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A goal programming approach for solving the random interval linear programming problem

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Abstract: This paper presents a goal programming model for solving linear programming problems involving random interval coefficients. In this model, a random interval with known characters is considered as the aspiration level (target) of the objective function. The original problem involving random interval parameters is transformed into a biobjective equivalent problem using the proposed model. Defining an auxiliary variable, an approach for solving the biobjective problem is presented. Two numerical examples are carried out to show the efficiency of the proposed model.

Key words: Random interval parameters, goal programming model, satisfactory solution, random interval programming, mean value

1. Introduction

The theory of probability is an important tool for describing the complexity of uncertain parameters. It has been applied in such different fields as economics [8], stochastic geometry [16], or dealing with imprecise information [3]. Moreover, some basic works about uncertain random variables were conducted by Wang et al. [19] and Zheng et al. [20]. The concept of a satisfactory solution for stochastic problems is based on transforming the stochastic objectives by using some statistical features such as variance, expected value, or quantiles. The obtained function, which can be replaced by the original stochastic objective, is called the deterministic equivalent function.

There are many approaches in the literature that can be used for solving stochastic programming problems [10, 18]. One of the most popular approaches for solving multiobjective stochastic programming problems is the goal programming (GP) model presented by Contini [2]. In his study, he first specified random variables with an arbitrary known distribution as the targets of stochastic objectives, then, minimizing the deviation of the objectives from their targets, he obtained a satisfactory solution. Of course a significant weakness is that the solution of a GP model is not necessarily a Pareto optimal solution. However, the GP approach gives a solution to a multiobjective problem with a given level of satisfaction.

Sometimes random data cannot be measured exactly and each value of a random variable is expressed as an interval. In other words, there are many probability events that can only be measured approximately. This kind of variable that simultaneously involves properties of random variables and intervals is called a random interval variable. However, there are many random interval phenomena for which we have no information about the probability of different values within intervals. In such cases, we face a set of interval data generated by

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a random process. These data can be treated as samples from the interval-valued random variable. Some studies about random intervals in statistics, engineering, and optimization problems exist in the literature; see [1, 3, 4, 14]. Moreover, some basic studies about the main characters of random intervals such as the range of the expected value, variance, covariance, and correlation coefficient have been done in the literature [5–7, 11, 12].

In this study, we propose a GP model for solving random interval linear programming (RILP) problems. In the presented model, we consider a random interval with known characters as the target of the objective function. The aim of this model is to get a solution that minimizes the deviation of the objective function from the target as much as possible. Considering the concept of the mean value, this deviation will be obtained as an interval. However, the problem of minimizing the interval function is not well defined, because the inequality relation between intervals is not a total order. Thus, the meaning of minimizing the interval objective should be defined. In this paper, we adopt a definition based on its end points. Applying this method, the original problem will be transformed into a biobjective problem. Solving the final problem by the minimax method, a satisfactory optimal solution is obtained.

The rest of this paper is organized as follows. In Section 2, some definitions and basic theorems about intervals are recalled. In Section 3, the random interval programming problem is introduced. Moreover, we present a GP model for solving RILP problems. Finally, two numerical examples are proposed to show that our method works successfully. In the final section, we present some concluding comments and future works.

2. Preliminaries

In this section, we recall some basic concepts of interval arithmetic that are used for dealing with problems involving interval parameters.

2.1. Interval variables

Definition 2.1 *An interval is defined by an ordered pair of brackets as*

$$\boldsymbol{a} = [a^L, a^R] = \{a : a^L \le a \le a^R, a \in \mathbb{R}\},$$

where a^L and a^R are the left and right limits of $\boldsymbol{a}$, respectively.

