Turkish Journal of Mathematics

http://journals.tubitak.gov.tr/math/

Research Article

Turk J Math
(2018) 42: 2664 – 2671
© TÜBİTAK
doi:10.3906/mat-1805-111

Derivations, generalized derivations, and *-derivations of period 2 in rings

Hesham NABIEL*
Department of Mathematics, Faculty of Science
Al-Azhar University, Nasr City, Cairo, Egypt

Received: 22.05.2018 • **Accepted/Published Online:** 06.08.2018 • **Final Version:** 27.09.2018

Abstract: The aim of this article is to discuss the existence of certain kinds of derivations and *-derivations that are of period 2. Moreover, we obtain the form of generalized reverse derivations and generalized left derivations of period 2.

Key words: Maps of period 2, derivations, generalized derivations, *-derivations, prime rings, semiprime rings

1. Introduction

Throughout this paper, R will represent an associative ring with center $Z(R)$. An ideal U of R is said to be central ideal if $U \subseteq Z(R)$. Given an integer $n \geq 2$, a ring R is said to be n-torsion free if for $x \in R$, $nx = 0$ implies $x = 0$. For $x, y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$. R is said to be domain if for $a, b \in R$, $ab = 0$ implies $a = 0$ or $b = 0$. A domain with identity is called a unital domain. R is said to be prime if for $a, b \in R$, $aRb = \{0\}$ implies $a = 0$ or $b = 0$, and is said to be semiprime if for $a \in R$, $aRa = \{0\}$ implies $a = 0$. Its clear that every domain is prime. An additive mapping $d : R \to R$ is called a *derivation* (*Jordan derivation*, respectively) if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$ ($d(x^2) = d(x)x + xd(x)$ for all $x \in R$, respectively). As in [9] by Bell and Daif and in [14] by Gölbaşi and Kaya, a right (left, respectively) *generalized derivation* F of R is an additive map of R associated with a derivation d of R such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$ ($F(xy) = xF(y) + d(x)y$ for all $x, y \in R$, respectively). If F is both a right and left generalized derivation with the same associated derivation, then F is said to be a generalized derivation. In [1] Aboubakr and González referred to a right (left, respectively) *generalized Jordan derivation* F of R to be an additive map of R associated with a Jordan derivation d of R such that $F(x^2) = F(x)x + xd(x)$ for all $x \in R$ ($F(x^2) = xF(x) + d(x)x$ for all $x \in R$, respectively). If F is both a right and left generalized Jordan derivation with the same associated Jordan derivation, then F is said to be a generalized Jordan derivation. An additive mapping $d : R \to R$ is called a *reverse derivation* (or sometimes *antiderivation*) if $d(xy) = d(y)x + yd(x)$ for all $x, y \in R$. The authors of [1] gave the following definition: a right (left, respectively) *generalized reverse derivation* F of R is an additive map of R associated with a reverse derivation d of R such that $F(xy) = F(y)x + yd(x)$ for all $x, y \in R$ ($F(xy) = yF(x) + d(y)x$ for all $x, y \in R$, respectively). If F is both a right and left generalized reverse derivation with the same associated reverse derivation, then F is said to be a generalized reverse derivation. In [13] Brešar and Vukman defined a *left*

*Correspondence: hnabiel@yahoo.com
2010 *AMS Mathematics Subject Classification:* 16N60, 16W10, 16U10, 17A36
This paper is a part of a PhD dissertation under the supervision of Professor MN Daif, Al-Azhar University, Cairo, Egypt.

derivation to be an additive mapping $d : R \to R$ satisfying $d(xy) = xd(y) + yd(x)$ for all $x, y \in R$. In [6] Ashraf and Ali gave the definition of a *generalized left derivation* to be an additive map F of R associated with a left derivation d of R such that $F(xy) = xF(y) + yd(x)$ for all $x, y \in R$. Reverse derivations and left derivations have been studied in some papers (see [1, 5, 13]). An additive bijective mapping g of R is called an anti-automorphism if $g(xy) = g(y)g(x)$ for all $x, y \in R$. An anti-automorphism $*$ of period 2 on a ring R is said to be an involution. A ring R equipped with an involution $*$ is called a $*$-ring or a ring with involution. An ideal U of R is called a $*$-ideal if $U^* = U$. In [12] Brešar defined a **-derivation* to be an additive map d of R satisfying $d(xy) = d(x)y^* + xd(y)$ for all $x, y \in R$. Accordingly, a *reverse* **-derivation* of R is an additive map d of R such that $d(xy) = d(y)x^* + yd(x)$ for all $x, y \in R$. In [4] Ali et al. gave the notion of a *left* **-derivation* of R to be an additive map d of R such that $d(xy) = xd(y) + y^*d(x)$ for all $x, y \in R$. For results on *-derivations, reverse *-derivations, left *-derivations, and their generalizations, see [2–4, 7, 12]. Let S be a nonempty subset of R and f a map of R. If $[x, f(x)] = 0$ for all $x \in S$, then f is said to be commuting on S, and if $[f(x), f(y)] = [x, y]$ for all $x, y \in S$, then f is said to be strong commutativity-preserving on S (see [8]).

