde{\vartheta},E)\rightarrow(Y_s,\widetilde{\vartheta}_s,E_s)$ is soft uniform continuous for every $s\in S$.

Remark 1 *For every* $\{e_s\}_{s\in S} \in \prod_{s\in S} E_s$*, we have the following equality:*

$$\left(\prod_{s\in S} X_s, \left(\prod_{s\in S} \widetilde{\vartheta}_s\right)_{\{e_s\}}\right) = \left(\prod_{s\in S} X_s, \prod_{s\in S} \left(\widetilde{\vartheta}_s\right)_{\{e_s\}}\right).$$

Let $(X, \widetilde{\vartheta}, E)$ be a soft uniform space and $(X, \widetilde{\vartheta}_e)$ be an ordinary space for every $e \in E$. If $1_X : X \to X$ and $j : \{e\} \to E$ are embedding mapping, then

$$(1_X, j) : (X, \widetilde{\vartheta}_e) \to (X, \widetilde{\vartheta}, E)$$

is a soft embedding mapping. Since

$$(1_X, j)^{-1} (F, E) = F(e), \text{ for every } (F, E) \in \widetilde{\tau},$$

the soft embedding function $(1_X, j)$ is soft uniform continuous. For this reason, we can give the following proposition.

Proposition 9 *The ordinary uniform space* $(X, \widetilde{\vartheta}_e)$ *is embedding to the soft uniform space* $(X, \widetilde{\vartheta}, E)$ *for every* $e \in E$*.*

5. Conclusion

The category of soft uniform spaces is a generalization of the category of uniform spaces. Therefore, the fundamental definitions and properties of soft uniform structure have been investigated. The relationship between soft uniform space and uniform space has been established.

References

[1] Acar U, Koyuncu F, Tanay B. Soft sets and soft rings. Comput Math Appl 2010; 59: 3458-3463.

[2] Aktaş H, Çağman N. Soft sets and soft groups. Inform Sciences 2007; 177: 2726-2735.

[3] Aygunoğlu A, Aygun H. Some notes on soft topological spaces. Neural Comput Appl 2011; 21: 113-119.

[4] Babitha KV, Sunil JJ. Soft set relations and functions. Comput Math Appl 2010; 60: 1840-1849.

[5] Bayramov S, Gunduz C. Soft locally compact spaces and soft paracompact spaces. J Math Sys Sci 2013; 3: 122-130.

[6] Çağman N, Karataş S, Enginoğlu S. Soft topology. Comput Math Appl 2011; 62: 351-358.

[7] Çetkin V, Aygün H. Uniformity structure in the context of soft set. Ann Fuzzy Math Inform 2013; 6: 69-76.

[8] Das S, Samanta SK. Soft real sets, soft real numbers and their properties. J Fuzzy Math 2012; 20: 551-576.

[9] Das S, Samanta SK. Soft metric. Ann Fuzzy Math Inform 2013; 6: 77-94.

[10] Feng F, Jun YB, Zhao F. Soft semirings. Comput Math Appl 2008; 56: 2621-2628.

[11] Gunduz C, Bayramov S. Intuitionistic fuzzy soft modules. Comput Math Appl 2011; 62: 2480-2486.

[12] Gunduz C, Bayramov S. Fuzzy soft modules. Inter Math For 2011; 6 517-527.

[13] Gunduz C, Sonmez A, Çakallı H. On soft mappings. http://arxiv.org/abs/1305.4545.

[14] Hussain S, Ahmad B. Some properties of soft topological spaces. Comput Math Appl 2011; 62: 4058-4067.

[15] Maji PK, Roy AR. An application of soft sets in a decision making problem. Comput Math Appl 2002; 44: 1077-1083.

[16] Maji PK, Bismas R, Roy AR. Soft set theory. Comput Math Appl 2003; 45: 555-562.

[17] Min WK. A note on soft topological spaces. Comput Math Appl 2011; 62: 3524-3528.

[18] Molodtsov D. Soft set theory-first results. Comput Math Appl 1999; 37: 19-31.

[19] Ozturk TY, Bayramov S. Category of chain complexes of soft modules. Inter Math For 2012; 7: 981-992.

[20] Ozturk TY, Gunduz C, Bayramov S. Inverse and direct systems of soft modules. Ann Fuzzy Math Inform 2013; 5: 73-85.

[21] Shabir M, İrfan AM. Soft ideals and generalized fuzzy ideals in semigroups. New Math Nat Comput 2009; 5: 599-615.

[22] Shabir M, Naz M. On soft topological spaces. Comput Math Appl 2011; 61: 1786-1799.

[23] Shabir M, Bashir A. Some properties of soft topological spaces. Comput Math Appl 2011; 62: 4058-4067.

[24] Sun QM, Zhang ZL, Jing L. Soft sets and soft modules. Lect Notes Comp Sci 2008; 59: 403-409.

[25] Zorlutuna İ, Akdağ M, Min WK, Atmaca S. Remarks on soft topological spaces. Ann Fuzzy Math Inform 2012; 3: 171-185.