The center (midpoint) and the radius of an interval **a** are defined respectively as

$$a^C := \frac{1}{2}(a^R + a^L), \quad a^W := \frac{1}{2}(a^R - a^L).$$

Definition 2.2 *Another representation of interval $\boldsymbol{a}$ is defined by its center point a^C and half-width length (or radius) a^W as follows:*

$$\boldsymbol{a} = < a^C, a^W > = \{a : a^C - a^W \le a \le a^C + a^W, a \in \mathbb{R}\}.$$

Definition 2.3 *Let $* \in \{+, -, ., \div\}$ be a binary operation on $\mathbb{R}$. If $\boldsymbol{a}$ and $\boldsymbol{b}$ are arbitrary closed intervals, then*

$$\boldsymbol{a} * \boldsymbol{b} = \{a * b : a \in \boldsymbol{a}, b \in \boldsymbol{b}\}.$$

In the case of division, it is supposed that $0 \notin \boldsymbol{b}$.

Let $\mathbf{a}$ and $\mathbf{b}$ be two intervals and $K \in \mathbb{R}$ be a constant. From Definitions (2.1) and (2.2) it can be explicitly shown that:

$$\mathbf{a}+\mathbf{b} = [a^L, a^R] + [b^L, b^R] = [a^L + b^L, a^R + b^R], \tag{2.1}$$

$$\begin{cases} K\mathbf{a} = K[a^L, a^R] = [Ka^L, Ka^R] & \text{if} \quad K \geq 0, \\ K\mathbf{a} = K[a^L, a^R] = [Ka^R, Ka^L] & \text{if} \quad K < 0, \end{cases} \tag{2.2}$$

$$\mathbf{a}-\mathbf{b} = [a^L, a^R] - [b^L, b^R] = [a^L - b^R, a^R - b^L], \tag{2.3}$$

$$K\mathbf{a} = K < a^C, a^W > = < Ka^C, \mid K \mid a^W > . \tag{2.4}$$

The possibly extended maximum of $\mathbf{a}$ and $\mathbf{b}$ is derived as

$$\mathbf{a} \vee \mathbf{b} = [a^L \vee b^L, a^R \vee b^R]. \tag{2.5}$$

The absolute value of $\mathbf{a}$ is defined as

$$\mid \mathbf{a} \mid = \begin{cases} [a^L, a^R], & a^L \geq 0, \\ [0, (-a^L) \vee a^R], & a^L < 0 < a^R, \\ [-a^R, -a^L], & a^R \leq 0, \end{cases} \tag{2.6}$$

where $a \vee b = max(a, b)$.

3. Random interval linear programming

In what follows, we first recall the concepts of random interval variables and random interval data, and then we present our GP model for solving RILP problems.

Generally, a random interval variable is a measurable function from a probability space to a collection of closed intervals. In other words, a random interval variable is a random variable taking interval values. The following definition can be used for describing random interval variables.

Definition 3.1 *([17]). Given a probability space* (Ω, A, P), $\boldsymbol{a}(\omega) = [a^L(\omega), a^R(\omega)]$ *is a random interval defined in* Ω *if* $a^L(\omega), a^R(\omega)$ *are random variables and for any* $\omega \in \Omega, a^L(\omega) \leq a^R(\omega)$. *In other words, if* (Ω, A, P) *is a probability space where* $\Omega = \{\omega_1, \omega_2, ..., \omega_m\}$, *then the function*

$$\xi(\omega) = \begin{cases} \mu_1, & if \quad \omega = \omega_1 \\ \mu_2, & if \quad \omega = \omega_2 \\ \vdots & \\ \mu_m, & if \quad \omega = \omega_m \end{cases},$$

where $\mu_1, \mu_2, ..., \mu_m$ *are intervals, is a random interval variable.*

Example 3.2 *Suppose that* $X(\omega)$ *is a random variable defined on* Ω *and* a *is a positive number; then* $\boldsymbol{I}(\omega) = \left[X(\omega)-a, X(\omega)+a\right]$ *is a random interval with random center* $X(\omega)$ *and (deterministic) width* $2a$. *More generally, if* $Y(\omega) \geq 0$ *is a random variable, then the random interval* $\boldsymbol{I}(\omega) = \left[X(\omega)-Y(\omega), X(\omega)+Y(\omega)\right]$ *has random center* $X(\omega)$ *and random width* $2Y(\omega)$.