In [9] Bell and Daif mentioned a map f on R of period 2 on a subset S of R to be a map satisfying $f^2(x) = x$ for all $x \in S$. Involutions are much studied examples. They proved in a semiprime $*$-ring R that $*$ is commuting on a $*$-ideal U of R if and only if $*$ is strong commutativity-preserving on R [[9], Theorem 1]. They also showed the following results:

Theorem 1.1 [*[9], Theorem 3*] *Let R be a semiprime ring and U a nonzero right ideal of R. Then R admits no derivation d of period 2 on U.*

Theorem 1.2 [*[9], Theorem 9*] *Let R be a (not necessarily commutative) unital domain and with char$(R) \neq 2$. If F is a right generalized derivation of period 2 on R, then F must be the identity map or its negative.*

Theorem 1.3 [*[9], Theorem 11*] *Let R be a prime ring with $Z(R) \neq \{0\}$ and with char$(R) \neq 2$. If F is a generalized derivation of period 2 on R with associated derivation d, then F is the identity map or its negative.*

Motivated by their results, we shall prove that a semiprime ring R cannot admit a reverse derivation that is of period 2 on a nonzero ideal of R and also cannot admit a left derivation that is of period 2 on a nonzero one-sided ideal of R. Furthermore, we show that a semiprime *-ring R cannot admit a *-derivation, a reverse *-derivation, or a left *-derivation of period 2 on a nonzero $*$-ideal U of R. Moreover, we shall discuss the form of generalized reverse derivations and generalized left derivations of period 2 in prime rings.

To prove our results, we need the following.

Lemma 1.1 [*[15], Lemma 1.1*] *Let R be a ring and U be a nonzero right ideal of R. Suppose that given $a \in U$, $a^n = 0$ for a fixed integer n, R has a nonzero nilpotent ideal.*

Lemma 1.2 [*[10], Remark(iii)*] *In a semiprime ring R, the center of a nonzero one-sided ideal is contained in $Z(R)$; in particular, any commutative one-sided ideal is contained in $Z(R)$.*

Theorem 1.4 [*[11], Theorem 1*] *Let R be a 2-torsion free semiprime ring and let $d : R \longrightarrow R$ be a Jordan derivation. In this case, d is a derivation.*

Theorem 1.5 [[16], Theorem 2.5] *Let R be a prime ring with $char(R) \neq 2$. Then every right generalized Jordan derivation on R is a right generalized derivation.*

Theorem 1.6 [[6], Proposition 3.1] *Let R be a prime ring with $char(R) \neq 2$. If R admits a generalized left derivation with associated Jordan left derivation d of R, then either $d = 0$ or R is commutative.*

Theorem 1.7 [[2], Theorem 2.1] *Let R be a semiprime $*$-ring. If R admits a generalized *-derivation F associated with a nonzero *-derivation d of R, then F maps R into $Z(R)$.*

Theorem 1.8 [[2], Theorem 2.3] *Let R be a semiprime $*$-ring. If R admits a generalized reverse *-derivation F associated with a nonzero reverse *-derivation d of R, then d maps R into $Z(R)$.*

2. Reverse and left derivations

Our aim in this section is to discuss the existence of reverse and left derivations of period 2 on suitable subsets of a semiprime ring R.

