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Existence and nonexistence of global solutions for nonlinear transmission acoustic problem

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Abstract: In this paper we consider a mixed problem for the nonlinear wave equations with transmission acoustic conditions, that is, the wave propagation over bodies consisting of two physically different types of materials, one of which is clamped. We prove the existence of a global solution. Under the condition of positive initial energy we show that the solution for this problem blows up in finite time.

Key words: Nonlinear wave equation, transmission acoustic conditions, locally reacting boundary, existence of global solutions, blow up result

1. Introduction

Let Ω be a bounded domain in R^n $(n \geq 1)$ with smooth boundary Γ_1, $\Omega_2 \subset \Omega$ is a subdomain with smooth boundary Γ_2, and $\Omega_1 = \Omega \backslash \Omega_2$ is a subdomain with boundary $\Gamma = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \varnothing$. The nonlinear transmission acoustic problem considered here is

$$u_{tt} - \Delta u + |u_t|^{q_1 - 1} u_t = |u|^{p-1} u \text{ in } \Omega_1 \times (0, \infty), \tag{1.1}$$

$$\upsilon_{tt} - \Delta \upsilon + |\upsilon_t|^{q_2 - 1} \upsilon_t = |\upsilon|^{p-1} \upsilon \text{ in } \Omega_2 \times (0, \infty), \tag{1.2}$$

$$M\delta_{tt} + D\delta_t + K\delta = -u_t \text{ on } \Gamma_2 \times (0, \infty), \tag{1.3}$$

$$u = 0 \text{ on } \Gamma_1 \times (0, \infty), \tag{1.4}$$

$$u = \upsilon, \delta_t = \frac{\partial u}{\partial \nu} - \frac{\partial \upsilon}{\partial \nu} \text{ on } \Gamma_2 \times (0, \infty), \tag{1.5}$$

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), x \in \bar{\Omega}_1, \tag{1.6}$$

$$\upsilon(x, 0) = \upsilon_0(x), \upsilon_t(x, 0) = \upsilon_1(x), x \in \bar{\Omega}_2, \tag{1.7}$$

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$$\delta(x,0)=\delta_0(x),\delta_t(x,0)=\frac{\partial u_0}{\partial \nu}-\frac{\partial \upsilon_0}{\partial \nu}\equiv \delta_1, x\in \bar{\Gamma}_2, \tag{1.8}$$

where ν is the unit outward normal vector to Γ; $M,D,K:\bar{\Gamma}_2\to R$, u_0, $u_1:\bar{\Omega}_1\to R$, υ_0, υ_1: $\bar{\Omega}_2\to R$, δ_0: $\bar{\Gamma}_2\to R$ are given functions; $p>1$, $q_i>1$, $i=1,2$ are constants.

Transmission problems arise in several applications of physics and biology. For example, problems like (1.1)–(1.8), called transmission acoustic problems, are related to the problem of two wave equations, which model the transverse acoustic vibrations of a membrane composed of two different materials Ω_1 and Ω_2.

Transmission problems were studied, for example, in [1, 2, 4, 13, 35, 37, 40]. The transmission problem for hyperbolic equations was investigated by Dautray and Lions [13], who proved the existence and regularity of solutions for the linear problem. Rivera and Oquendo [40] considered the transmission problem of viscoelastic waves and proved that the dissipation produced by the viscoelastic part can produce exponential decay of the solution. Bae [2] studied the transmission problem, in which one component is clamped and the other is in a viscoelastic fluid producing a dissipation mechanism on the boundary, and established a decay result. Bastos and Raposo [4] investigated the transmission problem with frictional damping and showed the well-posedness and exponential stability of the total energy.

Aliev and Mammadhasanov [1] studied the initial boundary value problem on longitudinal impact on a composite linear viscoelastic bar and established a well-posedness result by the method of dynamic regularization of transmission and boundary conditions.

In [19] the authors made a comparison of several boundary conditions, among which were the acoustic boundary conditions.

The acoustic boundary conditions were introduced by Beale and Rosencrans [7] and studied in [5, 6, 9–12, 14–18, 20, 22–24, 27, 30, 39, 41–43]. In [7] the authors derived

$$u_{tt}-\Delta u=0 \text{ in } \Omega\times(0,\infty),$$

$$\frac{\partial u}{\partial \nu}=\delta_t \text{ on } \Gamma\times(0,\infty),$$

$$m\delta_{tt}+d\delta_t+k\delta=-\rho_0 u_t \text{ on } \Gamma\times(0,\infty)$$

as a theoretical model for describing the acoustic wave motion into a fluid in $\Omega\times R^3$; here ρ_0, m, d, k are physical known quantities. The function $u(x,t)$ is the velocity potential of a fluid and $\delta(x,t)$ models the normal displacement of the point $x\in\Gamma$ at time t. To obtain the model, Beale and Rosencrans assumed that each point of surface Γ acts like a spring in response to excess pressure from the fluid in the interior and that each point of Γ does not influence the others. Surfaces of such type are called locally reacting; see Morse and Ingard [38]. See also [15] for a related model.

Similarly, in [12, 14] acoustic boundary conditions were coupled with homogeneous Dirichlet condition on a portion of the boundary. In [15, 18, 39] acoustic boundary conditions were imposed in the whole boundary. In [14] Frota et al. obtained decay results to a nonlinear wave equation when $n=1$; Cousin et al. [12] and Park and Park [42] obtained decay results when the coefficient in front of δ is zero.

In [17] Frota et al. considered acoustic boundary conditions in domains with nonlocally reacting boundary. A mixed problem for wave equations with nonlinear acoustic boundary conditions was considered by Gao et al.

[20] and Graber [22, 23]. In [27] Jeong et al. studied the global nonexistence of solutions for a quasilinear wave equation with acoustic boundary conditions.

Graber and Said-Houari [24] studied the stability of a structural acoustic wave equation with semilinear porous acoustic boundary conditions and obtained several results in local existence, global existence, the decay rate, and blow up results.

Bucci and Lasiecka [11] studied uniform stability properties of a strongly coupled system of partial differential equations of hyperbolic/parabolic type, which arises from the analysis and control of acoustic models with structural damping on an interface. In [30] Lasiecka obtained results on uniform stabilizability of a three-dimensional structural acoustic model describing the pressure in an acoustic clamber with flexible walls.

For the one wave equation

$$u_{tt} - \Delta u + |u_t|^{m-2} u_t = |u|^{p-2} u \text{ in } (0,T) \times \Omega, \tag{1.9}$$

with Dirichlet boundary conditions on $\partial\Omega$ in the absence of the damping term $|u_t|^{m-2} u_t$, the source term $|u|^{p-2} u$ causes finite-time blow-up of solutions with negative initial energy (see [3, 8, 28]). In contrast, in the absence of the source term, the damping term assures global existence for arbitrary initial data (see [25, 29]). The interaction between the damping and source terms was first considered by Levine [31, 32] for linear damping ($m = 2$). Levine showed that solutions with negative initial energy blow up in finite time. The main tool used in [25] and [31] is the "concavity method", which fails in the case of a nonlinear damping term ($m > 2$). Georgiev and Todorova [21] extended Levin's result to nonlinear damping case $m > 2$. In their work, the authors introduced a new method and determined relations between m and p for which there is a global existence and the relations between m and p for which there is a finite time blow up. Specifically, they showed that solutions with negative initial energy continue to exist globally if $m \geq p$ and blow up in finite time if $p > m$ and the initial energy is negative. Their method is based on the construction of a function L that satisfies the differential inequality

$$L'(t) \geq \omega L^{1+\nu}(t) \tag{1.10}$$

in $[0, \infty)$, where $\omega > 0$ and $\nu > 0$. The inequality (1.10) leads to a blow up of the solutions in finite time, provided that $L(0) > 0$.

The result in [21] was later generalized to an abstract setting and to unbounded domains by Levine and Serrin [34] and Levine and Park [33]. Vitillaro in [44] combined the arguments in [21] and [34] to extend these results to situations where damping is nonlinear and the solution has positive initial energy.