In real-world problems, statistical data are often expressed as a set of interval data generated by a random process. An important difference between a set of random interval data and a random interval variable is that each element of a set of random interval data has probability one, while for random interval variables, a probability distribution function is defined whose amount for each element is a real number expressed as $0 \leq p \leq 1$. In fact, the set of random interval data can be treated as samples from the interval-valued random variable. In such cases, instead of the exact value of sample statistics such as mean, variance, or covariance, we can only have an interval of them.

An important character for estimating the mean value of random interval data is defined as follows:

Definition 3.3 *Let* $\tilde{\boldsymbol{X}} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n\}$ *be a set of interval data such that its values are generated by a random process. The mean of* $\tilde{\boldsymbol{X}}$ *is an interval that can be obtained by using straightforward interval arithmetic:*

$$\boldsymbol{M} = \frac{\boldsymbol{x}_1 + \boldsymbol{x}_2 + ... + \boldsymbol{x}_n}{n}.$$

Indeed, $\boldsymbol{M} = \left[M^L, M^R\right]$ *where* $M^L = \dfrac{1}{n}\left(x_1^L + x_2^L + ... + x_n^L\right)$ *and* $M^R = \dfrac{1}{n}\left(x_1^R + x_2^R + ... + x_n^R\right)$.

3.1. A GP model for solving RILP problems

In this part, we present a GP approach for solving RILP problems. In this model, we assume coefficients and the target to be random intervals with known characters.

The basic approach of goal programming is to specify an aspiration level for each of the objectives and then seek a solution that minimizes the (weighted) sum of deviations of these objective functions from their respective goals.

The standard mathematical formulation of the GP model is given as follows:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{p} (\delta_i^+ + \delta_i^-) \\ \text{subject to:} \quad & \\ & f_i(x) - \delta_i^+ + \delta_i^- = g_i \quad (i = 1, 2, ..., p), \\ & x \in S, \delta_i^+, \delta_i^- \geq 0 \quad (i = 1, 2, ..., p), \end{aligned}$$

where δ_i^- and δ_i^+ are the positive and the negative deviations with respect to the targets g_i, respectively.

The GP approach for solving multiobjective linear programing problems with interval coefficients was investigated by Inuguichi et al. [9]. In this model, they specified an interval as the target of each objective function. Minimizing the sum of (the weighted sum) deviations of objective functions from their targets under the constraints of the original problem, they obtained an optimal solution.

The GP model for solving multiobjective stochastic programming problems was proposed by Contini [2]. In his study, he specified random variables with an arbitrary known distribution as the targets of random objectives, then via the concepts of the expected value and variance, he solved his GP model.

Consider the following form of RILP problem:

$$\begin{aligned} \text{minimize} \quad & z(x) = \tilde{\mathbf{c}}x \\ \text{subject to}: \quad & \\ & x \in S = \{x \in \mathbb{R}^n : Ax \le b, x \ge 0\}, \end{aligned} \tag{3.1}$$

where x is an n-dimensional decision variable column vector and A and b are an $m \times n$ matrix and m-dimensional column vector, respectively. Moreover, $\tilde{\mathbf{c}} = \left(\tilde{\mathbf{c}}_1, \tilde{\mathbf{c}}_2, ..., \tilde{\mathbf{c}}_n\right)$ is a vector of random interval data.