Theorem 2.1 *Let R be a 2-torsion free semiprime ring and U a nonzero right ideal of R. Then R admits no reverse derivation of period 2 on U.*

Proof Assuming that there is a reverse derivation d such that d is of period 2 on U, then $d(xy) = d(y)x+yd(x)$ for all $x, y \in R$. Therefore, $d(x^2) = d(x)x+xd(x)$ for all $x \in R$. By Theorem 1.4, we obtain that d is a derivation of period 2 on U, contrary to Theorem 1.1. □

Theorem 2.2 *Let R be a 3-torsion free semiprime ring and U a nonzero ideal of R. Then R admits no reverse derivation of period 2 on U.*

Proof Assume that there is a reverse derivation d such that d is of period 2 on U, i.e. $d^2(x) = x$ for all $x \in U$. Then for all $x, y \in U$, we have $xd(y) = d^2(xd(y)) = d(yx + d(y)d(x)) = d(x)y + xd(y) + xd(y) + d(x)y$, which implies

$$2d(x)y + xd(y) = 0 \quad \text{for all } x, y \in U. \tag{2.1}$$

Similarly, $d(x)y = d^2(d(x)y) = d(d(y)d(x) + yx) = xd(y) + d(x)y + d(x)y + xd(y)$ for all $x, y \in U$, which reduces to

$$2xd(y) + d(x)y = 0 \quad \text{for all } x, y \in U. \tag{2.2}$$

Adding (2.1) and (2.2) and using the 3-torsion freeness of R, we get $xd(y) + d(x)y = 0$ for all $x, y \in U$. Substituting in (2.1), we obtain $d(x)y = 0$ for all $x, y \in U$. Substituting in (2.2), we get $xd(y) = 0$ for $x, y \in U$. Therefore, $d(xy) = 0$, which implies $xy = 0$ for all $x, y \in U$. Then $x^2 = 0$ for all $x \in U$, contrary to Lemma 1.1 since R is semiprime.

□

Theorem 2.3 *Let R be a semiprime ring and U a nonzero one-sided ideal of R. Then R admits no left derivation of period 2 on U.*

Proof Suppose there exists a left derivation d on R that is of period 2 on U. For $x, y \in U$, we have $xy = d^2(xy) = d(xd(y) + yd(x)) = xy + d(y)d(x) + yx + d(x)d(y)$ for all $x, y \in U$. Thus,

$$d(y)d(x) + d(x)d(y) + yx = 0 \quad \text{for all } x, y \in U. \tag{2.3}$$

Similarly,

$$d(x)d(y) + d(y)d(x) + xy = 0 \quad \text{for all } x, y \in U. \tag{2.4}$$

By (2.3) and (2.4) we conclude that $xy = yx$ for all $x, y \in U$. That is, U is commutative. By Lemma 1.2, we get that U is a two-sided central ideal.

For $x, y \in U$, we have $xd(y) = d^2(xd(y)) = d(xy + d(y)d(x)) = xd(y) + yd(x) + d(y)x + d(x)y$, but U is central ideal, so $2d(x)y + d(y)x = 0$, and

$$2d(y)x + d(x)y = 0 \quad \text{for all } x, y \in U. \tag{2.5}$$

Thus,

$$d(y)x = d(x)y \quad \text{for all } x, y \in U. \tag{2.6}$$

Applying d for (2.6) we obtain $d(y)d(x) + xy = d(x)d(y) + yx$ for all $x, y \in U$, so

$$d(y)d(x) = d(x)d(y) \quad \text{for all } x, y \in U. \tag{2.7}$$

Recalling (2.4), we obtain

$$2d(y)d(x) + xy = 0 \quad \text{for all } x, y \in U. \tag{2.8}$$

Substituting yz for y in (2.8), $z \in U$, and using (2.8) we get $2d(z)yd(x) = 0$ for all $x, y, z \in U$. Therefore, $(2d(x)y)R(2d(x)y) = 0$ for all $x, y \in U$, but R is semiprime, so $2d(x)y = 0$ for all $x, y \in U$, and by (2.5) we obtain $d(y)x = 0$ for all $x, y \in U$. Applying d, we get $0 = d(d(y)x) = d(y)d(x) + xy$ for all $x, y \in U$, so $0 = 2d(y)d(x) + 2xy$ for all $x, y \in U$. By (2.4) and (2.7), we can see that $xy = 0$ for all $x, y \in U$, which is contrary to Lemma 1.1 since R is semiprime. □

3. Generalized reverse derivations and generalized left derivations

In this section we discuss the form of generalized reverse derivations and generalized left derivations that are of period 2.