Our main goal in this paper is to extend the above results on the wave equation (1.9) to our system (1.1)–(1.8). We study a mixed problem for the nonlinear wave equations with transmission acoustic conditions. To the best of our knowledge, there are no results on nonlinear wave equations with transmission acoustic conditions. We prove the existence of a global solution for the problem (1.1)–(1.8) under the condition $p \leq \min\{q_1, q_2\}$. For positive initial energy and the condition $p > \max\{q_1, q_2\}$ we give the blow up result.

Our paper is organized as follows. In Section 2 we introduce some notations, preliminaries, and a statement of main results; in Section 3 we prove the existence of a global solution, and the blow up result is proved in Section 4.

2. Preliminaries and main results

The inner product and norm in $L^2(\Omega_i)$, $i=1,2$, and $L^2(\Gamma_2)$ are denoted, respectively, by

$$(u,\,\upsilon)_i=\int_{\Omega_i}u(x)\,\upsilon(x)\,dx,\qquad \|u\|_i=\left(\int_{\Omega_i}(u(x))^2\,dx\right)^{\frac{1}{2}},\ \ i=1,2,$$

$$(\delta,\,\theta)_{\Gamma_2}=\int_{\Gamma_2}\delta(x)\,\theta(x)\,d\Gamma_2,\qquad \|\delta\|_{\Gamma_2}=\left(\int_{\Gamma_2}(\delta(x))^2\,d\Gamma_2\right)^{\frac{1}{2}}.$$

We define a closed subspace of the $H^1_{\Gamma_1}(\Omega_1)$ as

$$H^1_{\Gamma_1}(\Omega_1)=\left\{\ u\in H^1(\Omega_1):\ \gamma_0(u)=0\ \ a.\ e.\ on\ \Gamma_1\right\},$$

where $\gamma_0:\ H^1(\Omega_1)\to H^{1/2}(\Gamma)$ is the trace map of order zero and $H^{1/2}(\Gamma)$ is the Sobolev space of order $\frac{1}{2}$ defined over Γ, as introduced by Lions and Magenes [36]. Observe that the norm in $H^1_{\Gamma_1}(\Omega_1)$,

$$\|u\|_{H^1_{\Gamma_1}(\Omega_1)}=\left(\sum_{i=1}^{n}\int_{\Omega_1}\left(\frac{\partial u}{\partial x_i}\right)^2dx\right)^{\frac{1}{2}},$$

and the norm of the real Sobolev space $H^1(\Omega_1)$ are equivalent, because Poincaré's inequality holds in $H^1_{\Gamma_1}(\Omega_1)$. Thus, we consider $H^1_{\Gamma_1}(\Omega_1)$ with the above gradient norm.

In this section we give our main results on the existence and nonexistence of global solutions. First of all, we give the theorem on local existence of solutions and the regularity theorem for the problem (1.1)–(1.8), which were proved in [26] by combining the Galerkin method and the fixed point method (see [21]).

Theorem 2.1 *(local existence) Assume that*

$$M,\,D,\,K\in C\left(\bar{\Gamma}_2\right),\ M\geq 0,\ D>0,\ K\geq 0\ \ for\ \forall x\in\bar{\Gamma}_2,\tag{2.1}$$

$$p>1\ \ if\ \ n=1,2,\ 1<p\leq\frac{n}{n-2}\ \ if\ n\geq 3,\tag{2.2}$$

$$q_i>1\ \ if\ \ n=1,2,\ \ 1<q_i\leq\frac{n+2}{n-2}\ \ if\ n\geq 3,\tag{2.3}$$

and

$$(u_0,\,\upsilon_0\,,\,\delta_0)\in H^1_{\Gamma_1}(\Omega_1)\times H^1(\Omega_2)\times L^2(\Gamma_2),$$

$$(u_1,\,\upsilon_1\,,\,\delta_1)\in L^2(\Omega_1)\times L^2(\Omega_2)\times L^2(\Gamma_2),$$

$$u_0=\upsilon_0\ and\ \ u_1=\upsilon_1\ on\ \Gamma_2.$$

Then there exists $T>0$ such that the problem (1.1)–(1.8) has a unique solution (u,υ,δ), which satisfies

$$(u,\upsilon,\delta)\in L^\infty\left(0,T;H^1_{\Gamma_1}(\Omega_1)\times H^1(\Omega_2)\times L^2(\Gamma_2)\right),$$

$$u_t \in L^{\infty}\left(0,T;L^2\left(\Omega_1\right)\right) \cap L^{q_1+1}\left(\Omega_1 \times [0,T)\right),$$

$$\upsilon_t \in L^{\infty}\left(0,T;L^2\left(\Omega_2\right)\right) \cap L^{q_2+1}\left(\Omega_2 \times [0,T)\right), \delta_t \in L^{\infty}\left(0,T;L^2\left(\Gamma_2\right)\right).$$

Moreover, at least one of the following statements holds:

1) $\lim\limits_{t\to T-0}\left(\|u_t\|_1^2+\|\upsilon_t\|_2^2+\|\nabla u\|_1^2+\|\nabla \upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)=+\infty;$

2) $T=+\infty$.

Theorem 2.2 *(regularity result). Suppose that (2.1)–(2.3) hold. Let*

$$u_0 \in H^1_{\Gamma_1}\left(\Omega_1\right)\bigcap H^2\left(\Omega_1\right), u_1 \in H^1_{\Gamma_1}\left(\Omega_1\right)\bigcap L^{2q_1}\left(\Omega_1\right), \upsilon_0 \in H^2\left(\Omega_2\right), \upsilon_1 \in H^1\left(\Omega_2\right)\bigcap L^{2q_2}\left(\Omega_2\right),$$

$$u_0=\upsilon_0 \quad and\ u_1=\upsilon_1 \quad on\ \Gamma_2, \delta_0 \in L^2\left(\Gamma_2\right).$$

Then there exists $T>0$ such that the problem (1.1)–(1.8) has a unique solution (u,υ,δ), which satisfies

$$u,\ u_t \in L^{\infty}\left(0,T\,;H^1_{\Gamma_1}\left(\Omega_1\right)\right),\ u_{tt} \in L^{\infty}\left(0,T\,;L^2\left(\Omega_1\right)\right),$$

$$\upsilon,\ \upsilon_t \in L^{\infty}\left(0,T\,;H^1\left(\Omega_2\right)\right),\ \upsilon_{tt} \in L^{\infty}\left(0,T\,;L^2\left(\Omega_2\right)\right),$$

$$u\left(t\right) \in H\left(\Delta,\Omega_1\right),\ \upsilon\left(t\right) \in H\left(\Delta,\Omega_2\right) a.e.in\left(0,\ T\right),$$

$$\delta,\ \delta_t \in L^{\infty}\left(0,T\,;L^2\left(\Gamma_2\right)\right), \delta_{tt} \in L^2\left(0,T\,;L^2\left(\Gamma_2\right)\right),$$

and

$$\begin{aligned}
&u_{tt}-\Delta u+\left|u_t\right|^{q_1-1}u_t=\left|u\right|^{p-1}u \quad a.\ e.\ in\ \Omega_1\times(0,T),\\
&\upsilon_{tt}-\Delta\upsilon+\left|\upsilon_t\right|^{q_2-1}\upsilon_t=\left|\upsilon\right|^{p-1}\upsilon \quad a.\ e.\ in\ \Omega_2\times(0,T),\\
&M\delta_{tt}+D\delta_t+K\delta=-u_t\,,\ u=\upsilon\ \ a.\ e.\ on\ \Gamma_2\times(0,T),\\
&\left\langle\gamma_1\left(u\left(t\right)-\upsilon\left(t\right)\right),\ \gamma_0\left(\varphi\right)\right\rangle_{H^{-1/2}(\Gamma_2)\times H^{1/2}(\Gamma_2)}=\left(\delta_t\left(t\right),\ \gamma_0\left(\varphi\right)\right)_{\Gamma_2}\\
&\ for\ \forall\varphi\in H\left(\Delta,\ \Omega_1\right)\bigcup H\left(\Delta,\ \Omega_2\right)\ \ a.\ e.\ in\ \left(0,\ T\right),\\
&u\left(x,0\right)=u_0\left(x\right),\ u_t\left(x,0\right)=u_1\left(x\right)\ \ a.\ e.\ in\ \Omega_1,\\
&\upsilon\left(x,0\right)=\upsilon_0\left(x\right),\ \upsilon_t\left(x,0\right)=\upsilon_1\left(x\right)\ \ a.\ e.\ in\ \Omega_2,\\
&\delta\left(x,0\right)=\delta_0\left(x\right)\ \ a.\ e.\ on\,\Gamma_2.
\end{aligned}$$

The following theorem shows that the solution obtained in Theorem 2.1 is a global solution if $p \leq \min\left\{q_1,\ q_2\right\}$.