It is obvious that Problem (3.1) is not well-defined due to the randomness and intervalness of the coefficients involved in the objective function. In this situation, we cannot optimize the problem likewise in deterministic cases. Problem (3.1) can be regarded as a kind of stochastic programming problem. In order to handle such stochastic programming problems, there are several decision-making models such as the variance minimization model, the expectation optimization model, the fractile optimization model, and the probability maximization model [10, 18]. We use an extension of the expected value model and replace the objective function by its mean [18]. Hence, Problem (3.1) will be transformed into the following problem:

$$\begin{aligned} \text{minimize} \quad & E[z(x)] = E[\tilde{\mathbf{c}}x] \\ \text{subject to:} \quad & \\ & x \in S = \{x \in \mathbb{R}^n : Ax \le b, x \ge 0\}, \end{aligned} \tag{3.2}$$

where $E\left[\tilde{\mathbf{c}}x\right]$ denotes the mean of $\tilde{\mathbf{c}}x$. According to Definition (3.3), Problem (3.2) will be transformed into the following problem:

$$\begin{aligned} \text{minimize} \quad & E\big[z(x)\big] = \big[E(\tilde{c}^L)x, E(\tilde{c}^R)x\big] \\ \text{subject to:} \quad & \\ & x \in S = \{x \in \mathbb{R}^n : Ax \le b, x \ge 0\}, \end{aligned} \tag{3.3}$$

where $E(\tilde{c}^L) = (E(\tilde{c}_1^L), E(\tilde{c}_2^L), ..., E(\tilde{c}_n^L))$ and $E(\tilde{c}^R) = (E(\tilde{c}_1^R), E(\tilde{c}_2^R), ..., E(\tilde{c}_n^R))$ are two vectors of the mean of random data.

Consider now the random interval $\tilde{\mathbf{T}} = \left[\tilde{T}^L, \tilde{T}^R\right]$, $\tilde{T}^R > \tilde{T}^L$, with the mean $E(\tilde{\mathbf{T}}) = \left[E(\tilde{T}^L), E(\tilde{T}^R)\right]$ as the target of the objective function. Obviously, both $\left[E(\tilde{c}^L)x, E(\tilde{c}^R)x\right]$ and $E(\tilde{\mathbf{T}}\ )$ are intervals. According to Relation (2.3), the deviation $\big($subtraction of $\left[E(\tilde{c}^L)x, E(\tilde{c}^R)x\right]$ from $E\big(\tilde{\mathbf{T}}\ \big)\big)$ is an interval. This subtraction can be written as:

$$\begin{aligned} \mathbf{D}(x) &= |E(\tilde{\mathbf{T}}) - E(\tilde{\mathbf{c}})x| \\ &= |\big[E(\tilde{T}^L) - E(\tilde{c}^R)x, E(\tilde{T}^R) - E(\tilde{c}^L)x\big]|. \end{aligned}$$

From Relation (2.6), the following three cases may occur for $\mathbf{D}(x)$:

$$\mathbf{D}(x)=\begin{cases}\big[E\big(\tilde{T}^L\big)-E\big(\tilde{c}^R\big)x, E\big(\tilde{T}^R\big)-E\big(\tilde{c}^L\big)x\big],\\ \text{if}\quad E\big(\tilde{T}^L\big)-E\big(\tilde{c}^R\big)x\geq 0,\\ \big[0,\big(E\big(\tilde{c}^R\big)x-E\big(\tilde{T}^L\big)\big)\vee\big(E\big(\tilde{T}^R\big)-E\big(\tilde{c}^L\big)x\big)\big],\\ \text{if}\quad E\big(\tilde{T}^L\big)-E\big(\tilde{c}^R\big)x<0<E\big(\tilde{T}^R\big)-E\big(\tilde{c}^L\big)x,\\ \big[E\big(\tilde{c}^L\big)x-E\big(\tilde{T}^R\big),E\big(\tilde{c}^R\big)x-E\big(\tilde{T}^L\big)\big],\\ \text{if}\quad E\big(\tilde{T}^R\big)-E\big(\tilde{c}^L\big)x\leq 0.\end{cases}$$