Theorem 3.1 *Let R be a (not necessarily commutative) unital domain, with $char(R) \neq 2$. If F is a right generalized reverse derivation of period 2 on R associated with a reverse derivation d of R, then F must be the identity map or its negative.*

Proof Suppose there exists a right generalized reverse derivation F of period 2 on R associated with a reverse derivation d of R. Then $F(xy) = F(y)x + yd(x)$ for all $x, y \in R$. Therefore, $F(x^2) = F(x)x + xd(x)$ for all $x, y \in R$. By Theorem 1.5, F is a right generalized derivation that is of period 2 on U. By Theorem 1.2, we get the result. □

In a similar way we can prove the following theorem, by using Theorem 1.3.

Theorem 3.2 *Let R be a prime ring with $Z(R) \neq \{0\}$ and with $char(R) \neq 2$. If F is a generalized reverse derivation of period 2 on R with associated reverse derivation d on R, then F is the identity map or its negative.*

Theorem 3.3 *Let R be a (not necessarily commutative) unital domain, with $char(R) \neq 2$. If F is a generalized left derivation of period 2 on R associated with a left derivation d of R, then F must be the identity map or its negative.*

Proof By our assumption we have $F(xy) = xF(y) + yd(x)$ for all $x, y \in R$. By Theorem 1.6 we have $d = 0$ or R is commutative. If R is commutative, then $F(xy) = F(yx) = yF(x) + xd(y) = F(x)y + xd(y)$ for all $x, y \in R$. That is, F is a right generalized derivation of period 2 on R. Theorem 1.2 yields that F is the identity map or its negative.

Now assume that $d = 0$. Hence, $F(xy) = xF(y)$ for all $x, y \in R$. Note that $F(x) = xF(1) = xa$ for all $x \in R$, where $a = F(1)$. Since F is of period 2, we have $x = xa^2$, implying $x(1 - a^2) = 0$ for all $x \in R$. However, R is a domain, so $a = 1$ or $a = -1$. Hence, F is the identity map or its negative. □

4. *-Maps in rings with involution

In Theorem 1.7 Ali proved that the range of any generalized *-derivation of a semiprime ring R is contained in $Z(R)$. For the sake of completeness, we prove here a special case of his result.

Lemma 4.1 [*[2], Corollary 2.3*] *Let R be a semiprime $*$-ring. If R admits a *-derivation d, then $d(x) \in Z(R)$ for all $x \in R$.*

Proof For $x, y, z \in R$ we have

$$d(x(yz)) = d(x)z^*y^* + xd(y)z^* + xyd(z) \quad \text{for all } x, y, z \in R. \tag{4.1}$$

On the other hand,

$$d((xy)z) = d(x)y^*z^* + xd(y)z^* + xyd(z) \quad \text{for all } x, y, z \in R. \tag{4.2}$$

Comparing (4.1) and (4.2) we get $d(x)[y^*, z^*] = 0$ for all $x, y, z \in R$, which implies

$$d(x)[y, z] = 0 \quad \text{for all } x, y, z \in R. \tag{4.3}$$

Replacing y by $yd(x)$ in (4.3) and using (4.3) we obtain

$$d(x)R[d(x), z] = 0 \quad \text{for all } x, z \in R. \tag{4.4}$$

From (4.4) we get $[d(x), z]R[d(x), z] = 0$ for all $x, z \in R$. The semiprimeness of R completes our result. □

Similarly, we can get a special case of Theorem 1.8.

Lemma 4.2 [*[2], Corollary 2.5*] *Let R be a semiprime $*$-ring. If R admits a reverse *-derivation d, then $d(x) \in Z(R)$ for all $x \in R$.*

In the same vein, we can prove the following.

Lemma 4.3 *Let R be a semiprime $*$-ring. If R admits a left *-derivation d, then $d(x) \in Z(R)$ for all $x \in R$.*

Proof For $x, y, z \in R$ we have

$$d(x(yz)) = xyd(z) + xz^*d(y) + z^*y^*d(x) \quad \text{for all } x, y, z \in R. \tag{4.5}$$

On the other hand,

$$d((xy)z) = xyd(z) + z^*xd(y) + z^*y^*d(x) \quad \text{for all } x, y, z \in R. \tag{4.6}$$

Comparing (4.5) and (4.6) we get $[x, z^*]d(y) = 0$ for all $x, y, z \in R$, which implies

$$[x, z]d(y) = 0 \quad \text{for all } x, y, z \in R. \tag{4.7}$$

Replacing z by $d(y)z$ in (4.7) and using (4.7) we obtain

$$[x, d(y)]Rd(y) = 0 \quad \text{for all } x, y \in R. \tag{4.8}$$

Therefore, $[x, d(y)]R[x, d(y)] = 0$ for all $x, y \in R$. The semiprimeness of R completes our proof. □

Now we discuss the existence of *-derivations of period 2 in semiprime *-rings.