Theorem 2.3 *(global existence) If the assumptions of Theorem 2.1 hold and*

$$p \leq \min\ \left\{q_1,\ q_2\right\}, \tag{2.4}$$

then the local solution (u,υ,δ) is a global solution and T may be taken arbitrarily large.

Next we consider the problem (1.1)–(1.8) under the assumption

$$p > \max\{q_1, q_2\}, \tag{2.5}$$

in which case we show that the solution blows up in finite time.

Let s be a number with $2 < s < +\infty$ if $n = 1, 2$ and $2 \leq s \leq \frac{2n}{n-2}$ if $n \geq 3$. Then there exists the constant B_1 depending on Ω_1 and s such that

$$\|u\|_{L^s(\Omega_1)} \leq B_1 \|\nabla u\|_{L^2(\Omega_1)} \text{ for } \forall u \in H^1_{\Gamma_1}(\Omega_1). \tag{2.6}$$

By this and Phriedrich's inequality there exists the constant B depending on Ω_1, Ω_2, and p such that

$$\|u\|^{p+1}_{L^{p+1}(\Omega_1)} + \|\upsilon\|^{p+1}_{L^{p+1}(\Omega_2)} \leq B\left(\|\nabla u\|_1^2 + \|\nabla \upsilon\|_2^2\right)^{\frac{p+1}{2}} \tag{2.7}$$

for $\forall u \in H^1_{\Gamma_1}(\Omega_1)$ and $\forall\, \upsilon \in H^1(\Omega_2)$, which satisfy the condition $(1.5)_1$.

Now we define the following energy function associated with the solution (u, υ, δ) of the problem (1.1)–(1.8):

$$\begin{gathered} E(t) = \tfrac{1}{2}\left[\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|\upsilon_t\|_2^2 + \|\nabla \upsilon\|_2^2 + \left\|\sqrt{M}\delta_t\right\|^2_{\Gamma_2} + \left\|\sqrt{K}\delta\right\|^2_{\Gamma_2}\right] - \\ -\tfrac{1}{p+1}\left(|u|^{p+1}, 1\right)_1 - \tfrac{1}{p+1}\left(|\upsilon|^{p+1}, 1\right)_2. \end{gathered} \tag{2.8}$$

Multiplying equation (1.1) by u_t, (1.2) by υ_t, and (1.3) by δ_t and integrating over $\Omega_1 \times [0, t]$, $\Omega_2 \times [0, t]$, $\Gamma_2 \times [0, t]$, respectively, then summing them and integrating by parts (we will do this in Section 3), we can obtain

$$\frac{d}{dt}E(t) = -\left(\|u_t\|^{q_1+1}_{L^{q_1+1}(\Omega_1)} + \|\upsilon_t\|^{q_2+1}_{L^{q_2+1}(\Omega_2)} + \left\|\sqrt{D}\delta_t\right\|^2_{\Gamma_2}\right) \leq 0 \tag{2.9}$$

or

$$E(t) - E(0) = -\int_0^t \left(\|u_t\|^{q_1+1}_{L^{q_1+1}(\Omega_1)} + \|\upsilon_t\|^{q_2+1}_{L^{q_2+1}(\Omega_2)} + \left\|\sqrt{D}\delta_t\right\|^2_{\Gamma_2}\right) d\tau. \tag{2.10}$$

Thus, if (u, υ, δ) is a solution of the problem (1.1)–(1.8) then $E(t)$ is a nonincreasing function for $t > 0$. It follows from (2.6), (2.7), and (2.8) that

$$\begin{gathered} E(t) \geq \tfrac{1}{2}\left(\|\nabla u\|_1^2 + \|\nabla \upsilon\|_2^2\right) - \tfrac{1}{p+1}\int_{\Omega_1} |u|^{p+1}\,dx - \tfrac{1}{p+1}\int_{\Omega_2} |\upsilon|^{p+1}\,dx = \\ = \tfrac{1}{2}\left(\|\nabla u\|_1^2 + \|\nabla \upsilon\|_2^2\right) - \tfrac{1}{p+1}\|u\|^{p+1}_{L^{p+1}(\Omega_1)} - \tfrac{1}{p+1}\|\upsilon\|^{p+1}_{L^{p+1}(\Omega_2)} \geq \\ \geq \tfrac{1}{2}\left(\|\nabla u\|_1^2 + \|\nabla \upsilon\|_2^2\right) - \tfrac{B}{p+1}\left(\|\nabla u\|_1^2 + \|\nabla \upsilon\|_2^2\right)^{\frac{p+1}{2}} = g(\alpha), \end{gathered} \tag{2.11}$$

where

$$\alpha = \left(\|\nabla u\|_1^2 + \|\nabla \upsilon\|_2^2\right)^{\frac{1}{2}}, \tag{2.12}$$

$$g(\alpha) = \frac{1}{2}\alpha^2 - \frac{B}{p+1}\alpha^{p+1}. \tag{2.13}$$

Therefore,

$$g(0) = 0\,,\ g'(\alpha_1) = 0,$$

where $\alpha_1 = B^{-\frac{1}{p-1}}$, and since $p > 1$ then

$$g_{\max}(\alpha_1) = B^{-\frac{2}{p-1}}\left(\frac{1}{2} - \frac{1}{p+1}\right) = \alpha_1^2\left(\frac{1}{2} - \frac{1}{p+1}\right) = E_1 > 0. \tag{2.14}$$

Applying the idea of Vitillaro [44] we have the following lemma.

Lemma 2.4 *Let (u, υ, δ) be a solution of the problem (1.1)–(1.8). Assume that (2.1) and (2.2) hold. If*

$$0 < E(0) < E_1 \tag{2.15}$$

and

$$\left(\|\nabla u_0\|_1^2 + \|\nabla \upsilon_0\|_2^2\right)^{\frac{1}{2}} > \alpha_1, \tag{2.16}$$

then there exists $\alpha_2 > \alpha_1$ such that

$$\left(\|\nabla u\|_1^2 + \|\nabla \upsilon\|_2^2\right)^{\frac{1}{2}} \geq \alpha_2 \tag{2.17}$$

and

$$\int_{\Omega_1} |u|^{p+1}\,dx + \int_{\Omega_2} |\upsilon|^{p+1}\,dx \geq B\alpha_2^{p+1} \tag{2.18}$$

for $t \geq 0$.

Proof. Let $E(0) < E_1$. Then by (2.14) there exists a number α_2 such that $\alpha_2 > \alpha_1$ and $g(\alpha_2) = E(0)$. On the other hand, by (2.11), $g(\alpha) \leq E(t)$, and hence

$$g(\alpha_0) \leq E(0) = g(\alpha_2)\,, \tag{2.19}$$

where $\alpha_0 = \alpha|_{t=0} = \left(\|\nabla u_0\|_1^2 + \|\nabla \upsilon_0\|_2^2\right)^{\frac{1}{2}}$. Since by (2.16), $\alpha_0 > \alpha_1$ and $g'(\alpha) < 0$ if $\alpha > \alpha_1$, we obtain from (2.19) that $\alpha_0 \geq \alpha_2$.

Now, to establish (2.17) for such α_2, we suppose by contradiction that

$$\left(\|\nabla u(t_0, \cdot)\|_1^2 + \|\nabla \upsilon(t_0, \cdot)\|_2^2\right)^{\frac{1}{2}} < \alpha_2 \tag{2.20}$$

for some $t_0 > 0$. By the continuity of $\left(\|\nabla u(t, \cdot)\|_1^2 + \|\nabla \upsilon(t, \cdot)\|_2^2\right)^{\frac{1}{2}}$ we can suppose that

$$\alpha_1 < \left(\|\nabla u(t_0, \cdot)\|_1^2 + \|\nabla \upsilon(t_0, \cdot)\|_2^2\right)^{\frac{1}{2}} < \alpha_2.$$

From (2.11) and (2.20) we obtain

$$E(t_0) \geq g(\alpha\,|_{t_0}) = g\left(\left(\|\nabla u(t_0, \cdot)\|_1^2 + \|\nabla \upsilon(t_0, \cdot)\|_2^2\right)^{\frac{1}{2}}\right) > g(\alpha_2) = E(0)\,,$$

but it is impossible by (2.9). Therefore, (2.17) is proved.