Using deviation variables d^{L-},d^{L+},d^{R-}, and d^{R+}, we define the following relations:

$$E\big(\tilde{c}^R\big)x+d^{L-}-d^{L+}=E\big(\tilde{T}^L\big), \tag{3.4}$$

$$E\big(\tilde{c}^L\big)x+d^{R-}-d^{R+}=E\big(\tilde{T}^R\big), \tag{3.5}$$

$$d^{L-}d^{L+}=0,\quad d^{R-}d^{R+}=0. \tag{3.6}$$

The difference $E(\tilde{\mathbf{T}})-E(\tilde{\mathbf{c}})x$ is represented as

$$E(\tilde{\mathbf{T}})-E(\tilde{\mathbf{c}})x=\big[d^{L-}-d^{L+},d^{R-}-d^{R+}\big]. \tag{3.7}$$

Now we consider a representation of $\mathbf{D}(x)$ via deviational variables d^{L-},d^{L+},d^{R-}, and d^{R+}. The following three cases are possible:

(i) $d^{L-}=0$ and $d^{R-}=0$. Then

$$\mathbf{D}(x)=\big[d^{R+},d^{L+}\big]. \tag{3.8}$$

(ii) $d^{L-}=0$ and $d^{R+}=0$. Then

$$\mathbf{D}(x)=\big[0,d^{L+}\vee d^{R-}\big]. \tag{3.9}$$

(iii) $d^{L+}=0$ and $d^{R+}=0$. Then

$$\mathbf{D}(x)=\big[d^{L-},d^{R-}\big]. \tag{3.10}$$

Note that the case $d^{L+}=0$ and $d^{R-}=0$ cannot happen, because if we consider $d^{L+}=0$ and $d^{R-}=0$, then deviation $\mathbf{D}(x)$ will be obtained as $|\big[d^{L-},-d^{R+}\big]|$, which is not an interval.

It is clear that $d^{L-}d^{R+}=0$, because if we consider $d^{L-}>0$ and $d^{R+}>0$, then we have $E\big(\tilde{c}^R\big)x<E\big(\tilde{T}^L\big)$ and $E\big(\tilde{c}^L\big)x>E\big(\tilde{T}^R\big)$, which contradict $E\big(\tilde{c}^R\big)x>E\big(\tilde{c}^L\big)x$ and $E\big(\tilde{T}^R\big)>E\big(\tilde{T}^L\big)$. Thus, deviation $\mathbf{D}(x)$ is obtained as

$$\mathbf{D}(x)=\big[d^{L-}+d^{R+},d^{L+}\vee d^{R-}\big]. \tag{3.11}$$

The GP model with random interval coefficients and random interval target becomes the problem of minimizing $\mathbf{D}(x)$ under the constraints of Problem (3.1). However, $\mathbf{D}(x)$ is obtained as an interval. The inequality relation between intervals is not a total order. Thus, the meaning of minimizing $\mathbf{D}(x)$ should be defined. In this paper, we adopt a definition based on the end points. According to this definition, we minimize the lower and upper bound of $\mathbf{D}(x)$ under the constraints of the original problem, which gives the following biobjective programming problem. Hence, the GP problem can be formulated as follows:

$$\begin{aligned}
\text{minimize} \quad & \{d^{L-} + d^{R+}, d^{L+} \vee d^{R-}\} \\
\text{subject to:} \quad & \\
& E(\tilde{c}^R)x + d^{L-} - d^{L+} = E(\tilde{T}^L), \\
& E(\tilde{c}^L)x + d^{R-} - d^{R+} = E(\tilde{T}^R), \\
& x \in S = \{x \in \mathbb{R}^n : Ax \le b, x \ge 0\}, \\
& d^{L-}d^{L+} = 0, \ d^{R-}d^{R+} = 0, \\
& d^{R-}, d^{R+}, d^{L-}, d^{L+} \ge 0.
\end{aligned} \tag{3.12}$$

The following theorem shows that there is no need to add the constraints $d^{L-}d^{L+} = 0$ and $d^{R-}d^{R+} = 0$ to this model.