Lemma 4.4 *Let R be a semiprime $*$-ring and U be a nonzero one-sided ideal of R. If R admits a *-derivation d that is of period 2 on U, then U is a two-sided central ideal of R.*

Proof For all $x \in U$ and $r \in R$ we have by Lemma 4.1 that $d(d(x))r = rd(d(x))$. Therefore,

$$xr = rx \quad \text{for all } x \in U, r \in R. \tag{4.9}$$

That is, U is a two-sided central ideal. □

A *-derivation d commutes with $*$ if $d(x^*) = d(x)^*$.

Theorem 4.1 *Let R be a semiprime $*$-ring and U a nonzero one-sided $*$-ideal of R. Then R admits no *-derivation d that commutes with $*$ and is of period 2 on U.*

Proof Assume that R has a *-derivation d such that $d^2(x) = x$ for all $x \in U$. Then $xy = d^2(xy) = d(d(x)y^* + xd(y)) = xy + 2d(x)d(y^*) + xy$ for all $x, y \in U$, which yields

$$2d(x)d(y^*) + xy = 0 \quad \text{for all } x, y \in U. \tag{4.10}$$

Since $xd(y) \in U$ for all $x, y \in U$, we have $xd(y) = d^2(xd(y)) = d(d(x)d(y^*) + xy) = xd(y) + 2d(x)y^* + xd(y)$ for all $x, y \in U$, which implies

$$2d(x)y^* + xd(y) = 0 \quad \text{for all } x, y \in U. \tag{4.11}$$

Replacing x by y^*x in (4.10) and using (4.10), we get $2d(y^*)x^*d(y^*) = 0$ for all $x, y \in U$. Since $d(y^*)$ is in the center of R, we have $2d(y^*)Rx^*d(y^*) = 0$. This yields
$(2x^*d(y^*))R(2x^*d(y^*)) = 0$. However, R is semiprime, and hence $2x^*d(y^*) = 0$ for all $x, y \in U$, and since U is a $*$-ideal we obtain $2x^*d(y) = 0$ for all $x, y \in U$. By Lemma 4.1 we get $2d(y)x^* = 0$ for all $x, y \in U$, and using (4.11) we obtain

$$xd(y) = 0 \quad \text{for all } x, y \in U. \tag{4.12}$$

Applying d on (4.12) gives $0 = d(xd(y)) = d(x)d(y^*) + xy$ for all $x, y \in U$. Therefore, $2d(x)d(y^*) + 2xy = 0$, and by (4.10) we get $xy = 0$ for all $x, y \in U$, which implies $x^2 = 0$ for all $x \in U$, contrary to Lemma 1.1 since R is semiprime. □

In a similar manner we obtain the following result for reverse *- derivations.

Lemma 4.5 *Let R be a semiprime $*$-ring and U a nonzero one-sided ideal of R. If R admits a reverse *-derivation d that is of period 2 on U, then U is a two-sided central ideal of R.*

Theorem 4.2 *Let R be a semiprime $*$-ring and U a nonzero one sided $*$-ideal of R. Then R admits no reverse *-derivation d that commutes with $*$ and is of period 2 on U.*

Proof Assume that R has a reverse *-derivation d such that $d^2(x) = x$ for all $x \in U$. Since $d(x) \in Z(R)$ for all $x \in U$, by Lemma 4.2, and d is of period 2 on U, we have $xy = d^2(xy) = d(d(y)x^* + yd(x)) = d(x^*d(y) + d(x)y) = yx + 2d(y)d(x^*) + yx$ for all $x, y \in U$. By Lemma 4.5 this yields