Using (2.8), (2.9), (2.10), (2.12), (2.13), and (2.17) we get

$$\frac{1}{p+1}\int_{\Omega_1}|u|^{p+1}\,dx+\frac{1}{p+1}\int_{\Omega_2}|\upsilon|^{p+1}\,dx=$$

$$=\frac{1}{2}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla\upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)-E(t)\geq$$

$$\geq\frac{1}{2}\left(\|\nabla u\|_1^2+\|\nabla\upsilon\|_2^2\right)-E(t)=\frac{1}{2}\alpha^2-E(t)\geq\frac{1}{2}\alpha_2^2-E(t)\geq\frac{1}{2}\alpha_2^2-E(0)=$$

$$=\frac{1}{2}\alpha_2^2-g(\alpha_2)=\frac{B}{p+1}\alpha_2^{p+1},$$

from which we obtain (2.18).

Lemma 2.4 is proved.

Theorem 2.5 *Assume that* (2.1)–(2.2) *and* (2.5) *hold and* $K\geq 1$. *If*

$$(u_0,\ \upsilon_0,\ \delta_0)\in H^1_{\Gamma_1}(\Omega_1)\times H^1(\Omega_2)\times L^2(\Gamma_2),$$

$$(u_1,\ \upsilon_1,\ \delta_1)\in L^2(\Omega_1)\times L^2(\Omega_2)\times L^2(\Gamma_2),$$

then the solution of the problem (1.1)–(1.8) *with initial data satisfying* (2.15) *and* (2.16) *blows up in finite time.*

3. Proof of Theorem 2.3

Approximating the initial data with the smooth ones and using Theorems 2.1 and 2.2, we can show that all the operations performed below are valid.

Let $(u,\,\upsilon,\,\delta)$ be a weak solution to the transmission acoustic problem (1.1)–(1.8). Multiplying equation (1.1) by u_t, (1.2) by υ_t, and (1.3) by δ_t and integrating over Ω_1, Ω_2, Γ_2, respectively, then summing them and integrating by parts, we have

$$\frac{1}{2}\frac{d}{dt}\|u_t\|_1^2-\left(\frac{\partial u}{\partial\nu},\ u_t\right)_{\Gamma_2}+\frac{1}{2}\frac{d}{dt}\|\nabla u\|_1^2+\left(|u_t|^{q_1+1},\ 1\right)_1=\frac{1}{p+1}\frac{d}{dt}\left(|u|^{p+1},\ 1\right)_1,$$

$$\frac{1}{2}\frac{d}{dt}\|\upsilon_t\|_2^2+\left(\frac{\partial\upsilon}{\partial\nu},\ \upsilon_t\right)_{\Gamma_2}+\frac{1}{2}\frac{d}{dt}\|\nabla\upsilon\|_2^2+\left(|\upsilon_t|^{q_2+1},\ 1\right)_2=\frac{1}{p+1}\frac{d}{dt}\left(|\upsilon|^{p+1},\ 1\right)_2,$$

$$\frac{1}{2}\frac{d}{dt}\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2+\frac{1}{2}\frac{d}{dt}\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2=-(u_t,\,\delta_t)_{\Gamma_2},$$

from which, using $(1.5)_1$, we obtain

$$\frac{1}{2}\frac{d}{dt}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla\upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)-$$

$$-\frac{1}{p+1}\frac{d}{dt}\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]-\left(\frac{\partial u}{\partial\nu}-\frac{\partial\upsilon}{\partial\nu},\ u_t\right)_{\Gamma_2}+$$

$$+(u_t\,,\ \delta_t\,)_{\Gamma_2}+\left(|u_t|^{q_1+1},\ 1\right)_1+\left(|\upsilon_t|^{q_2+1},\ 1\right)_2+\left\|\sqrt{D}\delta_t\right\|^2_{\Gamma_2}=0$$

or by $(1.5)_2$

$$\frac{1}{2}\frac{d}{dt}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla \upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|^2_{\Gamma_2}+\left\|\sqrt{K}\delta\right\|^2_{\Gamma_2}\right)-$$

$$-\frac{1}{p+1}\frac{d}{dt}\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]+$$

$$+\left(|u_t|^{q_1+1}\ ,\ 1\right)_1+\left(|\upsilon_t|^{q_2+1}\ ,\ 1\right)_2+\left\|\sqrt{D}\delta_t\right\|^2_{\Gamma_2}=0\,. \tag{3.1}$$

Note that, using (2.8), equality (3.1) can be written as (2.9).

Integrating (3.1) from 0 to t we obtain

$$\frac{1}{2}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla \upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|^2_{\Gamma_2}+\left\|\sqrt{K}\delta\right\|^2_{\Gamma_2}\right)+$$

$$+\frac{1}{p+1}\left(|u|^{p+1},1\right)_1+\frac{1}{p+1}\left(|\upsilon|^{p+1},1\right)_2+$$

$$+\int_0^t\left[\left(|u_t|^{q_1+1}\ ,\ 1\right)_1+\left(|\upsilon_t|^{q_2+1}\ ,\ 1\right)_2+\left\|\sqrt{D}\delta_t\right\|^2_{\Gamma_2}\right]d\tau=$$

$$=\frac{1}{2}\left(\|u_1\|_1^2+\|\nabla u_0\|_1^2+\|\upsilon_1\|_2^2+\|\nabla \upsilon_0\|_2^2+\left\|\sqrt{M}\delta_1\right\|^2_{\Gamma_2}+\left\|\sqrt{K}\delta_0\right\|^2_{\Gamma_2}\right)+$$

$$+\frac{1}{p+1}\left(|u_0|^{p+1},1\right)_1+\frac{1}{p+1}\left(|\upsilon_0|^{p+1},1\right)_2+$$

$$+2\int_0^t\left(|u|^{p-1}\ u,\ u_t\right)_1d\tau+2\int_0^t\left(|\upsilon|^{p-1}\ \upsilon,\ \upsilon_t\right)_2d\tau. \tag{3.2}$$

Let us estimate the last two terms of (3.2). Using the Hölder inequality with exponents $\rho=\frac{q_1+1}{q_1}$ and $\rho'=q_1+1\ \left(\frac{1}{\rho}+\frac{1}{\rho'}=1\right)$, we have

$$\int_0^t\left(|u|^{p-1}\ u,\ u_t\right)_1d\tau\le\int_0^t\int_{\Omega_1}|u|^p\ u_t\,dx\,d\tau\le$$

$$\le\left(\int_0^t\int_{\Omega_1}|u|^{\frac{p(q_1+1)}{q_1}}dx\,d\tau\right)^{\frac{q_1}{q_1+1}}\left(\int_0^t\int_{\Omega_1}|u_t|^{q_1+1}dx\,d\tau\right)^{\frac{1}{q_1+1}},$$

from which, using the Young inequality $\left(ab\le\frac{a^\rho}{\rho\eta^\rho}+\frac{\eta^{\rho'}b^{\rho'}}{\rho'}\ ,\ \frac{1}{\rho}+\frac{1}{\rho'}=1\right)$ with the parameter $\eta=\mu_1^{\frac{1}{q_1+1}}$, we obtain

$$\int_0^t\left(|u|^{p-1}\ u,\ u_t\right)_1d\tau\le\frac{q_1}{(q_1+1)\ \mu_1^{\frac{1}{q_1}}}\int_0^t\int_{\Omega_1}|u|^{\frac{p(q_1+1)}{q_1}}dx\,d\tau+$$

$$+\frac{\mu_1}{q_1+1}\int_0^t\int_{\Omega_1}|u_t|^{q_1+1}\,dx\,d\tau. \tag{3.3}$$

