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Corrigendum and addendum to “Modules whose p-submodules are direct summands”

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Abstract: This paper is written to correct the proof of Lemma 2.1 (i) in [1] and to add some decomposition results for the class of PD-modules defined in [1].

Key words: PD-modules

In line 14 of the Introduction replace “[...] a sublattice of the lattice of submodules of M” with “closed under intersections.”

In the proof of Lemma 2.1 (i) replace “Observe that $(M/N_1) \oplus (M/N_2) \cong M/(N_1 \cap N_2)$.” with “Define the homomorphism $\alpha : M \to M/(N_1 \cap N_2)$ by $\alpha(m) = (m + N_1, m + N_2)$. Observe that $M/(N_1 \cap N_2) \cong \alpha(M) \leq (M/N_1) \oplus (M/N_2)$. Hence $\alpha(M)$ is nonsingular, as $(M/N_1) \oplus (M/N_2)$ is nonsingular. Thus [...]”

In the statement of Proposition 2.4, insert “M_1 is a p-submodule of M such that” before “and [...]”.

In the proof of Proposition 3.6 replace ”[3, Lemma 4.11]” with ”[3, Lemma 4.13]”.

Moreover, we obtain the following decomposition results with respect to the second singular submodule $Z_2(M)$ of M for the class of PD-modules.

Proposition 1 *Let M be a PD-module and K a p-submodule of M. Then $M = Z_2(M) \oplus T \oplus Y$, where $K = Z_2(M) \oplus T$ and Y are PD-modules.*

Proof Let M be a PD-module and K a p-submodule of M. Then $M = K \oplus K'$ for some $K' \leq M$. Since $K \trianglelefteq_p M$, K and K' are PD-modules by [1, Proposition 3.6]. Recall that $Z_2(M) \subseteq K$, as M/K is nonsingular. Since $Z_2(M) \trianglelefteq_p M$ and $Z_2(M) \subseteq K$, $Z_2(M) \trianglelefteq_p K$. Moreover, $Z(K/Z_2(M)) = 0$ yields that $Z_2(M)$ is a p-submodule of K. It follows that $K = Z_2(M) \oplus T$ for some $T \leq K$. Therefore, $M = K \oplus K' = Z_2(M) \oplus T \oplus K'$. Hence, $K' = Y$; Y is the desired direct summand. □

Corollary 2 *M is a PD-module if and only if $M = Z_2(M) \oplus Y$, where $Z_2(M)$ and Y are PD-modules.*

Proof It is clear from Proposition 1. □

References

[1] Kara Y. Modules whose p-submodules are direct summands. Turk J Math 2018; 42: 28-33.

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