Theorem 3.4 *The constraints $d^{L-}d^{L+} = 0$ and $d^{R-}d^{R+} = 0$ of Problem (3.12) satisfy other constraints and can be omitted.*

Proof Let $\hat{x}, \hat{d}^{L-}, \hat{d}^{L+}, \hat{d}^{R-}$, and $\hat{d}^{R+}$ be a Pareto optimal solution of (3.12), i.e. there does not exist another $\bar{x}, \bar{d}^{L-}, \bar{d}^{L+}, \bar{d}^{R-}$, or $\bar{d}^{R+}$ such that $\bar{d}^{L-} + \bar{d}^{L+} \le \hat{d}^{L-} + \hat{d}^{L+}$ and $\bar{d}^{L+} \vee \bar{d}^{R-} \le \hat{d}^{L+} \vee \hat{d}^{R-}$ with a strict inequality holding for at least one. Suppose $\hat{d}^{L-}\hat{d}^{L+} \neq 0$. We consider $d_0^{L-} \ge 0$ and $d_0^{L+} \ge 0$ such that $d_0^{L-}d_0^{L+} = 0$ and $\hat{d}^{L-} - \hat{d}^{L+} = d_0^{L-} - d_0^{L+}$. Obviously, d_0^{L-} and d_0^{L+} satisfy the constraints of (3.12) and the relations $\hat{d}^{L-} > d_0^{L-}$ and $\hat{d}^{L+} > d_0^{L+}$ hold, because for $d_0^{L-}d_0^{L+} = 0$ two cases are possible:

1. If $d_0^{L+} = 0$ and $d_0^{L-} > 0$, then

   $$\hat{d}^{L-} - \hat{d}^{L+} = d_0^{L-} - d_0^{L+} = d_0^{L-},$$

   and in addition we have $\hat{d}^{L-}, \hat{d}^{L+} > 0$. Hence, relation $\hat{d}^{L-} > d_0^{L-}$ is satisfied.

2. If $d_0^{L-} = 0$ and $d_0^{L+} > 0$, then

   $$\hat{d}^{L-} - \hat{d}^{L+} = d_0^{L-} - d_0^{L+} = -d_0^{L+},$$

   and we have $\hat{d}^{L-}, \hat{d}^{L+} > 0$. Hence, relation $\hat{d}^{L+} > d_0^{L+}$ is satisfied.

According to these facts, the following relations,

$$\hat{d}^{L-} + \hat{d}^{R+} > d_0^{L-} + \hat{d}^{R+},$$

$$\hat{d}^{L+} \vee \hat{d}^{R-} \geq d_0^{L+} \vee \hat{d}^{R-},$$

obviously are satisfied. This contradicts the Pareto optimality of $\hat{x}$ and $\hat{d}^{L-}, \hat{d}^{L+}, \hat{d}^{R-}$, and $\hat{d}^{R+}$. Hence,

$$\hat{d}^{L-}\hat{d}^{L+} = 0.$$

In the case of $d^{R-}d^{R+} = 0$, we can prove it in the same manner. □

In order to obtain a Pareto optimal solution of biobjective problem (3.12), we should transform it into a single-objective programming problem. There are many approaches that can be used for solving biobjective programming problems. One of the most popular is the minimax method. In this method, we minimize an auxiliary variable, which is the upper bound of two objectives. Therefore, Problem (3.12) is converted to the following equivalent problem:

$$\begin{aligned}
\text{minimize} \quad & \lambda \\
\text{subject to:} \quad & \\
& E(\tilde{c}^R)x + d^{L-} - d^{L+} = E(\tilde{T}^L), \\
& E(\tilde{c}^L)x + d^{R-} - d^{R+} = E(\tilde{T}^R), \\
& x \in S = \{x \in \mathbb{R}^n : Ax \leq b, x \geq 0\}, \\
& d^{L-} + d^{R+} \leq \lambda, \\
& d^{L+} \vee d^{R-} \leq \lambda, \\
& d^{R-}, d^{R+}, d^{L-}, d^{L+}, \lambda \geq 0.
\end{aligned} \tag{3.13}$$