By the condition (2.4) and the Young inequality with exponents $\rho=\frac{(p+1)\,q_1}{p\,(q_1+1)},\ \rho'=\frac{(p+1)\,q_1}{q_1-p}\ \left(\frac{1}{\rho}+\frac{1}{\rho'}=1\right)$ we have

$$|u|^{\frac{p(q_1+1)}{q_1}}\le\frac{p\,(q_1+1)}{(p+1)\,q_1}|u|^{p+1}+\frac{q_1-p}{(p+1)\,q_1}.$$

Using this in (3.3) we get

$$\int_0^t\left(|u|^{p-1}\,u,\ u_t\right)_1 d\tau\le\frac{p\mu_1^{-\frac{1}{q_1}}}{p+1}\int_0^t\int_{\Omega_1}|u|^{p+1}\,dx\,d\tau+\frac{(q_1-p)\,\mu_1^{-\frac{1}{q_1}}T\,mes\,\Omega_1}{(q_1+1)\,(p+1)}+$$

$$+\frac{\mu_1}{q_1+1}\int_0^t\int_{\Omega_1}|u_t|^{q_1+1}\,dx\,d\tau. \tag{3.4}$$

Similarly, we have

$$\int_0^t\left(|\upsilon|^{p-1}\,\upsilon,\ \upsilon_t\right)_2 d\tau\le\frac{p\mu_2^{-\frac{1}{q_2}}}{p+1}\int_0^t\int_{\Omega_2}|\upsilon|^{p+1}\,dx\,d\tau+\frac{(q_2-p)\,\mu_2^{-\frac{1}{q_2}}T\,mes\,\Omega_2}{(q_2+1)\,(p+1)}+$$

$$+\frac{\mu_2}{q_2+1}\int_0^t\int_{\Omega_2}|\upsilon_t|^{q_2+1}\,dx\,d\tau. \tag{3.5}$$

From (3.2), (3.4), and (3.5) we conclude that

$$\frac{1}{2}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla\upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)+$$

$$+\frac{1}{p+1}\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]+\left(1-\frac{2\mu_1}{q_1+1}\right)\int_0^t\left(|u_t|^{q_1+1}\,,\ 1\right)_1 d\tau+$$

$$+\left(1-\frac{2\mu_2}{q_2+1}\right)\int_0^t\left(|\upsilon_t|^{q_2+1}\,,\ 1\right)_2 d\tau+\int_0^t\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2 d\tau\le$$

$$\le\ \frac{1}{2}\left(\|u_1\|_1^2+\|\nabla u_0\|_1^2+\|\upsilon_1\|_2^2+\|\nabla\upsilon_0\|_2^2+\left\|\sqrt{M}\delta_1\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta_0\right\|_{\Gamma_2}^2\right)+$$

$$+\frac{1}{p+1}\left[\left(|u_0|^{p+1},1\right)_1+\left(|\upsilon_0|^{p+1},1\right)_2\right]+\frac{T}{p+1}\sum_{i=1}^{2}\frac{(q_i-p)\,\mu_i^{-\frac{1}{q_i}}mes\,\Omega_i}{q_i+1}+$$

$$+\frac{p}{p+1}\max\left\{\mu_1^{-\frac{1}{q_1}},\ \mu_2^{-\frac{1}{q_2}}\right\}\int_0^t\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]d\tau \tag{3.6}$$

and μ_1 , μ_2 are chosen such that

$$1-\frac{2\mu_1}{q_1+1}>0\,,\ 1-\frac{2\mu_2}{q_2+1}>0.$$

By Gronwall's inequality and (3.6) we obtain

$$\frac{1}{2}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla\upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)+$$

$$+\frac{1}{p+1}\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]+\left(1-\frac{2\mu_1}{q_1+1}\right)\int_0^t\left(|u_t|^{q_1+1}\ ,\ 1\right)_1d\tau+$$

$$+\left(1-\frac{2\mu_2}{q_2+1}\right)\int_0^t\left(|\upsilon_t|^{q_2+1}\ ,\ 1\right)_2d\tau+\int_0^t\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2d\tau\le C_T,$$

where C_T depends on $\|u_1\|_1$, $\|\upsilon_1\|_2$, $\|\nabla u_0\|_1$, $\|\nabla\upsilon_0\|_2$, $\|\delta_0\|_{\Gamma_2}$, $\|\delta_1\|_{\Gamma_2}$ and on $T>0$, which is arbitrary. By the standard continuation argument, the local solution (u,υ,δ) obtained in Theorem 2.1 is global.

Theorem 2.3 is proved.

4. Proof of Theorem 2.5

Let

$$H(t)=E_1-E(t)\ ,\ \ t\ge0. \tag{4.1}$$

From (2.9) we have

$$H'(t)=-E'(t)=\|u_t\|_{L^{q_1+1}(\Omega_1)}^{q_1+1}+\|\upsilon_t\|_{L^{q_2+1}(\Omega_2)}^{q_2+1}+\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2\ge0\,,\ t\ge0. \tag{4.2}$$

Therefore, from (2.15), (4.1), and (4.2) we obtain that

$$0<H(0)\le H(t)\,,\ \ t\ge0. \tag{4.3}$$

By (2.8) and (4.1) we have

$$H(t)=$$

$$=E_1-\frac{1}{2}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla\upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)+$$

$$+\frac{1}{p+1}\left(|u|^{p+1},1\right)_1+\frac{1}{p+1}\left(|\upsilon|^{p+1},1\right)_2\le E_1-\frac{1}{2}\left(\|\nabla u\|_1^2+\|\nabla\upsilon\|_2^2\right)+$$

$$+\frac{1}{p+1}\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right],$$

from which, using (2.12) and (2.17),

$$H(t)\le E_1-\frac{1}{2}\alpha_2^2+\frac{1}{p+1}\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]$$

or since $\alpha_2 > \alpha_1$, then by (2.14)

$$H(t) \leq \frac{1}{p+1}\left[\left(|u|^{p+1}, 1\right)_1 + \left(|\upsilon|^{p+1}, 1\right)_2\right]. \tag{4.4}$$

We define $\Theta(t)$ as follows:

$$\Theta(t) = H^{1-\sigma}(t) + \varepsilon\left(\int_{\Omega_1} u\, u_t dx + \int_{\Omega_2} \upsilon\, \upsilon_t dx - \int_{\Gamma_2} u\,\delta\, d\Gamma_2\right) \tag{4.5}$$

for $\varepsilon > 0$ to be chosen later and for σ satisfying

$$0 < \sigma \leq \min\left\{\frac{p-1}{2\,(p+1)}, \frac{p-q_1}{q_1\,(p+1)}, \frac{p-q_2}{q_2\,(p+1)}\right\}. \tag{4.6}$$

Note that $p-1>0,\ p-q_1>0,\ p-q_2>0$ by (2.2) and (2.5).

Differentiating (4.5) and taking into account (1.1) and (1.2) yields

$$\Theta'(t) = (1-\sigma)\,H^{-\sigma}(t)\,H'(t) + \varepsilon\int_{\Omega_1} u_t^2\,dx + \varepsilon\int_{\Omega_2} \upsilon_t^2 dx +$$

$$+\varepsilon\left(\int_{\Gamma_2}\frac{\partial u}{\partial\nu}u\,d\Gamma_2 - \int_{\Omega_1}|\nabla u|^2\,dx\right) -$$

$$-\varepsilon\int_{\Omega_1}|u_t|^{q_1-1}\,u_t\,u dx + \varepsilon\int_{\Omega_1}|u|^{p+1}\,dx + \varepsilon\left(-\int_{\Gamma_2}\frac{\partial\upsilon}{\partial\nu}\upsilon\,d\Gamma_2 - \int_{\Omega_2}|\nabla\upsilon|^2\,dx\right) -$$

$$-\varepsilon\int_{\Omega_2}|\upsilon_t|^{q_2-1}\,\upsilon_t\,\upsilon\,dx + \varepsilon\int_{\Omega_2}|\upsilon|^{p+1}\,dx - \varepsilon\int_{\Gamma_2}u\,\delta_t\,d\Gamma_2 - \varepsilon\int_{\Gamma_2}\delta\,u_t\,d\Gamma_2,$$