$$2d(y)d(x^*) + yx = 0 \quad \text{for all } x, y \in U. \tag{4.13}$$

Replacing y by y^*y in (4.13), we have $2d(y^*y)d(x^*) + y^*yx = 0$ for all $x, y \in U$, and by Lemma 4.5 we get $2d(yy^*)d(x^*) + y^*yx = 0$ for all $x, y \in U$. Thus, $2(d(y^*)y^* + y^*d(y))d(x^*) + y^*yx = 0$ for all $x, y \in U$, and using (4.13) we obtain $2d(y^*)y^*d(x^*) = 0$ for all $x, y \in U$. Since $d(y^*)$ is in the center of R, we have $2d(y^*)Rx^*d(y^*) = 0$ for all $x, y \in U$, which implies that $(2y^*d(x^*))R(2y^*d(x^*)) = 0$ for all $x, y \in U$. Hence, $2y^*d(x^*) = 0$ for all $x, y \in U$, and since U is a $*$-ideal we obtain

$$2yd(x^*) = 0 \quad \text{for all } x, y \in U. \tag{4.14}$$

Since $xd(y) \in U$ for $x, y \in U$, we have $xd(y) = d(y)x + 2yd(x^*) + d(y)x$, using Lemma 4.5 and Lemma 4.2. Therefore, $2yd(x^*) + d(y)x = 0$ for all $x, y \in U$. Using (4.14) we obtain

$$d(y)x = 0 \quad \text{for all } x, y \in U. \tag{4.15}$$

Applying d on (4.15) and using (4.13) we get $xy = 0$ for all $x, y \in U$, which implies $x^2 = 0$ for all $x \in U$, contrary to Lemma 1.1 since R is semiprime. □

Lemma 4.6 *Let R be a semiprime *-ring and U be a nonzero one-sided ideal of R. If R admits a left *-derivation d that is of period 2 on U, then U is a two-sided central ideal.*

Proof Follows from Lemma 4.3 . □

Theorem 4.3 *Let R be a semiprime $*$-ring and U a nonzero one-sided $*$-ideal of R. Then R admits no left *-derivation d that commutes with $*$ and is of period 2 on U.*

Proof Assuming that R has a left *-derivation d such that $d^2(x) = x$ for all $x \in U$, then $d(xy) = xd(y) + y^*d(x)$ for all $x, y \in R$. We have by Lemma 4.6 that $d(xy) = d(x)y^* + xd(y)$ for all $x, y \in R$. Thus, d is a *-derivation that is of period 2 on U, which contradicts Theorem 4.1.

□

References

[1] Aboubakr A, González S. Generalized reverse derivations on semiprime rings. Siberian Math J 2015; 56: 199-205.

[2] Ali S. On generalized *-derivations in $*$-rings. Palestine J Math 2012; 1: 32-37.

[3] Ali S, Alhazmi H, Khan AN. On Jordan (θ, ϕ)-biderivations in rings with involution. Br J Math Comput Sci 2016; 17: 1-7.

[4] Ali S, Dar NA, Pagon D. On Jordan *-mappings in rings with involution. J Egyptian Math Soc 2016; 24: 15-19.

[5] Ashraf M. On Lie ideals and Jordan left derivations of prime rings. Archivum Mathematicum 2000; 36: 201-206.

[6] Ashraf M, Ali S. On generalized Jordan left derivations in rings. Bull Korean Math Soc 2008; 45: 253-261.

[7] Ashraf M, Ali S, Khan A. Generalized (α, β)*-derivations and related mappings in semiprime $*$-rings. Asian-Eur J Math 2012; 5: 1-10.

[8] Bell HE, Daif MN. On commutativity and strong commutativity-preserving maps. Canad Math Bull 1994; 37: 443-447.

[9] Bell HE, Daif MN. On maps of period 2 on prime and semiprime rings. Internat J Math and Math Sci 2014: 1-4.

[10] Bell HE, Martindale WS 3rd. Centralizing mappings of semiprime rings. Canad Math Bull 1987; 30: 92-101.

[11] Brešar M. Jordan derivations on semiprime rings. P Am Math Soc 1998; 104: 1003-1006.

[12] Brešar M, Vukman J. On some additive mappings in rings with involution. Aequationes Math 1989; 38: 178-185.

[13] Brešar M, Vukman J. On left derivations and related mappings. P Am Math Soc 1990; 110: 7-16.

[14] Gölbaşi Ö, Kaya K. On Lie ideals with generalized derivations. Siberian Math J 2006; 47: 862-866.

[15] Herstein IN. Topics in Ring Theory. Lecture Notes in Mathematics. Chicago, IL, USA: University of Chicago, 1965.

[16] Jing W, Lu S. Generalized Jordan derivations on prime rings and standard operator algebras. Taiwanese J Math 2003; 7: 605-613.