Instead of the constraint $d^{L+} \vee d^{R-} \leq \lambda$ of Problem (3.13), we can use its equivalent convex constraints. Problem (3.13) will thus be converted into the following convex programming problem:

$$\begin{aligned}
\text{minimize} \quad & \\
\text{subject to:} \quad & \\
& E(\tilde{c}^R)x + d^{L-} - d^{L+} = E(\tilde{T}^L), \\
& E(\tilde{c}^L)x + d^{R-} - d^{R+} = E(\tilde{T}^R), \\
& x \in S = \{x \in \mathbb{R}^n : Ax \leq b, x \geq 0, \}, \\
& d^{L-} + d^{R+} \leq \upsilon_1, \\
& d^{L+} \leq \lambda, \\
& d^{R-} \leq \lambda, \\
& d^{R-}, d^{R+}, d^{L-}, d^{L+}, \lambda \geq 0.
\end{aligned} \tag{3.14}$$

Problem (3.14) is a deterministic equivalent linear programming problem of (3.1) and can be solved by convex techniques. Moreover, the optimal solution of (3.14) is a satisfactory solution of (3.1).

Example 3.5 *Consider the random interval linear programming problem defined as*

$$\begin{aligned} \text{minimize} \quad & z(x) = \tilde{\boldsymbol{c}}x \\ \text{subjectto}: & \\ & a_1 x \leq b_1, \\ & a_2 x \leq b_2, \\ & x \geq 0, \end{aligned} \tag{3.15}$$

where $x = (x_1, x_2, x_3, x_4)^T$ *is the decision variable vector and* $(b_1, b_2) = (27, 25)$ *is a crisp vector. The coefficient vectors* $a_i, i = 1, 2$ *are given in Table 1. The coefficients* $\tilde{c}^L$ *and* $\tilde{c}^R$ *are two vectors of random*

Table 1. The coefficient vectors a_i.

a_1	7	3	4	6
a_2	5	6	7	9

data with means $(6, 1, 2, 3)^T$ *and* $(7, 4, 6, 5)^T$, *respectively. Considering the random interval* $\tilde{\boldsymbol{T}} = [\tilde{T}^L, \tilde{T}^R]$ *with* $E[\tilde{T}^L, \tilde{T}^R] = [33, 43]$ *as the target of the objective function, we have:*

$$\begin{aligned} \text{minimize} \quad & \lambda \\ \text{subject to}: & \\ & 6x_1 + x_2 + 2x_3 + 3x_4 + d^{L-} - d^{L+} = 43, \\ & 7x_1 + 4x_2 + 6x_3 + 5x_4 + d^{R-} - d^{R+} = 33, \\ & 7x_1 + 3x_2 + 4x_3 + 6x_4 \leq 27, \\ & 5x_1 + 6x_2 + 7x_3 + 9x_4 \leq 25, \\ & d^{L-} + d^{R+} \leq \lambda, \\ & d^{R-} \leq \lambda, \\ & d^{L+} \leq \lambda, \\ & x_1, x_2, x_3, x_4, \lambda \geq 0. \end{aligned} \tag{3.16}$$

Using CVX (a MATLAB-based modeling system for convex optimization) to solve this problem, we obtain the following satisfactory solution:

$$\begin{aligned} x^* &= \big(x_1^*, x_2^*, x_3^*, x_4^*\big) = \big(3.8571, 0.000000, 0.000000, 0.000000\big), \\ d^{L+} &= 0.000000, \quad d^{L-} = 19.8571, \quad d^{R+} = 0.000000, \quad d^{R-} = 6.0000, \\ \lambda &= 19.8571. \end{aligned}$$

The optimal solution of Problem (3.16) is a satisfactory solution for Problem (3.15). This solution is obtained by minimizing the upper and lower bound of deviation under the constraints of the original problem. Indeed, we use the end points to obtain the optimal solution. This definition is based on order relation "LR". There are some other orders defined in the literature that can be used instead of order LR [15]. The solution of this problem depends on how we treat the interval objective function of problem, i.e. which order we choose.