or since by (1.5),

$$\int_{\Gamma_2}\frac{\partial u}{\partial\nu}u\,d\Gamma_2 - \int_{\Gamma_2}\frac{\partial\upsilon}{\partial\nu}\upsilon\,d\Gamma_2 = \int_{\Gamma_2}\left(\frac{\partial u}{\partial\nu} - \frac{\partial\upsilon}{\partial\nu}\right)u\,d\Gamma_2 = \int_{\Gamma_2}\delta_t\,u\,d\Gamma_2,$$

then we obtain

$$\Theta'(t) = (1-\sigma)\,H^{-\sigma}(t)\,H'(t) + \varepsilon\int_{\Omega_1} u_t^2\,dx + \varepsilon\int_{\Omega_2}\upsilon_t^2\,dx -$$

$$-\varepsilon\int_{\Omega_1}|\nabla u|^2\,dx - \varepsilon\int_{\Omega_2}|\nabla\upsilon|^2\,dx -$$

$$-\varepsilon\int_{\Omega_1}|u_t|^{q_1-1}\,u_t\,u\,dx - \varepsilon\int_{\Omega_2}|\upsilon_t|^{q_2-1}\,\upsilon_t\,\upsilon\,dx + \varepsilon\int_{\Omega_1}|u|^{p+1}\,dx +$$

$$+\varepsilon\int_{\Omega_2}|\upsilon|^{p+1}dx - \varepsilon\int_{\Gamma_2}\delta\,u_t\,d\Gamma_2. \tag{4.7}$$

Using the Hölder inequality with exponents $\rho = \frac{q_1+1}{q_1}$ and $\rho' = q_1+1$ $\left(\frac{1}{\rho}+\frac{1}{\rho'}=1\right)$ we have

$$\int_{\Omega_1}|u_t|^{q_1-1}\,u_t\,u\,dx \leq \int_{\Omega_1}|u_t|^{q_1}\,|u|\,dx \leq$$

$$\leq \left(\int_{\Omega_1} |u_t|^{q_1+1} \, dx \right)^{\frac{q_1}{q_1+1}} \left(\int_{\Omega_1} |u|^{q_1+1} \, dx \right)^{\frac{1}{q_1+1}},$$

from which, by the Young inequality $\left(ab \leq \frac{a^\rho}{\rho \eta^\rho} + \frac{\eta^{\rho'} b^{\rho'}}{\rho'}, \ \frac{1}{\rho} + \frac{1}{\rho'} = 1 \right)$ with parameter $\eta = \left[P_1 H^{-\sigma}(t) \right]^{-\frac{q_1}{q_1+1}}$ (P_1 is a large constant to be fixed later), we obtain

$$\left| \int_{\Omega_1} |u_t|^{q_1-1} \, u_t \, u \, dx \right| \leq \frac{q_1}{q_1+1} P_1 H^{-\sigma}(t) \int_{\Omega_1} |u_t|^{q_1+1} \, dx +$$

$$+ \frac{1}{(q_1+1) \left(P_1 H^{-\sigma}(t) \right)^{q_1}} \int_{\Omega_1} |u|^{q_1+1} \, dx. \tag{4.8}$$

Similarly, we have

$$\left| \int_{\Omega_2} |\upsilon_t|^{q_2-1} \, \upsilon_t \, \upsilon \, dx \right| \leq \frac{q_2}{q_2+1} P_2 H^{-\sigma}(t) \int_{\Omega_2} |\upsilon_t|^{q_2+1} \, dx +$$

$$+ \frac{1}{(q_2+1) \left(P_2 H^{-\sigma}(t) \right)^{q_2}} \int_{\Omega_2} |\upsilon|^{q_2+1} \, dx. \tag{4.9}$$

By (4.2) from (4.8) and (4.9), we get

$$\left| \varepsilon \int_{\Omega_1} |u_t|^{q_1-1} \, u_t \, u \, dx + \varepsilon \int_{\Omega_2} |\upsilon_t|^{q_2-1} \, \upsilon_t \, \upsilon \, dx \right| \leq$$

$$\leq \varepsilon P H^{-\sigma}(t) \left[\int_{\Omega_1} |u_t|^{q_1+1} \, dx + \int_{\Omega_2} |\upsilon_t|^{q_2+1} \, dx \right] +$$

$$+ \frac{\varepsilon}{\left(P_1 H^{-\sigma}(t) \right)^{q_1}} \int_{\Omega_1} |u|^{q_1+1} \, dx + \frac{\varepsilon}{\left(P_2 H^{-\sigma}(t) \right)^{q_2}} \int_{\Omega_2} |\upsilon|^{q_2+1} \, dx \leq$$

$$\leq \varepsilon P H^{-\sigma}(t) H'(t) + \varepsilon P_1^{-q_1} H^{\sigma q_1}(t) \int_{\Omega_1} |u|^{q_1+1} \, dx +$$

$$+ \varepsilon P_2^{-q_2} H^{\sigma q_2}(t) \int_{\Omega_2} |\upsilon|^{q_2+1} \, dx, \tag{4.10}$$

where $P = \max \{ P_1, P_2 \}$.

Taking into account (4.4) and the obvious inequalities

$$\int_{\Omega_1} |u|^{q_1+1} \, dx \leq k_1 \left(\int_{\Omega_1} |u|^{p+1} \, dx \right)^{\frac{q_1+1}{p+1}} \leq$$

$$\leq k_1 \left(\int_{\Omega_1} |u|^{p+1} \, dx + \int_{\Omega_2} |\upsilon|^{p+1} \, dx \right)^{\frac{q_1+1}{p+1}},$$

$$\int_{\Omega_2} |\upsilon|^{q_2+1}\, dx \le k_2 \left(\int_{\Omega_2} |\upsilon|^{p+1}\, dx \right)^{\frac{q_2+1}{p+1}} \le$$

$$\le k_2 \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx \right)^{\frac{q_2+1}{p+1}},$$

with $k_1 = |\Omega_1|^{\frac{p-q_1}{p+1}}$, $k_2 = |\Omega_2|^{\frac{p-q_2}{p+1}}$, from (4.10) we get

$$\begin{gathered}\left|\varepsilon \int_{\Omega_1} |u_t|^{q_1-1}\, u_t\, u\, dx + \varepsilon \int_{\Omega_2} |\upsilon_t|^{q_2-1}\, \upsilon_t\, \upsilon\, dx\right| \le \varepsilon P H^{-\sigma}(t)\, H'(t) + \\ + \frac{\varepsilon P_1^{-q_1} k_1}{(p+1)^{\sigma q_1}} \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx \right)^{\sigma q_1 + \frac{q_1+1}{p+1}} + \\ + \frac{\varepsilon P_2^{-q_2} k_2}{(p+1)^{\sigma q_2}} \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx \right)^{\sigma q_2 + \frac{q_2+1}{p+1}}.\end{gathered} \tag{4.11}$$

By (2.8) and (4.1) we have

$$\begin{gathered}-\|\nabla u\|_1^2 - \|\nabla \upsilon\|_2^2 = 2H(t) - 2E_1 + \|u_t\|_1^2 + \|\upsilon_t\|_2^2 + \left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2 + \left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2 - \\ - \frac{2}{p+1} \int_{\Omega_1} |u|^{p+1}\, dx - \frac{2}{p+1} \int_{\Omega_2} |\upsilon|^{p+1}\, dx.\end{gathered} \tag{4.12}$$

From (2.18) we get

$$-2\varepsilon E_1 \ge -\frac{2\varepsilon E_1}{B\alpha_2^{p+1}} \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx \right). \tag{4.13}$$

Since $g'(\alpha_1) = 0$, then by (2.13), we obtain that $\alpha_1 - B\alpha_1^p = 0$ or

$$\alpha_1^2 = B\alpha_1^{p+1}. \tag{4.14}$$

Taking into account (2.14) and (4.14) in (4.13) yields

$$\begin{gathered}-2\varepsilon E_1 \ge -2\varepsilon \alpha_1^2 \left(\frac{1}{2} - \frac{1}{p+1} \right) \frac{1}{B\alpha_2^{p+1}} \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx \right) = \\ = -\frac{\varepsilon(p-1)}{p+1} \left(\frac{\alpha_1}{\alpha_2} \right)^{p+1} \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx \right).\end{gathered} \tag{4.15}$$