Example 3.6 *Consider the random interval linear programming problem defined as*

$$\begin{array}{ll} \text{minimize} & z(x) = \tilde{\boldsymbol{c}}x \\ \text{subject to :} & \\ & a_1 x \leq b_1, \\ & a_2 x \leq b_2, \\ & x \geq 0, \end{array}$$

where $x = \big(x_1, x_2, x_3\big)^T$ *is the decision variable vector and* $\big(b_1, b_2\big) = \big(28, 35\big)$ *is a crisp vector. The coefficient vectors* $a_i, i = 1, 2$ *are given in Table 2.*

Table 2. The coefficient vectors a_i.

a_1	3	8	4
a_2	4	1	7

The coefficient vectors $\tilde{c}^L$ *and* $\tilde{c}^R$ *are two vectors of random data with means* $(1, 2, 4)^T$ *and* $(4, 3, 7)^T$*, respectively. Considering the random interval* $\tilde{\boldsymbol{T}} = [\tilde{T}^L, \tilde{T}^R]$ *with* $E\big[\tilde{T}^L, \tilde{T}^R\big] = \big[30, 34\big]$ *as the target of the objective function, we have:*

$$\begin{array}{ll} \text{minimize} & \lambda \\ \text{subjectto :} & \\ & x_1 + 2x_2 + 4x_3 + d^{L-} - d^{L+} = 34, \\ & 4x_1 + 3x_2 + 7x_3 + d^{R-} - d^{R+} = 30, \\ & 3x_1 + 8x_2 + 4x_3 \leq 28, \\ & 4x_1 + x_2 + 7x_3 \leq 35, \\ & d^{L-} + d^{R+} \leq \lambda, \\ & d^{L+} \leq \lambda, \\ & d^{R-} \leq \lambda, \\ & x_1, x_2, x_3, \lambda \geq 0. \end{array} \tag{3.17}$$

Using CVX to solve this problem, we obtain a satisfactory solution as follows:

$$\begin{aligned} x^* &= (x_1^*, x_2^*, x_3^*) = \big(0.000000, 1.727273, 3.545455\big), \\ d^{L+} &= 0.000000, \quad d^{L-} = 16.363636, \quad d^{R+} = 0.000000, \quad d^{R-} = 0.000000, \\ \lambda &= 16.363636. \end{aligned}$$

The optimal solution of Problem (3.18) is a satisfactory solution for Problem (3.17).

4. Conclusion

In this paper, we have proposed a GP model for solving RILP problems. In the presented model, we first used the mean of random interval data for transforming the original problem into an interval one. Considering a random interval as the target of the objective function, we minimized the subtraction of the mean value of the random interval function from the mean value of its target. However, this subtraction has been obtained as an interval. To transform this problem into an equivalent well-defined minimization problem, we have adopted the order relation "LR". Introducing an auxiliary variable, we have solved the final problem by the minimax method. Moreover, we have presented two examples and solved them by the presented method.

In addition to the mean value, there are many variables such as variance and covariance that can be used as a transforming tool for solving random interval programming problems in future works. Some practical problems such as transportation problems, financial problems, and supply chain and multiobjective programming problems involving random interval parameters can be investigated. Moreover, in this paper we have considered random interval parameters only in the objective function. In future works, they can be considered for parameters in technical matrix A and limited resources b. Moreover, it should be noted that the optimization problem involving random interval parameters can be considered as a kind of interval programming or stochastic programming problem. An extension of the methods used for solving these problems can be considered for solving random interval programming problems.

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