Multiplying (4.12) by ε and using (4.15) we have

$$-\varepsilon \|\nabla u\|_1^2 - \varepsilon \|\nabla \upsilon\|_2^2 \ge$$

$$\ge 2\varepsilon H(t) - \left[\frac{\varepsilon(p-1)}{p+1} \left(\frac{\alpha_1}{\alpha_2} \right)^{p+1} + \frac{2\varepsilon}{p+1} \right] \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx \right) +$$

$$+\varepsilon \|u_t\|_1^2 + \varepsilon \|\upsilon_t\|_2^2 + \varepsilon \left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2 + \varepsilon \left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2.$$

Using this and (4.11) from (4.7) we get

$$\Theta'(t) \geq (1-\sigma-\varepsilon P) H^{-\sigma}(t) H'(t) + 2\varepsilon H(t) +$$
$$+2\varepsilon \int_{\Omega_1} u_t^2\, dx + 2\varepsilon \int_{\Omega_2} \upsilon_t^2\, dx - \varepsilon \int_{\Gamma_2} \delta\, u_t\, d\Gamma_2 +$$
$$+\varepsilon \left(1 - \tfrac{p-1}{p+1}\left(\tfrac{\alpha_1}{\alpha_2}\right)^{p+1} - \tfrac{2}{p+1}\right)\left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx\right) +$$
$$+\varepsilon \left\| \sqrt{M}\delta_t \right\|^2_{\Gamma_2} + \varepsilon \left\| \sqrt{K}\delta \right\|^2_{\Gamma_2} - \tag{4.16}$$
$$-\tfrac{\varepsilon\, k_1}{P_1^{q_1}(p+1)^{\sigma q_1}} \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx\right)^{\sigma q_1 + \frac{q_1+1}{p+1}} -$$
$$-\tfrac{\varepsilon\, k_2}{P_2^{q_2}(p+1)^{\sigma q_2}} \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx\right)^{\sigma q_2 + \frac{q_2+1}{p+1}}.$$

Since $\alpha_2 > \alpha_1$ and $p > 1$, then

$$1 - \frac{p-1}{p+1}\left(\frac{\alpha_1}{\alpha_2}\right)^{p+1} - \frac{2}{p+1} = \frac{p-1}{p+1}\left(1 - \left(\frac{\alpha_1}{\alpha_2}\right)^{p+1}\right) = \beta > 0. \tag{4.17}$$

Since by (4.6), $0 < \sigma\, q_i + \frac{q_i+1}{p+1} \leq 1, i = 1, 2$, then using the algebraic inequality

$$z^{\nu} \leq z + 1 \leq \left(1 + \frac{1}{a}\right)(z + a), \forall z \geq 0,\ 0 < \nu \leq 1,\ a > 0, \tag{4.18}$$

we obtain

$$\left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx\right)^{\sigma\, q_i + \frac{q_i+1}{p+1}} \leq$$

$$\leq \left(1 + \frac{1}{H(0)}\right)\left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx + H(0)\right),$$

or denoting $1 + \frac{1}{H(0)} = \gamma$, by (4.3) we get

$$\left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx\right)^{\sigma q_i + \frac{q_i+1}{p+1}} \leq$$

$$\leq \gamma \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx + H(t)\right), i = 1, 2. \tag{4.19}$$

Using (4.17) and (4.19) in (4.16), we have

$$\Theta'(t) \geq (1-\sigma-\varepsilon P)\, H^{-\sigma}(t)\, H'(t) + 2\varepsilon H(t) + 2\varepsilon \int_{\Omega_1} u_t^2\, dx +$$
$$+2\varepsilon \int_{\Omega_2} \upsilon_t^2 dx - \varepsilon \int_{\Gamma_2} \delta\, u_t\, d\Gamma_2 +$$
$$+\varepsilon\beta \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx\right) + \varepsilon \left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2 + \varepsilon \left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2 -$$
$$-\varepsilon\gamma \left(\frac{k_1}{P_1^{q_1}(p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2}(p+1)^{\sigma q_2}}\right) \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx + H(t)\right)$$

or since $-\varepsilon \int_{\Gamma_2} \delta\, u_t\, d\Gamma_2 \geq -\frac{\varepsilon}{2} \int_{\Gamma_2} \delta^2\, d\Gamma_2 - \frac{\varepsilon}{2} \int_{\Gamma_2} u_t^2\, d\Gamma_2$ and $K \geq 1$, then

$$\begin{gathered}\Theta'(t) \geq (1-\sigma-\varepsilon P)\, H^{-\sigma}(t)\, H'(t) + \\ +\frac{3\varepsilon}{2} \int_{\Omega_1} u_t^2 dx + 2\varepsilon \int_{\Omega_2} \upsilon_t^2 dx + \frac{\varepsilon}{2} \int_{\Gamma_2} \delta^2 d\Gamma_2 + \\ +\varepsilon \left[2 - \gamma \left(\frac{k_1}{P_1^{q_1}(p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2}(p+1)^{\sigma q_2}}\right)\right] H(t) + \varepsilon \left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2 + \\ +\varepsilon \left[\beta - \gamma \left(\frac{k_1}{P_1^{q_1}(p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2}(p+1)^{\sigma q_2}}\right)\right] \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx\right).\end{gathered} \tag{4.20}$$

We choose P_1, P_2 large enough such that

$$\begin{gathered}\theta_1 = 2 - \gamma \left(\frac{k_1}{P_1^{q_1}(p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2}(p+1)^{\sigma q_2}}\right) > 0, \\ \theta_2 = \beta - \gamma \left(\frac{k_1}{P_1^{q_1}(p+1)^{\sigma q_1}} + \frac{k_2}{P_2^{q_2}(p+1)^{\sigma q_2}}\right) > 0.\end{gathered}$$

Then we choose ε small enough such that $1-\sigma-\varepsilon P > 0$ and

$$\Theta(0) = H^{1-\sigma}(0) + \varepsilon \left(\int_{\Omega_1} u_0 u_1 dx + \int_{\Omega_2} \upsilon_0 \upsilon_1 dx - \int_{\Gamma_2} u_0 \delta_0 d\Gamma_2\right) > 0 \tag{4.21}$$

or

$$\varepsilon < \min\left\{\frac{1-\sigma}{P}, \frac{H^{1-\sigma}(0)}{\left|\int_{\Omega_1} u_0 u_1 dx + \int_{\Omega_2} \upsilon_0 \upsilon_1 dx - \int_{\Gamma_2} u_0 \delta_0 d\Gamma_2\right|}\right\}.$$

Then from (4.20) we have

$$\begin{gathered}\Theta'(t) \geq \varepsilon \left(\int_{\Omega_1} u_t^2\, dx + \int_{\Omega_2} \upsilon_t^2 dx\right) + \frac{\varepsilon}{2} \int_{\Gamma_2} \delta^2 d\Gamma_2 + \varepsilon\theta_1 H(t) + \\ +\varepsilon\theta_2 \left(\int_{\Omega_1} |u|^{p+1}\, dx + \int_{\Omega_2} |\upsilon|^{p+1}\, dx\right).\end{gathered} \tag{4.22}$$

Therefore, $\Theta(t)$ is a nondecreasing function for $t \geq 0$. Using (4.21), we obtain that

$$\Theta(t) \geq \Theta(0) > 0, t \geq 0.$$

Since $0 < \sigma < 1$, it is obvious that $\frac{1}{1-\sigma} > 1$. Then using the inequality

$$(a+b)^r \leq C(a^r + b^r), a, b \geq 0, r > 0, C > 0,$$

from (4.5) we get

$$\Theta^{\frac{1}{1-\sigma}}(t) \le C_0 \left[H(t) + \left(\int_{\Omega_1} u\, u_t\, dx + \int_{\Omega_2} \upsilon\, \upsilon_t dx - \int_{\Gamma_2} u\delta\ d\Gamma_2 \right)^{\frac{1}{1-\sigma}} \right], \tag{4.23}$$

where C_0 is a positive constant.

On the other hand, from Hölder and Young inequalities we obtain that

$$\begin{aligned}
&\left(\int_{\Omega_1} u\, u_t\, dx + \int_{\Omega_2} \upsilon\, \upsilon_t\, dx - \int_{\Gamma_2} u\delta\ d\Gamma_2 \right)^{\frac{1}{1-\sigma}} \le \\
&\le \Bigg[|\Omega_1|^{\frac{p-1}{2(p+1)}} \left(\int_{\Omega_1} |u|^{p+1}\ dx \right)^{\frac{1}{p+1}} \left(\int_{\Omega_1} u_t^2\, dx \right)^{\frac{1}{2}} + \\
&+\ |\Omega_2|^{\frac{p-1}{2(p+1)}} \left(\int_{\Omega_2} |\upsilon|^{p+1}\ dx \right)^{\frac{1}{p+1}} \left(\int_{\Omega_2} \upsilon_t^2\, dx \right)^{\frac{1}{2}} + \\
&+ |\Omega_1|^{\frac{p-1}{2(p+1)}} \left(\int_{\Omega_1} |u|^{p+1}\ dx \right)^{\frac{1}{p+1}} \left(\int_{\Gamma_2} \delta^2\, d\Gamma_2 \right)^{\frac{1}{2}} \Bigg]^{\frac{1}{1-\sigma}} \le
\end{aligned}$$

$$\begin{aligned}
&\le |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left(\int_{\Omega_1} |u|^{p+1}\ dx \right)^{\frac{1}{(p+1)(1-\sigma)}} \left(\int_{\Omega_1} u_t^2\, dx \right)^{\frac{1}{2(1-\sigma)}} + \\
&+ |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left(\int_{\Omega_2} |\upsilon|^{p+1}\ dx \right)^{\frac{1}{(p+1)(1-\sigma)}} \left(\int_{\Omega_2} \upsilon_t^2\, dx \right)^{\frac{1}{2(1-\sigma)}} + \\
&+ |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left(\int_{\Omega_1} |u|^{p+1}\ dx \right)^{\frac{1}{(p+1)(1-\sigma)}} \left(\int_{\Gamma_2} \delta^2\ d\Gamma_2 \right)^{\frac{1}{2(1-\sigma)}} \le
\end{aligned} \tag{4.24}$$

$$\begin{aligned}
&\le |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\tfrac{1-2\sigma}{2(1-\sigma)} \left(\int_{\Omega_1} |u|^{p+1}\ dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} + \tfrac{1}{2(1-\sigma)} \int_{\Omega_1} u_t^2\, dx \right] + \\
&+ |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\tfrac{1-2\sigma}{2(1-\sigma)} \left(\int_{\Omega_2} |\upsilon|^{p+1}\ dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} + \tfrac{1}{2(1-\sigma)} \int_{\Omega_2} \upsilon_t^2\, dx \right] + \\
&+ |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\tfrac{1-2\sigma}{2(1-\sigma)} \left(\int_{\Omega_1} |u|^{p+1}\ dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} + \tfrac{1}{2(1-\sigma)} \int_{\Gamma_2} \delta^2\ d\Gamma_2 \right].
\end{aligned}$$

Since by (4.6), $0 < \frac{2}{(p+1)(1-2\sigma)} \le 1$, then using (4.18) we obtain

$$\begin{aligned}
&\left(\int_{\Omega_1} |u|^{p+1}\ dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} \le \left(1 + \tfrac{1}{H(0)} \right) \left(\int_{\Omega_1} |u|^{p+1}\ dx + H(0) \right) \le \\
&\le \left(1 + \tfrac{1}{H(0)} \right) \left(\int_{\Omega_1} |u|^{p+1}\ dx + H(t) \right) = \gamma \left(\int_{\Omega_1} |u|^{p+1}\ dx + H(t) \right).
\end{aligned} \tag{4.25}$$

Similarly, we have

$$\left(\int_{\Omega_2} |\upsilon|^{p+1}\ dx \right)^{\frac{2}{(p+1)(1-2\sigma)}} \le \gamma \left(\int_{\Omega_2} |\upsilon|^{p+1}\ dx + H(t) \right). \tag{4.26}$$

From (4.24)–(4.26) we get

$$\left(\int_{\Omega_1} u\, u_t\, dx + \int_{\Omega_2} \upsilon\, \upsilon_t\, dx - \int_{\Gamma_2} u\delta\; d\Gamma_2\right)^{\frac{1}{1-\sigma}} \leq$$
$$\leq |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)}\gamma \left(\int_{\Omega_1} |u|^{p+1}\; dx + H(t)\right) + \frac{1}{2(1-\sigma)} \int_{\Omega_1} u_t^2\, dx\right] +$$
$$+ |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)}\gamma \left(\int_{\Omega_2} |\upsilon|^{p+1}\; dx + H(t)\right) + \frac{1}{2(1-\sigma)} \int_{\Omega_2} \upsilon_t^2\, dx\right] +$$
$$+ |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \left[\frac{1-2\sigma}{2(1-\sigma)}\gamma \left(\int_{\Omega_1} |u|^{p+1}\; dx + H(t)\right) + \frac{1}{2(1-\sigma)} \int_{\Gamma_2} \delta^2\; d\Gamma_2\right].$$

Using this in (4.23), we obtain

$$\Theta^{\frac{1}{1-\sigma}}(t) \leq C_0 \left[1 + 2|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)} + |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)}\right] H(t) +$$
$$+ C_0 |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{1-\sigma} \int_{\Omega_1} |u|^{p+1}\; dx + C_0 |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)} \int_{\Omega_2} |\upsilon|^{p+1}\; dx +$$
$$+ C_0 \frac{|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)} \int_{\Omega_1} u_t^2\, dx + C_0 \frac{|\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)} \int_{\Omega_2} \upsilon_t^2\, dx + C_0 \frac{|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)} \int_{\Gamma_2} \delta^2\, d\Gamma_2$$

or

$$\Theta^{\frac{1}{1-\sigma}}(t) \leq C_1 \left(H(t) + \int_{\Omega_1} |u|^{p+1}\; dx + \right.$$
$$\left. + \int_{\Omega_2} |\upsilon|^{p+1}\; dx + \int_{\Omega_1} u_t^2\, dx + \int_{\Omega_2} \upsilon_t^2\, dx + \int_{\Gamma_2} \delta^2\, d\Gamma_2\right), \tag{4.27}$$

where

$$C_1 = C_0 \max\left\{1 + 2|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)} + \right.$$
$$+ |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)},\;\; |\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{1-\sigma},$$
$$\left. |\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}} \frac{(1-2\sigma)\gamma}{2(1-\sigma)},\;\; \frac{|\Omega_1|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)},\;\; \frac{|\Omega_2|^{\frac{p-1}{2(p+1)(1-\sigma)}}}{2(1-\sigma)}\right\}.$$

Putting $C_2 = \varepsilon \min\{1; \theta_1; \theta_2\}$, from (4.22) we get

$$\Theta'(t) \geq C_2 \left(H(t) + \int_{\Omega_1} |u|^{p+1}\; dx + \int_{\Omega_2} |\upsilon|^{p+1}\; dx + \int_{\Omega_1} u_t^2 dx + \int_{\Omega_2} \upsilon_t^2 dx\right). \tag{4.28}$$

It follows from (4.27) and (4.28) that

$$\Theta'(t) \geq C_3 \Theta^{\frac{1}{1-\sigma}}(t),$$

where $C_3 = \frac{C_2}{C_1}$. Integrating both sides of this inequality over $[0, t]$ yields

$$\Theta(t) \geq \left(\Theta^{-\frac{\sigma}{1-\sigma}}(0) - \frac{C_3\sigma}{1-\sigma} t\right)^{-\frac{1-\sigma}{\sigma}}.$$

Noting that $\Theta(0) > 0$, there exists $T_1 = \frac{(1-\sigma)\,\Theta^{-\frac{\sigma}{1-\sigma}}(0)}{C_3\sigma}$ such that $\Theta(t) \to +\infty$ as $t \to T_1$. In other words, the solution of problem (1.1)–(1.8) blows up in finite time.

Theorem 2.5 is proved.

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